Exercise 15.1

1. Find the circumference and area of a circle of radius of 4.2 cm.
Solution:

Given,
Radius (r) = 4.2 cm
We know that,
Circumference of a circle = $2\pi r$
$= 2 \times (22/7) \times 4.2 = 26.4\ cm^2$
Area of a circle = πr^2
$= (22/7) \times 4.2^2$
$= 22 \times 0.6 \times 4.2 = 55.44\ cm^2$
Hence, the circumference and area of the circle is 26.4 cm^2 and 55.44 cm^2.

2. Find the circumference of a circle whose area is 301.84 cm^2.
Solution:

Given,
Area of the circle = 301.84 cm^2
We know that,
Area of a Circle = $\pi r^2 = 301.84\ cm^2$
$(22/7) \times r^2 = 301.84$
$r^2 = 13.72 \times 7 = 96.04$
$r = \sqrt{96.04} = 9.8$
So, the radius is = 9.8 cm.
Now, Circumference of a circle = $2\pi r$
$= 2 \times (22/7) \times 9.8 = 61.6$ cm
Hence, the circumference of the circle is 61.6 cm.

3. Find the area of a circle whose circumference is 44 cm.
Solution:

Given,
Circumference = 44 cm
We know that,
Circumference of a circle = $2\pi r = 44$ cm
$2 \times (22/7) \times r = 44$
$r = 7$ cm
Now, Area of a Circle = πr^2
$= (22/7) \times 7 \times 7$
$= 154\ cm^2$
Hence, area of the Circle = 154 cm^2

4. The circumference of a circle exceeds the diameter by 16.8 cm. Find the circumference of the circle.

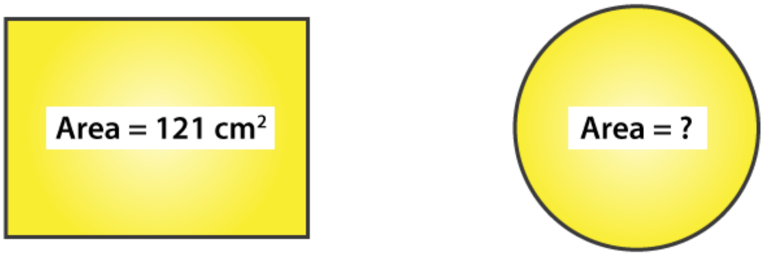
Solution:

Let the radius of the circle be r cm
So, the diameter (d) = 2r [As radius is half the diameter]
We know that,
Circumference of a circle (C) = $2\pi r$
From the question,
Circumference of the circle exceeds its diameter by 16.8 cm
C = d + 16.8
$2\pi r = 2r + 16.8$ [d = 2r]
$2\pi r - 2r = 16.8$
$2r(\pi - 1) = 16.8$
$2r(3.14 - 1) = 16.8$
r = 3.92 cm
Thus, radius = 3.92 cm
Now, the circumference of the circle (C) = $2\pi r$
$C = 2 \times 3.14 \times 3.92$
$= 24.64$ cm
Hence, the circumference of the circle is 24.64 cm.

5. A horse is tied to a pole with 28 m long string. Find the area where the horse can graze.
Solution:

Given,
Length of the string (l) = 28 m
Area the horse can graze is the area of the circle with a radius equal to the length of the string.
We know that,
Area of a Circle = πr^2
$= (22/7) \times 28 \times 28 = 2464\ m^2$
Hence, the area of the circle which is same as the area the horse can graze is $2464\ m^2$

6. A steel wire when bent in the form of a square encloses an area of 121 cm^2. If the same wire is bent in the form of a circle, find the area of the circle.
Solution:



Given,
Area of the square = $a^2 = 121\ cm^2$

We know that,
Area of the circle = πr^2
Area of a square = a^2
121 cm^2 = a^2
So, a = 11 cm
Thus, each side of the square = 11 cm
Now, the perimeter of the square = 4a
= 4 × 11 = 44 cm
From the question, its understood that
Perimeter of the square = Circumference of the circle
We know that, circumference of a circle (C) = $2\pi r$
$4a = 2\pi r$
$44 = 2(22/7)r$
r = 7 cm
Now, area of the Circle = πr^2
= (22/7) × 7 × 7 = 154 cm^2
Hence, the area of the circle is 154 cm^2.

7. The circumference of two circles are in the ratio of 2:3. Find the ratio of their areas.
Solution:

Let's consider the radius of two circles C_1 and C_2 be r_1 and r_2.
We know that, Circumference of a circle (C) = $2\pi r$
And their circumference will be $2\pi r_1$ and $2\pi r_2$.
So, their ratio is = $r_1 : r_2$
Given, circumference of two circles is in a ratio of 2: 3
$r_1 : r_2$ = 2: 3
Then, the ratios of their areas is given as

$$= \pi r_1^2 : \pi r_2^2$$

$$= \left(\frac{r1}{r2}\right)^2$$

$$= \left(\frac{2}{3}\right)^2$$

= 4/9
Hence, ratio of their areas = 4: 9.

8. The sum of the radii of two circles is 140 cm and the difference of their circumference is 88 cm. Find the diameters of the circles.
Solution:

Let the radii of the two circles be r_1 and r_2.
And, the circumferences of the two circles be C_1 and C_2.

We know that, circumference of circle (C) = $2\pi r$
Given,
Sum of radii of two circle i.e., $r_1 + r_2 = 140$ cm ... (i)
Difference of their circumference,
$C_1 – C_2 = 88$ cm
$2\pi r_1 - 2\pi r_2 = 88$ cm
$2(22/7)(r_1 – r_2) = 88$ cm
$(r_1 – r_2) = 14$ cm
$r_1 = r_2 + 14$..... (ii)
Substituting the value of r_1 in equation (i), we have,
$r_2 + r_2 + 14 = 140$
$2r_2 = 140 - 14$
$2r_2 = 126$
$r_2 = 63$ cm
Substituting the value of r_2 in equation (ii), we have,
$r_1 = 63 + 14 = 77$ cm
Therefore,
Diameter of circle 1 = $2r_1 = 2 \times 77 = 154$ cm
Diameter of circle 2 = $2r_2 = 2 \times 63 = 126$ cm

9. Find the radius of a circle whose circumference is equal to the sum of the circumferences of two circles of radii 15cm and 18cm.
Solution:

Given,
Radius of circle 1 = $r_1 = 15$ cm
Radius of circle 2 = $r_2 = 18$ cm
We know that, the circumference of a circle (C) = $2\pi r$
So, $C_1 = 2\pi r_1$ and $C_2 = 2\pi r_2$
Let the radius be r of the circle which is to be found and its circumference (C)
Now, from the question
$C = C_1 + C_2$
$2\pi r = 2\pi r_1 + 2\pi r_2$
$r = r_1 + r_2$ [After dividing by 2π both sides]
$r = 15 + 18$
$r = 33$ cm
Thus, the radius of the circle = 33 cm

10. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having its area equal to the sum of the areas of two circles.
Solution:

Given,
Radii of the two circles are 6 cm and 8 cm
Area of circle with radius 8 cm = $\pi (8)^2 = 64\pi$ cm^2
Area of circle with radius 6cm = $\pi (6)^2 = 36\pi$ cm^2

Sum of areas = $64\pi + 36\pi = 100\pi$ cm^2
Let the radius of the circle be x cm
Area of the circle = 100π cm^2 (from above)
$\pi x^2 = 100\pi$
$x = \sqrt{100} = 10$ cm
Therefore, the radius of the circle is 10 cm.

11. The radii of two circles are 19 cm and 9 cm respectively. Find the radius and area of the circle which has circumferences is equal to sum of the circumference of two circles.
Solution:
Given,
Radius of circle 1 = r_1 = 19 cm
Radius of circle 2 = r_2 = 9 cm
We know that, the circumference of a circle (C) = $2\pi r$
So, $C_1 = 2\pi r_1$ and $C_2 = 2\pi r_2$
Let the radius be r of the circle which is to be found and its circumference (C)
Now, from the question
$C = C_1 + C_2$
$2\pi r = 2\pi r_1 + 2\pi r_2$
$r = r_1 + r_2$ [After dividing by 2π both sides]
$r = 19 + 9$
$r = 28$ cm
Thus, the radius of the circle = 28 cm
So, the area of required circle = $\pi r^2 = (22/7) \times 28 \times 28 = 2464$ cm^2

12. The area of a circular playground is 22176 m^2. Find the cost of fencing this ground at the rate of ₹50 per metre.
Solution:

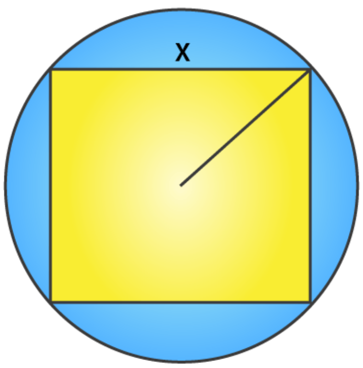
Given,
Area of the circular playground = 22176 m^2
And the cost of fencing per metre = ₹50
If the radius of the ground is taken as r.
Then, its area = πr^2
$\pi r^2 = 22176$
$r^2 = 22176(7/22) = 7056$
Taking square root on both sides, we have
r = 84 m
We know that, fencing is done only on the circumference of the ground
Circumference of the ground = $2\pi r = 2(22/7)84 = 528$ m
So, the cost of fencing 528 m = ₹50 x 528 = ₹26400
Therefore, the cost of fencing the ground is ₹26400.

13. The side of a square is 10 cm. Find the area of the circumscribed and inscribed circles.
Solution:

For circumscribed circle:
Radius = diagonal of square/ 2
Diagonal of the square = side x √2
= 10√2 cm
Radius = (10 × 1.414)/ 2 = 7.07 cm
Thus, the radius of the circumcircle = 7.07 cm
Then, its area is = πr^2 = (22/7) × 7.07 × 7.07 = 157.41 cm^2

Therefore, the Area of the Circumscribed circle is 157.41 cm^2
For inscribed circle:
Radius = side of square/ 2
= 10/ 2 = 5 m
Then, its area is = πr^2 = 3.14 × 5 × 5 = 78.5 cm^2
Thus, the area of the circumscribed circle is 157.41 cm^2 and the area of the inscribed circle is 78.5 cm^2.

14. If a square is inscribed in a circle, find the ratio of areas of the circle and the square.
Solution:



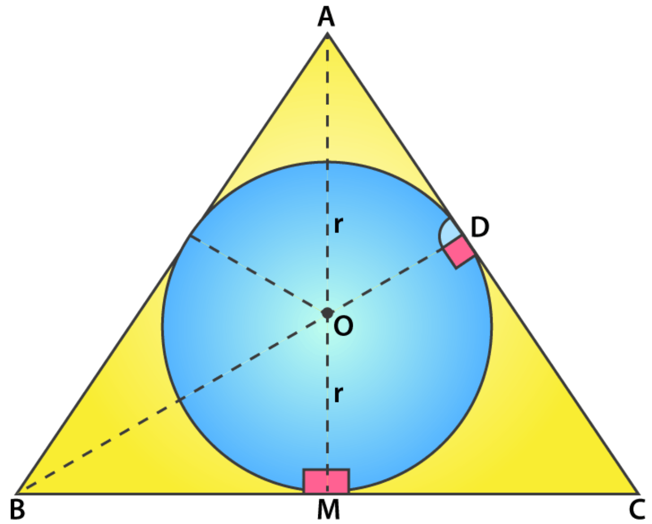
Let side of square be x cm which is inscribed in a circle.
Given,
Radius of circle (r) = 1/2 (diagonal of square)
= 1/2(x√2)
r = x/√2
We know that, area of the square = x^2
And, the area of the circle = πr^2

$$= \pi\left(\frac{x}{\sqrt{2}}\right)^2 = \frac{\pi * x^2}{2}$$

$$\frac{\text{area of circle}}{\text{area of square}} = \frac{\pi}{2} = \pi : 2$$

Therefore, the ratio of areas of the circle and the square = π : 2

15. The area of circle inscribed in an equilateral triangle is 154 cm^2. Find the perimeter of the triangle.
Solution:



Let the circle inscribed in the equilateral triangle be with a centre O and radius r.
We know that, Area of a Circle = πr^2
But, the given that area is 154 cm^2.
$(22/7) \times r^2 = 154$
$r^2 = (154 \times 7)/22 = 7 \times 7 = 49$
r = 7 cm
From the figure seen above, we infer that
At point M, BC side is tangent and also at point M, BM is perpendicular to OM.
We know that,
In an equilateral triangle, the perpendicular from vertex divides the side into two halves.
BM = ½ x BC
Consider the side of the equilateral triangle be x cm.

$$BM = \frac{1}{2}x = \frac{x}{2}$$

$$OB^2 = BM^2 + MO^2$$

$$OB = \sqrt{r^2 + \frac{x^2}{4}} = \sqrt{49 + \frac{x^2}{4}}$$

$$BD = \frac{\sqrt{3}}{2}(\text{side}) = \frac{\sqrt{3}}{2}x = OB + OD$$

$$\frac{\sqrt{3}}{2}x - r = \sqrt{49 + \frac{x^2}{4}}, r = 7$$

After solving the above equation we get,

$x = 14\sqrt{3}$ cm

$\text{Perimeter} = 3x = 3 \times 14\sqrt{3} = 42\sqrt{3}$ cm

Therefore, the perimeter of the triangle is found to be $42\sqrt{3}$ cm = 42(1.73) = 72.7 cm