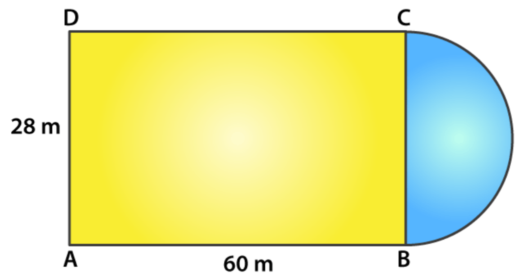
Exercise 15.4 Page No: 15.56

1. A plot is in the form of a rectangle ABCD having semi-circle on BC as shown in Fig.15.64. If AB = 60m and BC = 28m, find the area of the plot.

Solution:



Given, ABCD is a rectangle

So, AB = CD = 60 m

And, BC = AD = 28 m

For the radius of the semi-circle = BC/2 = 28/2 = 14 m

Now,

Area of the plot = Area of rectangle ABCD + Area of semi – circle

$= (l \times b) + \frac{1}{2} \pi r^2$

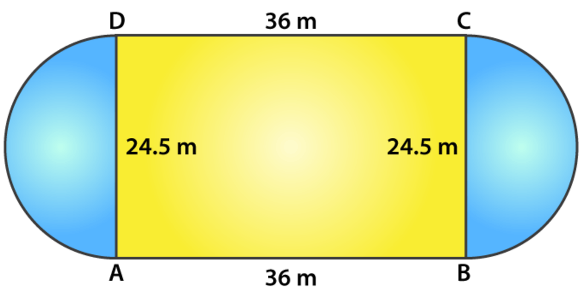
$= (60 \times 28) + \frac{1}{2} (22/7)(14)^2$

$= 1680 + 308$

$= 1988 \text{ cm}^2$

2. A play ground has the shape of a rectangle, with two semi-circles on its smaller sides as diameters, added to its outside. If the sides of the rectangle are 36m and 24.5m, find the area of the playground.

Solution:



Given,

Length of rectangle = 36 m

Breadth of rectangle = 24.5 m

Radius of the semi-circle = breath/2 = 24.5/2 = 12.25 m

Now,

Area of the playground = Area of the rectangle + 2 x area of semi-circles

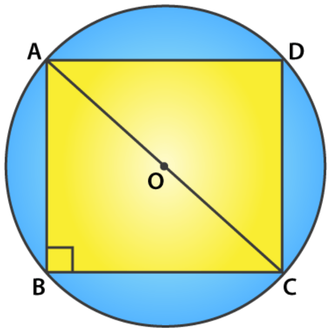
$= l \times b + 2 \times \frac{1}{2} (\pi r^2)$

$= (36 \times 24.5) + (22/7) \times 12.25^2$

$= 882 + 471.625 = 1353.625$

Thus, the area of the playground is 1353.625 m^2

3. Find the area of the circle in which a square of area 64 cm^2 is inscribed.
Solution:

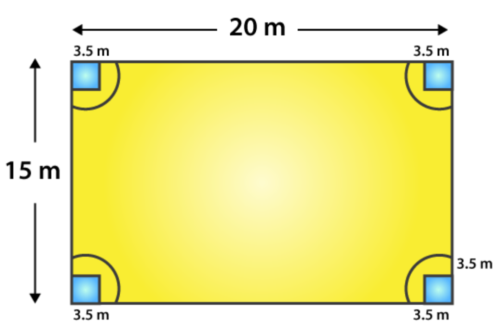


Given,
Area of square inscribed the circle = 64 cm^2
$Side^2 = 64$
Side = 8 cm
So, AB = BC = CD = DA = 8 cm

In triangle ABC, by Pythagoras theorem we have
$AC^2 = AB^2 + BC^2$
$AC^2 = 8^2 + 8^2$
$AC^2 = 64 + 64 = 128$
$AC = \sqrt{128} = 8\sqrt{2}$ cm

Now, as angle B = 90^o and AC being the diameter of the circle
The radius is $AC/2 = 8\sqrt{2}/2 = 4\sqrt{2}$ cm
Thus, the area of the circle = $\pi r^2 = 3.14(4\sqrt{2})^2$
= 100.48 cm^2

4. A rectangular piece is 20m long and 15m wide. From its four corners, quadrants of radii 3.5m have been cut. Find the area of the remaining part.
Solution:

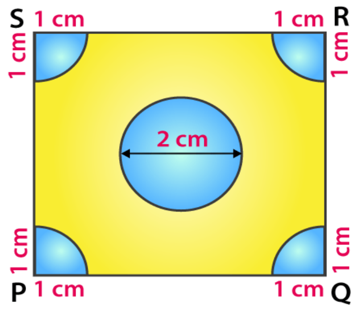


Given,
Length of the rectangle = 20 m

Breadth of the rectangle = 15 m
Radius of the quadrant = 3.5 m
So,
Area of the remaining part = Area of the rectangle – 4 x Area of one quadrant

$= (l \times b) - 4 \times (¼ \times \pi r^2)$
$= (l \times b) - \pi r^2$
$= (20 \times 15) - (22/7)(3.5)^2$
$= 300 - 38.5$
$= 261.5\ m^2$

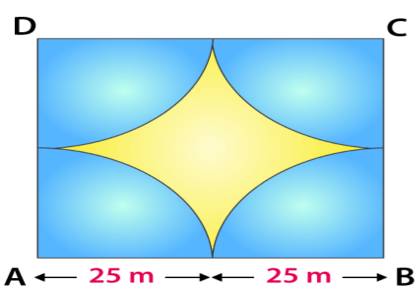
5. In fig. 15.73, PQRS is a square of side 4 cm. Find the area of the shaded square.
Solution:



We know that, each quadrant is a sector of 90° in a circle of 1 cm radius. In other words its 1/4th of a circle.
So, its area = $¼ \times \pi r^2$
$= ¼ \times (22/7)(1)^2 = 22/28\ cm^2$
And, the area of the square = $side^2$ [Given, side = 4 cm]
$= 4^2 = 16\ cm^2$
Area of the circle = $\pi r^2 = \pi(1)^2 = 22/7\ cm^2$ [Given, diameter = 2 cm, so radius = 1cm]
Thus,
The area of the shaded region = area of the square – area of the circle – 4 x area of a quadrant

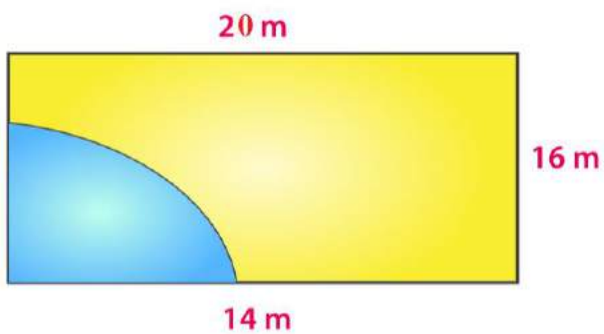
$= 16 - 22/7 - (4 \times 22/28)$
$= 16 - 22/7 - 22/7 = 16 - 44/7$
$= 68/7\ cm^2$

6. Four cows are tethered at four corners of a square plot of side 50m, so that they just cannot reach one another. What area will be left un-grazed?
Solution:



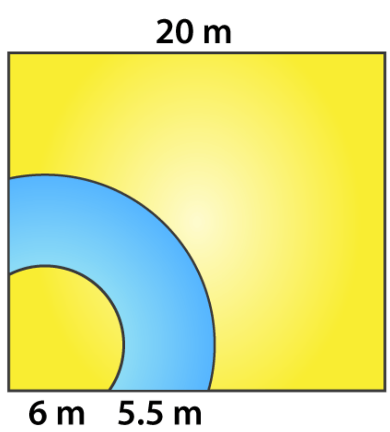
Given,
Side of square plot = 50 m
Radius of a quadrant = 25 m
So, we can tell
Area of plot left un-grazed = Area of the plot – 4 x (area of a quadrant)
= $\text{Side}^2 - 4 \times (¼ \times \pi r^2)$
= $50^2 - 22/7 \times (25)^2$
= 2500 – 1964.28
= 535.72 m^2

7. A cow is tied with a rope of length 14 m at the corner of a rectangle field of dimensions 20 m x 16 m, find the area of the field in which the cow can graze.
Solution:



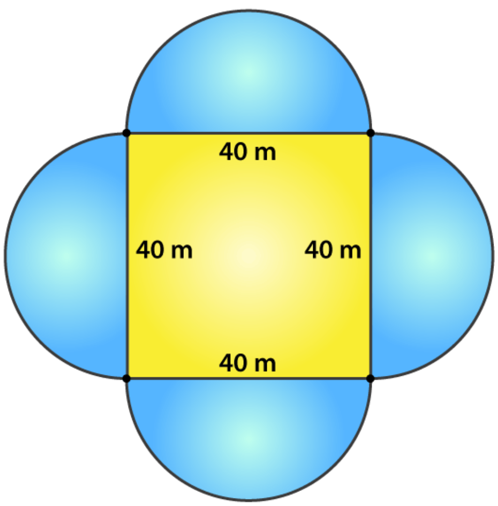
The dotted portion indicated the area over which the cow can graze.
It's clearly seen that, the shaded area is the area of a quadrant of a circle of radius equal to the length of the rope.
Thus, the required area = $¼ \times \pi r^2$
= ¼ x 22/7 x 14 x 14
= 154
Hence, the area of the field in which the cow can graze is 154 cm^2

8. A calf is tied with a rope of length 6 m at the corner of a square grassy lawn of side 20 m. If the length of the rope is increased by 5.5 m, find the increase in area of the grassy lawn in which the calf can graze.
Solution:



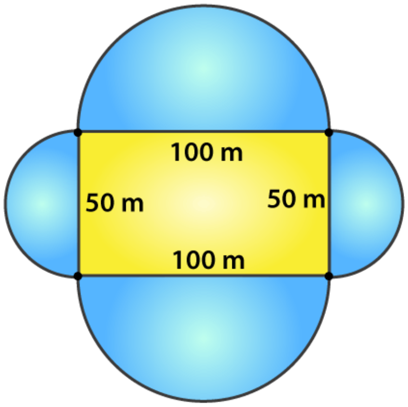
Given,
The initial length of the rope = 6 m
Then the rope is said to be increased by 5.5m
So, the increased length of the rope = (6 + 5.5) = 11.5 m
We know that, the corner of the lawn is a quadrant of a circle.
Thus,
The required area = $\frac{1}{4} \times \pi(11.5)^2 - \frac{1}{4} \times \pi(5.5)^2$
$= \frac{1}{4} \times 22/7\ (11.5^2 - 6^2)$
$= \frac{1}{4} \times 22/7\ (132.25 - 36)$
$= \frac{1}{4} \times 22/7 \times 96.25$
$= 75.625\ cm^2$

9. A square tank has its side equal to 40 m. There are four semi-circular grassy plots all around it. Find the cost of turfing the plot at Rs 1.25 per square meter.
Solution:



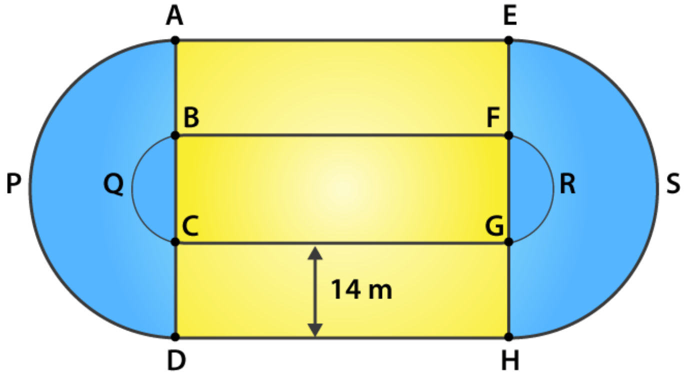
Given,
Side of the square tank = 40 m
And, the diameter of the semi-circular grassy plot = side of the square tank = 40 m
Radius of the grassy plot = 40/2 = 20 m
Then,
The area of the four semi-circular grassy plots = $4 \times \frac{1}{2} \pi r^2$
$= 4 \times \frac{1}{2} (3.14)(20)^2$
$= 2512\ m^2$
Rate of turfing the plot = Rs 1.25 per m^2
So, the cost for 2512 m^2 = (1.25 x 2512) = Rs 3140

10. A rectangular park is 100 m by 50 m. It is surrounded by semi-circular flower beds all round. Find the cost of levelling the semi-circular flower beds at 60 paise per square meter.
Solution:



Given,
Length of the park = 100 m and the breadth of the park = 50 m
The radius of the semi-circular flower beds = half of the corresponding side of the rectangular park
Radius of the bigger flower bed = 100/2 = 50 m
And the radius of the smaller flower bed = 50/2 = 25 m
Total area of the flower beds = 2[Area of bigger flower bed + Area of smaller flower bed]
$= 2[\frac{1}{2} \pi(50)^2 + \frac{1}{2} \pi(25)^2]$
$= \pi[(50)^2 + (25)^2]$
$= 3.14 \times [2500 + 625]$
$= 9812.5 \text{ m}^2$
Now, rate of levelling flower bed = 60 paise per m^2
Therefore,
The total cost of levelling = 9812.5 x 60 = 588750 paise
= Rs 5887.50

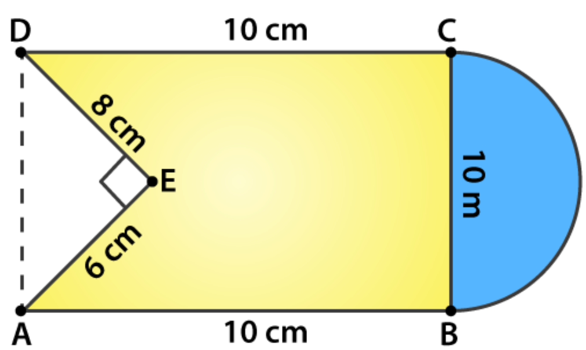
11. The inner perimeter of a running track (show in Fig.) is 400 m. The length of each of the straight portion is 90 m and the ends are semi-circles. If the track is everywhere 14 m wide, find the area of the track. Also, find the length of the outer running track.
Solution:



Let the radius of the inner semi-circle = r
And that of the outer semi-circle = R

Given,
Length of the straight portion = 90 m
Width of the track = 14 m
The inner perimeter of the track = 400 m
But,
Inner perimeter of the track = BF + FRG + GC + CQB = 400
$90 + \pi r + 90 + \pi r = 400$
$2 \pi r + 180 = 400$
$2 \times 22/7 \times r = 220$
$r = 35$ m
So, the radius of the outer semi-circle = 35 + 14 = 49 m
Now,
Area of the track = 2[Area of the rectangle AEFB + Area of semi-circle APD – Area of semi-circle BQC]
$= 2[90 \times 14 + ½ \times 22/7 \times 49^2 – ½ \times 22/7 \times 35^2]$
$= 2[1260 + 11 \times 7 \times 49 – 11 \times 5 \times 35]$
$= 2 [1260 + 3773 - 1925]$
$= 3 \times 3108$
$= 6216$ m^2
Thus,
The length of outer running track = AE + APD + DH + HSE
$= 90 + \pi R + 90 + \pi R$
$= 180 + 2 \pi R$
$= 180 + 2 \times 22/7 \times 49$
$= 180 + 308$
$= 488$ m

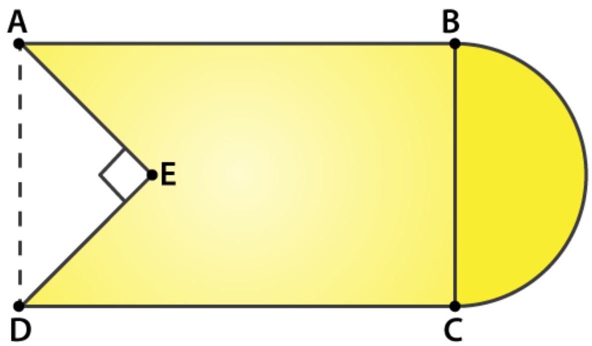
12. Find the area of Fig. 15.76 in square cm, correct to one place of decimal.
Solution:



The radius of the semi-circle = 10/2 = 5 cm
It's seen that,
The area of figure = Area of square + Area of semi-circle – Area of triangle AED
$= 10 \times 10 + ½ \pi r^2 – ½ \times 6 \times 8$
$= 100 + ½ (22/7)(5)^2 - 24$

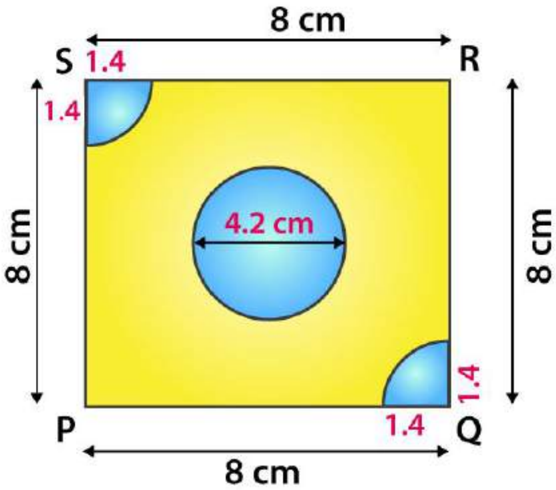
= (700 + 275 – 168)/7
= (807)/7
= 115.3 cm^2

13. From a rectangular region ABCD with AB = 20 cm, a right angle AED with AE = 9 cm and DE = 12 cm, is cut off. On the other end, taking BC as diameter, a semicircle is added on outside the region. Find the area of the shaded region. (π = 22/7)
Solution:



Given,
Length of the rectangle ABCD = 20 cm
AE = 9 cm and DE = 12 cm
The radius of the semi-circle = BC/ 2 or AD/2
Now, using Pythagoras theorem in triangle AED
AD = $\sqrt{(AE^2 + ED^2)} = \sqrt{(9^2 + 12^2)}$
= $\sqrt{(81 + 144)}$
= $\sqrt{(225)}$ = 15 cm
So, the area of the rectangle = 20 x 15 = 300 cm^2
And, the area of the triangle AED = ½ x 12 x 9 = 54 cm^2
The radius of the semi-circle = 15/2 = 7.5 cm
Area of semi-circle = ½ $\pi(15/2)^2$ = ½ x 3.14 x 7.5^2 = 88.31 cm^2
Thus,
The area of the shaded region = Area of the rectangle ABCD + Area of semi-circle – Area of triangle AED
= 300 + 88.31 – 54
= 334.31 cm^2

14. From each of the two opposite corners of a square of side 8 cm, a quadrant of a circle of radius 1.4 cm is cut. Another circle of radius 4.2 cm is also cut from the center as shown in Fig. Find the area of the remaining (shaded) portion of the square. (Use π = 22/7)
Solution:



Given,
Side of the square = 8 cm
Radius of circle = 4.2 cm
Radius of the quadrant = 1.4 cm
Thus,
Area of the shaded potion = Area of square – Area of circle – 2 x Area of the quadrant

$= \text{side}^2 - \pi r^2 - 2 \times \frac{1}{2} \pi r^2$

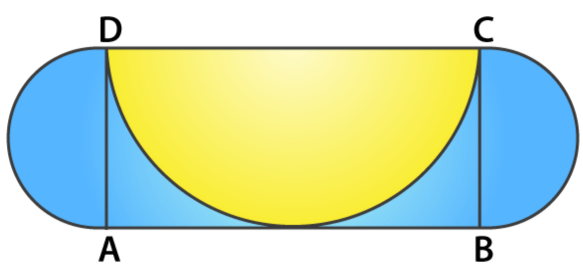
$= 8^2 - \pi(4.2)^2 - 2 \times \frac{1}{2} \pi(1.4)^2$

$= 64 - 22/7(4.2 \times 4.2) - 22/7(1.4 \times 1.4)$

$= 64 - 388.08/7 - 21.56/7$

$= 5.48 \text{ cm}^2$

15. ABCD is a rectangle with AB = 14 cm and BC = 7 cm. Taking DC, BC and AD as diameters, three semi-circles are drawn as shown in the figure. Find the area of the shaded region.
Solution:



Given,
ABCD is a rectangle with AB = 14 cm and BC = 7 cm
It’s seen that,
The area of shaded region = Area of rectangle ABCD + 2 x area of semi-circle with AD and BC as diameters – area of the semi-circle with DC as diameter

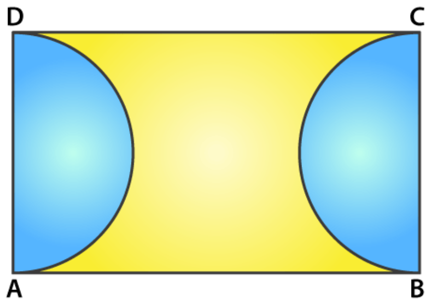
$= 14 \times 7 + 2 \times \frac{1}{2} \pi(7/2)^2 - \frac{1}{2} \pi(7)^2$

$= 98 + 22/7 \times (7/2)^2 - \frac{1}{2} (22/7)(7)^2$

$= 98 + 77/2 - 77$

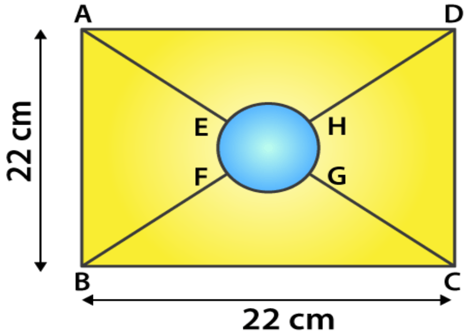
$= 59.5 \text{ cm}^2$

16. ABCD is rectangle, having AB = 20 cm and BC = 14 cm. Two sectors of 180° have been cut off. Calculate:
(i) the area of the shaded region.
(ii) the length of the boundary of the shaded region.
Solution:



Given,
Length of the rectangle = AB = 20 cm
Breadth of the rectangle = BC = 14 cm
(i) Area of the shaded region = Area of rectangle – 2 x Area of the semi-circle
= l x b – 2 x ½ πr^2
= 20 x 14 – (22/7) x 7^2
= 280 – 154
= 126 cm^2
(ii) Length of the boundary of the shaded region = 2 x AB + 2 x circumference of a semi-circle
= 2 x 20 + 2 x πr
= 40 + 2 x (22/7) x 7
= 40 + 44
= 84 cm

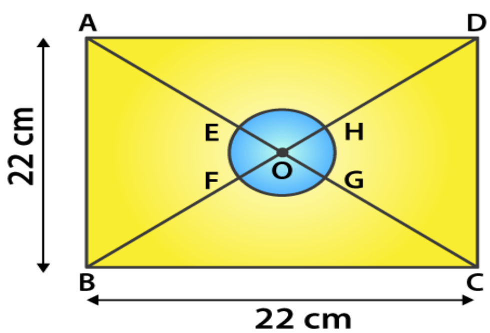
17. The square ABCD is divided into five equal parts, all having same area. The central part is circular and the lines AE, GC, BF and HD lie along the diagonals AC and BD of the square. If AB = 22 cm, find:
(i) the circumference of the central part. (ii) the perimeter of the part ABEF.
Solution:



Given,
Side of the square = 22 cm = AB
Let the radius of the centre part be r cm.

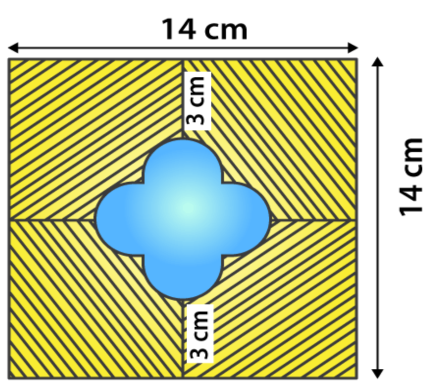
Then, area of the circle = 1/5 x area of the square
$\pi r^2 = 1/5 \times 22^2$
$22/7 \times r^2 = (22 \times 22)/5$
r = 154/5 = 5.55 cm
(i) Circumference of central part = 2πr = 2(22/7)(5.55) = 34.88 cm

(ii) Let O be the center of the central part. Then, its clear that O is also the center of the square as well.
Now, in triangle ABC
By Pythagoras theorem
$AC^2 = AB^2 + BC^2 = 22^2 + 22^2 = 2 \times 22^2$
AC = 22√2
Since diagonals of a square bisect each other
AO = ½ AC = ½ (22√2) = 11√2 cm



And,
AE = BF = OA – OE = 11√2 – 5.55 = 15.51 – 5.55 = 9.96 cm
EF = ¼(Circumference of the circle) = 2πr/4
= ½ x 22/7 x 5.55 = 8.72 cm
Thus, the perimeter of the part ABEF = AB + AE + EF + BF
= 22 + 2 x 9.96 + 8.72
= 50.64 cm

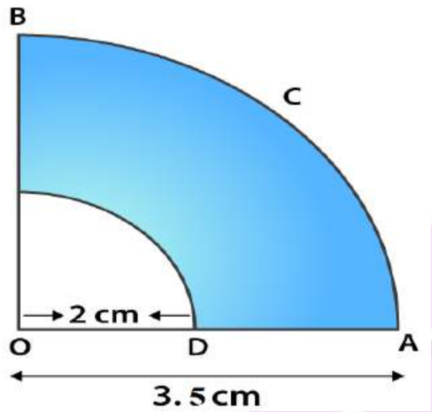
18. In figure, find the area of the shaded region. (Use π = 3.14)
Solution:



The side of the square = 14 cm
So, it's area = $14^2 = 196\ cm^2$

Let's assume the radius of each semi-circle be r cm.
Then,
r + 2r + r = 14 – 3 – 3
4r = 8
r = 2
The radius of each semi-circle is 2 cm.
Area of 4 semi-circles = (4 x ½ x 3.14 x 2 x 2) = 25.12 cm^2
Now, length of side of the smaller square = 2r = 2 x 2 = 4 cm
Thus, the area of smaller square = 4x4 = 16 cm^2
Area of unshaded region = Area of 4 semi-circles + Area of smaller square
= (25.12 + 16) = 41.12 cm^2
Therefore,
The area of shaded region = Area of square ABCD – Area of unshaded region
= (196 – 41.12) = 154.88 cm^2

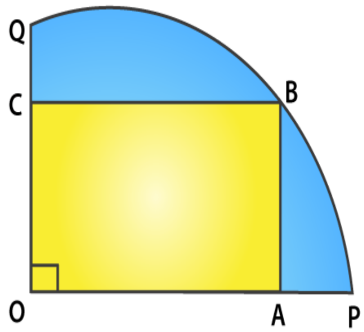
19. OACB is a quadrant of a circle with center O and radius 3.5 cm. If OD = 2 cm, find the area of the (i) quadrant OACB (ii) shaded region.
Solution:



Given,
Radius of small quadrant, r = 2 cm
Radius of big quadrant, R = 3.5 cm
(i) Area of quadrant OACB = ¼ πR^2
= ¼ (22/7)$(3.5)^2$
= 269.5/28 = 9.625 cm^2

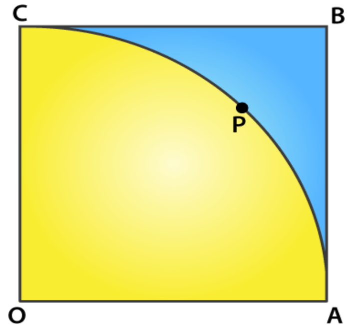
(ii) Area of shaded region = Area of big quadrant – Area of small quadrant
= ¼ $\pi(R^2 - r^2)$
= ¼ (22/7)$(3.5^2 - 2^2)$
= ¼ (22/7)(12.25 - 4)
= ¼ (22/7)(8.25)
= 6.482 cm^2

20. A square OABC is inscribed in a quadrant OPBQ of a circle. If OA = 21 cm, find the area of the shaded region.
Solution:



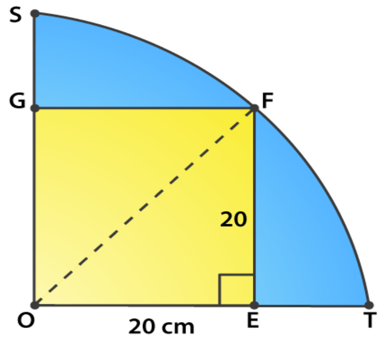
Given,
Side of the square = 21 cm = OA
Area of the square = $OA^2 = 21^2 = 441\ cm^2$
Diagonal of the square OB = $\sqrt{2}$ OA = $21\sqrt{2}$ cm
And, from the fig. its seen that
The diagonal of the square is equal to the radius of the circle, r = $21\sqrt{2}$ cm
So, the area of the quadrant = $\frac{1}{4}\pi r^2 = \frac{1}{4}(22/7)(21\sqrt{2})^2 = 693\ cm^2$
Thus,
The area of the shaded region = Area of the quadrant – Area of the square
= 693 – 441
= $252\ cm^2$

21. OABC is a square of side 7 cm. If OAPC is a quadrant of a circle with centre O, then find the area of the shaded region.
Solution:



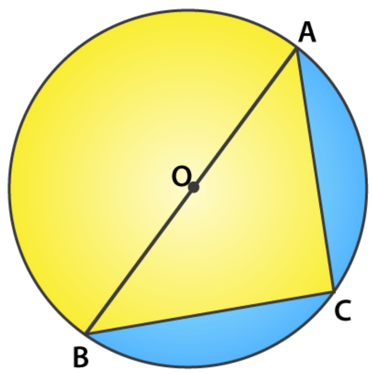
Given,
OABC is a square of side 7 cm
So, OA = AB = BC = OC = 7 cm
Area of square OABC = $side^2 = 7^2 = 49\ cm^2$
And given, OAPC is a quadrant of a circle with centre O.
So, the radius of the quadrant = OA = OC = 7 cm
Area of the quadrant OAPC = $90/360 \times \pi r^2$
= $\frac{1}{4} \times (22/7) \times 7^2$
= $77/2 = 38.5\ cm^2$
Thus,
Area of shaded portion = Area of square OABC – Area of quadrant OAPC
= $(49 – 38.5) = 10.5\ cm^2$

22. OE = 20 cm. In sector OSFT, square OEFG is inscribed. Find the area of the shaded region.
Solution:



It's seen that, OEFG is a square of side 20 cm.
So its diagonal = √2 side = 20√2 cm
And, the radius of the quadrant = diagonal of the square
Radius of the quadrant = 20√2 cm
So,
Area of the shaded portion = Area of quadrant – Area of square
$= \frac{1}{4}\pi r^2 - side^2$
$= \frac{1}{4}(22/7)(20\sqrt{2})^2 - (20)^2$
$= \frac{1}{4}(22/7)(800) - 400$
$= 400 \times 4/7 = 1600/7 = 228.5\ cm^2$

23. Find the area of the shaded region in Fig., if AC = 24 cm, BC = 10 cm and O is the center of the circle
Solution:



Given,
AC = 24 cm and BC = 10 cm
Since, AB is the diameter of the circle
Angle ACB = 90°
So, using Pythagoras theorem
$AB^2 = AC^2 + BC^2 = 24^2 + 10^2 = 576 + 100 = 676$
AB = √676 = 26 cm
Thus, the radius of the circle = 26/2 = 13 cm

The area of shaded region = Area of semi-circle – Area of triangle ACB

$= \frac{1}{2} \pi r^2 – \frac{1}{2} \times b \times h$

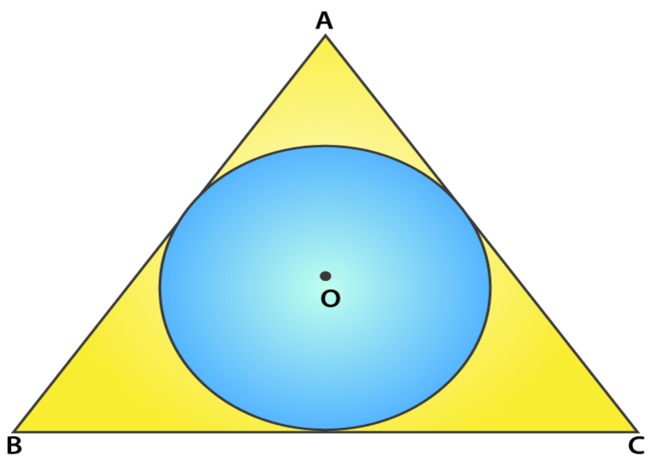
$= \frac{1}{2} (22/7)13^2 – \frac{1}{2} \times 10 \times 24$

$= 265.33 – 120$

$= 145.33 \text{ cm}^2$

24. A circle is inscribed in an equilateral triangle ABC of side 12 cm, touching its sides (fig.,). Find the radius of the inscribed circle and the area of the shaded part.

Solution:



Given,

An equilateral triangle of side = 12 cm

Area of the equilateral triangle = $\sqrt{3}/4(\text{side})^2$

$= \sqrt{3}/4(12)^2 = 36\sqrt{3} \text{ cm}^2$

Perimeter of triangle ABC = 3 x 12 = 36 cm

So,

The radius of incircle = Area of triangle/ ½ (perimeter of triangle)

$= 36\sqrt{3}/ \frac{1}{2} \times 36$

$= 2\sqrt{3}$ cm

Therefore,

Area of the shaded part = Area of equilateral triangle – Area of circle

$= 36\sqrt{3} - \pi r^2$

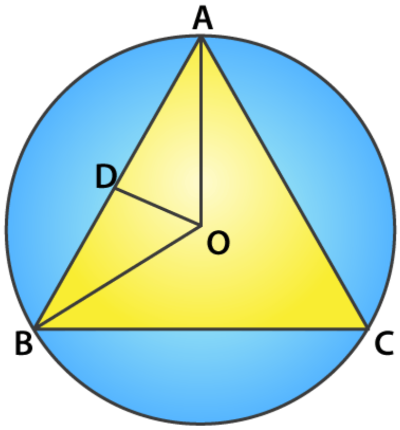
$= 36(1.732) – (3.14)(2\sqrt{3})^2$

$= 62.352 – 37.68$

$= 24.672 \text{ cm}^2$

25. In fig., an equilateral triangle ABC of side 6 cm has been inscribed in a circle. Find the area of the shaded region. (Take π = 3.14).

Solution:



Given,
Side of the equilateral triangle = 6 cm
And,
The area of the equilateral triangle = $\sqrt{3}/4(\text{side})^2$
$= \sqrt{3}/4(6)^2$
$= \sqrt{3}/4(36)$
$= 9\sqrt{3}\ \text{cm}^2$
Let us mark the center of the circle as O, OA and OB are the radii of the circle.
In triangle BOD,
$\sin 60^o = BD/ OB$
$\sqrt{3}/2 = 3/ OB$
$OB = 2\sqrt{3}\ \text{cm} = r$

Therefore,
The area of shaded region = Area of the circle – area of the equilateral triangle
$= \pi r^2 - 9\sqrt{3}$
$= 3.14 \times (2\sqrt{3})^2 - 9\sqrt{3}$
$= 3.14 \times 12 – 9 \times 1.732$
$= 37.68 – 15.588$
$= 22.092\ \text{cm}^2$