

Exercise 16.1

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1. How many balls, each of radius 1 cm, can be made from a solid sphere of lead of radius 8 cm?

Solution:

Given,

A solid sphere of radius, $R = 8$ cm

With this sphere, we have to make spherical balls of radius $r = 1$ cm

Let's assume that the number of balls made as n

Then, we know that

$$\text{Volume of the sphere} = \frac{4}{3} \pi r^3$$

The volume of the solid sphere = sum of the volumes of n spherical balls.

$$n \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$n \times \frac{4}{3} \pi (1)^3 = \frac{4}{3} \pi (8)^3$$

$$n = 8^3 = 512$$

Therefore, 512 balls can be made of radius 1 cm each with a solid sphere of radius 8 cm.

2. How many spherical bullets each of 5 cm in diameter can be cast from a rectangular block of metal 11dm x 1 m x 5 dm?

Solution:

Given,

A metallic block of dimension 11dm x 1m x 5dm

The diameter of each bullet = 5 cm

We know that,

$$\text{Volume of the sphere} = \frac{4}{3} \pi r^3$$

$$\text{Since, } 1 \text{ dm} = 10^{-1} \text{ m} = 0.1 \text{ m}$$

$$\text{The volume of the rectangular block} = 1.1 \times 1 \times 0.5 = 0.55 \text{ m}^3$$

$$\text{Radius of the bullet} = \frac{5}{2} = 2.5 \text{ cm}$$

Let the number of bullets made from the rectangular block be n .

Then from the question,

The volume of the rectangular block = sum of the volumes of the n spherical bullets

$$0.55 = n \times \frac{4}{3} \pi (2.5)^3$$

Solving for n , we have

$$n = 8400$$

Therefore, 8400 can be cast from the rectangular block of metal.

3. A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of the two of the balls are 2 cm and 1.5 cm respectively. Determine the diameter of the third ball?

Solution:

Given,

Radius of the spherical ball = 3 cm

We know that,

$$\text{The volume of the sphere} = \frac{4}{3} \pi r^3$$

$$\text{So, it's volume (V)} = \frac{4}{3} \pi r^3$$

That the ball is melted and recast into 3 spherical balls.

$$\text{Volume } (V_1) \text{ of first ball} = \frac{4}{3} \pi 1.5^3$$

$$\text{Volume } (V_2) \text{ of second ball} = \frac{4}{3} \pi 2^3$$

Let the radius of the third ball = r cm

$$\text{Volume of third ball } (V_3) = \frac{4}{3} \pi r^3$$

Volume of the spherical ball is equal to the volume of the 3 small spherical balls.

$$V = V_1 + V_2 + V_3$$

$$\frac{4}{3} \pi r^3 = \frac{4}{3} \pi 1.5^3 + \frac{4}{3} \pi 2^3 + \frac{4}{3} \pi r^3$$

Now,

Cancelling out the common part from both sides of the equation we get,

$$(3)^3 = (2)^3 + (1.5)^3 + r^3$$

$$r^3 = 3^3 - 2^3 - 1.5^3 \text{ cm}^3$$

$$r^3 = 15.6 \text{ cm}^3$$

$$r = (15.6)^{1/3} \text{ cm}$$

$$r = 2.5 \text{ cm}$$

$$\begin{aligned} \text{As diameter} &= 2 \times \text{radius} = 2 \times 2.5 \text{ cm} \\ &= 5.0 \text{ cm.} \end{aligned}$$

Thus, the diameter of the third ball is 5 cm

4. 2.2 cubic dm of brass is to be drawn into a cylindrical wire of 0.25 cm in diameter. Find the length of the wire?

Solution:

Given,

2.2 dm³ of brass is to be drawn into a cylindrical wire of Diameter = 0.25 cm

$$\begin{aligned} \text{So, radius of the wire } (r) &= d/2 \\ &= 0.25/2 = 0.125 \times 10^{-2} \text{ cm} \end{aligned}$$

Now, 1 cm = 0.01 m

So, 0.1cm = 0.001 m

Let the length of the wire be (h)

We know that,

$$\text{Volume of the cylinder} = \pi r^2 h$$

It's understood that,

$$\text{Volume of cylindrical wire} = \text{Volume of brass of } 2.2 \text{ dm}^3$$

$$\frac{22}{7} (0.125 \times 10^{-2})^2 \times h = 2.2 \times 10^{-3}$$

$$h = 448 \text{ m}$$

Therefore, the length of the cylindrical wire drawn is 448 m

5. What length of a solid cylinder 2 cm in diameter must be taken to recast into a hollow cylinder of length 16 cm, external diameter 20 cm and thickness 2.5 mm?

Solution:

Given,

Diameter of the solid cylinder = 2 cm

Length of hollow cylinder = 16 cm

The solid cylinder is recast into a hollow cylinder of length 16 cm, external diameter of 20 cm and thickness of 2.5 mm = 0.25 cm

We know that,

Volume of a cylinder = $\pi r^2 h$

Radius of the solid cylinder = 1 cm

So,

Volume of the solid cylinder = $\pi 1^2 h = \pi h \text{ cm}^3$

Let's assume the length of the solid cylinder as h

And,

Volume of the hollow cylinder = $\pi h(R^2 - r^2)$

Thickness of the cylinder = (R - r)

0.25 = 10 - r

So, the internal radius of the cylinder is 9.75 cm

Volume of the hollow cylinder = $\pi \times 16(100 - 95.0625)$

Hence, it's understood that

Volume of the solid cylinder = volume of the hollow cylinder

$\pi h = \pi \times 16(100 - 95.06)$

$h = 79.04 \text{ cm}$

Therefore, the length of the solid cylinder is 79.04 cm.

6. A cylindrical vessel having diameter equal to its height is full of water which is poured into two identical cylindrical vessels with diameter 42 cm and height 21 cm which are filled completely. Find the diameter of the cylindrical vessel?

Solution:

Given,

The diameter of the cylinder = the height of the cylinder

$\Rightarrow h = 2r$, where h – height of the cylinder and r – radius of the cylinder

We know that,

Volume of a cylinder = $\pi r^2 h$

So, volume of the cylindrical vessel = $\pi r^2 2r = 2\pi r^3$ (as $h = 2r$)..... (i)

Now,

Volume of each identical vessel = $\pi r^2 h$

Diameter = 42 cm, so the radius = 21 cm

Height = 21 cm

So, the volume of two identical vessels = $2 \times \pi 21^2 \times 21$ (ii)

Since the volumes on equation (i) and (ii) are equal

On equating both the equations, we have

$2\pi r^3 = 2 \times \pi 21^2 \times 21$

$r^3 = (21)^3$

$r = 21 \text{ cm}$

So, $d = 42 \text{ cm}$

Therefore, the diameter of the cylindrical vessel is 42 cm.

7. 50 circular plates each of diameter 14 cm and thickness 0.5 cm are placed one above the other to form a right circular cylinder. Find its total surface area.

Solution:

Given,

50 circular plates each with diameter 14 cm

Radius of circular plates = 7cm

Thickness of plates = 0.5 cm

As these plates are one above the other, the total thickness of all the plates = $0.5 \times 50 = 25$ cm

$$\begin{aligned}\text{So, the total surface area of the right circular cylinder formed} &= 2\pi r \times h + 2\pi r^2 \\ &= 2\pi r (h + r) \\ &= 2(22/7) \times 7 \times (25 + 7) \\ &= 2 \times 22 \times 33 = 1408 \text{ cm}^2\end{aligned}$$

Therefore, the total surface area of the cylinder is 1408 cm^2

8. 25 circular plates, each of radius 10.5 cm and thickness 1.6 cm, are placed one above the other to form a solid circular cylinder. Find the curved surface area and the volume of the cylinder so formed.

Solution:

Given,

250 circular plates each with radius 10.5 cm and thickness of 1.6 cm.

As the plates are placed one above the other, the total height becomes = $1.6 \times 25 = 40$ cm

We know that,

$$\begin{aligned}\text{Curved surface area of a cylinder} &= 2\pi rh \\ &= 2\pi \times 10.5 \times 40 = 2640 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{And, volume of the cylinder} &= \pi r^2 h \\ &= \pi \times 10.5^2 \times 40 = 13860 \text{ cm}^3\end{aligned}$$

Therefore,

The curved surface area of the cylinder is 2640 cm^2 and the volume of the cylinder is 13860 cm^3

9. Find the number of metallic circular discs with 1.5 cm base diameter and of height 0.2 cm to be melted to form a right circular cylinder of height 10 cm and diameter 4.5 cm.

Solution:

Given,

Radius of each circular disc = $r = 1.5/2 = 0.75$ cm

Height of each circular disc = $h = 0.2$ cm

Radius of cylinder = $R = 4.5/2 = 2.25$ cm

Height of cylinder = $H = 10$ cm

So, the number of metallic discs required is given by n

$n = \text{Volume of cylinder} / \text{volume of each circular disc}$

$$n = \pi R^2 H / \pi r^2 h$$

$$n = (2.25)^2 (10) / (0.75)^2 (0.2)$$

$$n = 3 \times 3 \times 50 = 450$$

Therefore, 450 metallic discs are required.

10. How many spherical lead shots each of diameter 4.2 cm can be obtained from a solid rectangular lead piece with dimensions 66 cm × 42 cm × 21 cm.

Solution:

Given,

Radius of each spherical lead shot = $r = 4.2/2 = 2.1$ cm

The dimensions of the rectangular lead piece = 66 cm × 42 cm × 21 cm

So, the volume of a spherical lead shot = $\frac{4}{3} \pi r^3$
 $= \frac{4}{3} \times \frac{22}{7} \times 2.1^3$

And, the volume of the rectangular lead piece = 66 × 42 × 21

Thus,

The number of spherical lead shots = Volume of rectangular lead piece / Volume of a spherical lead shot
 $= \frac{66 \times 42 \times 21}{(\frac{4}{3} \times \frac{22}{7} \times 2.1^3)}$
 $= 1500$

11. How many spherical lead shots of diameter 4 cm can be made out of a solid cube of lead whose edge measures 44 cm.

Solution:

Given,

The radius of each spherical lead shot = $r = 4/2 = 2$ cm

Volume of each spherical lead shot = $\frac{4}{3} \pi r^3 = \frac{4}{3} \pi 2^3$ cm³

Edge of the cube = 44 cm

Volume of the cube = 44³ cm³

Thus,

Number of spherical lead shots = Volume of cube / Volume of each spherical lead shot
 $= \frac{44 \times 44 \times 44}{(\frac{4}{3} \pi 2^3)}$
 $= 2541$

12. Three cubes of a metal whose edges are in the ratio 3: 4: 5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. Find the edges of the three cubes.

Solution:

Let the edges of three cubes (in cm) be 3x, 4x and 5x respectively.

So, the volume of the cube after melting will be = $(3x)^3 + (4x)^3 + (5x)^3$
 $= 9x^3 + 64x^3 + 125x^3 = 216x^3$

Now, let a be the edge of the new cube so formed after melting

Then we have,

$$a^3 = 216x^3$$

$$a = 6x$$

We know that,

$$\text{Diagonal of the cube} = \sqrt{(a^2 + a^2 + a^2)} = a\sqrt{3}$$

$$\text{So, } 12\sqrt{3} = a\sqrt{3}$$

$$a = 12 \text{ cm}$$

$$x = 12/6 = 2$$

Thus, the edges of the three cubes are 6 cm, 8 cm and 10 cm respectively.

13. A solid metallic sphere of radius 10.5 cm is melted and recast into a number of smaller cones, each of radius 3.5 cm and height 3 cm. Find the number of cones so formed.

Solution:

Given,

Radius of metallic sphere = $R = 10.5$ cm

So, its volume = $\frac{4}{3} \pi R^3 = \frac{4}{3} \pi (10.5)^3$

We also have,

Radius of each cone = $r = 3.5$ cm

Height of each cone = $h = 3$ cm

And, its volume = $\frac{1}{3} \pi r^2 h = \frac{1}{3} \pi (3.5)^2 (3)$

Thus,

$$\begin{aligned} \text{The number of cones} &= \text{Volume of metallic sphere} / \text{Volume of each cone} \\ &= \frac{4}{3} \pi (10.5)^3 / \frac{1}{3} \pi (3.5)^2 (3) \\ &= 126 \end{aligned}$$

14. The diameter of a metallic sphere is equal to 9 cm. It is melted and drawn into a long wire of diameter 2 mm having uniform cross-section. Find the length of the wire.

Solution:

Given,

Radius of the sphere = $9/2$ cm

So, its volume = $\frac{4}{3} \pi r^3 = \frac{4}{3} \pi (9/2)^3$

And, the radius of the wire = 2 mm = 0.2 cm

Let the length of the wire = h cm

So, the volume of wire = $\pi r^2 h = \pi (0.2)^2 h$

Then, according to the question we have

Volume of wire = Volume of sphere

$$\pi (0.2)^2 h = \frac{4}{3} \pi (9/2)^3$$

$$h = 4 \times 729 / (3 \times 8 \times 0.01) = 12150 \text{ cm}$$

Therefore, the length of the wire = 12150 cm

15. An iron spherical ball has been melted and recast into smaller balls of equal size. If the radius of each of the smaller balls is $1/4$ of the radius of the original ball, how many such balls are made? Compare the surface area, of all the smaller balls combined together with that of the original ball.

Solution:

Let the radius of the big ball be x cm

The, the radius of the small ball = $x/4$ cm

And, let the number of balls = n

Then according to the question, we have

Volume of n small balls = Volume of the big ball

$$n \times \frac{4}{3} \pi \left(\frac{x}{4}\right)^3 = \frac{4}{3} \pi x^3$$

$$n \times \left(\frac{x^3}{64}\right) = x^3$$

$$n = 64$$

Therefore, the number of small balls = 64

Next,

$$\begin{aligned} \text{Surface area of all small balls/ surface area of big ball} &= 64 \times 4\pi\left(\frac{x}{4}\right)^2 / 4\pi(x)^2 \\ &= 64/16 = 4/1 \end{aligned}$$

Thus, the ratio of the surface area of the small balls to that of the original ball is 4:1

16. A copper sphere of radius 3 cm is melted and recast into a right circular cone of height 3 cm. Find the radius of the base of the cone?

Solution:

Given,

Radius of the copper sphere = 3 cm

We know that,

$$\begin{aligned} \text{Volume of the sphere} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \pi \times 3^3 \dots\dots (i) \end{aligned}$$

Also, given that the copper sphere is melted and recasted into a right circular cone

Height of the cone = 3 cm

We know that,

$$\begin{aligned} \text{Volume of the right circular cone} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi \times r^2 \times 3 \dots\dots (ii) \end{aligned}$$

On comparing equation (i) and (ii) we have,

$$\frac{4}{3} \pi \times 3^3 = \frac{1}{3} \pi \times r^2 \times 3$$

$$r^2 = 36$$

$$r = 6 \text{ cm}$$

Therefore, the radius of the base of the cone is 6 cm.

17. A copper rod of diameter 1 cm and length 8 cm is drawn into a wire of length 18 m of uniform thickness. Find the thickness of the wire?

Solution:

Given,

Diameter of the copper wire = 1 cm

So, radius of the copper wire = $\frac{1}{2}$ cm = 0.5 cm

Length of the copper rod = 8 cm

We know that,

$$\begin{aligned} \text{Volume of the cylinder} &= \pi r^2 h \\ &= \pi \times 0.5^2 \times 8 \dots\dots\dots (i) \end{aligned}$$

Length of the wire = 18 m = 1800 cm

$$\begin{aligned} \text{Volume of the wire} &= \pi r^2 h \\ &= \pi r^2 \times 1800 \dots\dots (ii) \end{aligned}$$

On equating both the equations, we have

$$\pi \times 0.5^2 \times 8 = \pi r^2 \times 1800$$

$$r^2 = \frac{2}{1800} = \frac{1}{900}$$

$$r = 1/30 \text{ cm}$$

Therefore, the diameter of the wire is $1/15 \text{ cm}$ i.e. 0.67 mm which is the thickness of the wire.

18. The diameters of internal and external surfaces of a hollow spherical shell are 10cm and 6 cm respectively. If it is melted and recast into a solid cylinder of length of $8/3$, find the diameter of the cylinder?

Solution:

Given,

Internal diameter of the hollow sphere = 6 cm

So, the internal radius of the hollow sphere = $6/2 \text{ cm} = 3 \text{ cm} = r$

External diameter of the hollow sphere = 10 cm

So, the external radius of the hollow sphere = $10/2 \text{ cm} = 5 \text{ cm} = R$

We know that,

$$\begin{aligned} \text{Volume of the hollow spherical shell} &= \frac{4}{3} \pi \times (R^3 - r^3) \\ &= \frac{4}{3} \pi \times (5^3 - 3^3) \quad \dots\dots (i) \end{aligned}$$

And given, the length of the solid cylinder = $8/3 \text{ cm}$

Let the radius of the solid cylinder be $r \text{ cm}$

We know that,

$$\begin{aligned} \text{Volume of the cylinder} &= \pi \times r^2 \times h \\ &= \pi \times r^2 \times 8/3 \quad \dots\dots (ii) \end{aligned}$$

Now equating both (i) and (ii), we have

$$\frac{4}{3} \pi \times 5^3 - 3^3 = \pi \times r^2 \times 8/3$$

$$\frac{4}{3} \times (125 - 27) = r^2 \times 8/3$$

$$98/2 = r^2$$

$$r^2 = 49$$

$$r = 7$$

So, $d = 7 \times 2 = 14 \text{ cm}$

Therefore, the diameter of the cylinder is 14 cm

19. How many coins 1.75 cm in diameter and 2 mm thick must be melted to form a cuboid 11 cm x 10 cm x 7 cm?

Solution:

Given,

Diameter of the coin = 1.75 cm

So, its radius = $1.75/2 = 0.875 \text{ cm}$

Thickness or the height = 2 mm = 0.2 cm

We know that,

$$\begin{aligned} \text{Volume of the cylinder (V}_1) &= \pi r^2 h \\ &= \pi 0.875^2 \times 0.2 \end{aligned}$$

And, the volume of the cuboid (V_2) = $11 \times 10 \times 7 \text{ cm}^3$

Let the number of coins needed to be melted be n .

So, we have

$$V_2 = V_1 \times n$$

$$11 \times 10 \times 7 = \pi 0.875^2 \times 0.2 \times n$$

$$11 \times 10 \times 7 = 22/7 \times 0.875^2 \times 0.2 \times n$$

On solving we get, $n = 1600$

Therefore, the number of coins required are 1600

20. The surface area of a solid metallic sphere is 616 cm^2 . It is melted and recast into a cone of height 28 cm. Find the diameter of the base of the cone so formed.

Solution:

Given,

The height of the cone = 28 cm

Surface area of the solid metallic sphere = 616 cm^2

We know that,

Surface area of the sphere = $4\pi r^2$

So, $4\pi r^2 = 616$

$$r^2 = 49$$

$$r = 7$$

Radius of the solid metallic sphere = 7 cm

Let's assume r to be the radius of the cone

We know that,

$$\begin{aligned} \text{Volume of the cone} &= \frac{1}{3} \pi r^2 h \\ &= \frac{1}{3} \pi r^2 (28) \dots\dots (i) \end{aligned}$$

$$\begin{aligned} \text{Volume of the sphere} &= \frac{4}{3} \pi r^3 \\ &= \frac{4}{3} \pi 7^3 \dots\dots\dots (ii) \end{aligned}$$

On equating equations (i) and (ii), we have

$$\frac{1}{3} \pi r^2 (28) = \frac{4}{3} \pi 7^3$$

Eliminating the common terms, we get

$$r^2 (28) = 4 \times 7^3$$

$$r^2 = 49$$

$$r = 7$$

So, diameter of the cone = $7 \times 2 = 14 \text{ cm}$

Therefore, the diameter of the base of the cone is 14 cm

21. A cylindrical bucket, 32 cm high and 18 cm of radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Solution:

Given,

Height of the cylindrical bucket = 32 cm

Radius of the cylindrical bucket = 18 cm

Height of conical heap = 24 cm

We know that,

Volume of cylinder = $\pi \times r^2 \times h$

And, volume of cone = $\frac{1}{3} \pi \times r^2 \times h$

Then, from the question

Volume of the conical heap = Volume of the cylindrical bucket

$$\frac{1}{3} \pi \times r^2 \times 24 = \pi \times 18^2 \times 32$$

$$r^2 = 18^2 \times 4$$

$$r = 18 \times 2 = 36 \text{ cm}$$

Now,

Slant height of the conical heap (l) is given by

$$l = \sqrt{(h^2 + r^2)}$$

$$l = \sqrt{(24^2 + 36^2)} = \sqrt{1872}$$

$$l = 43.26 \text{ cm}$$

Therefore, the radius and slant height of the conical heap are 36 cm and 43.26 cm respectively.

22. A solid metallic sphere of radius 5.6 cm is melted and solid cones each of radius 2.8 cm and height 3.2 cm are made. Find the number of such cones formed.

Solution:

Let the number of cones made be n

Given,

Radius of metallic sphere = 5.6 cm

Radius of the cone = 2.8 cm

Height of the cone = 3.2 cm

We know that,

$$\text{Volume of a sphere} = \frac{4}{3} \pi \times r^3$$

$$\text{So, } V_1 = \frac{4}{3} \pi \times 5.6^3$$

And,

$$\text{Volume of cone} = \frac{1}{3} \pi \times r^2 \times h$$

$$V_2 = \frac{1}{3} \pi \times 2.8^2 \times 3.2$$

Thus, the number of cones (n) = Volume of the sphere / Volume of the cone

$$n = \frac{\frac{4}{3} \pi \times 5.6^3}{\frac{1}{3} \pi \times 2.8^2 \times 3.2}$$

$$n = \frac{(4 \times 5.6^3)}{(2.8^2 \times 3.2)}$$

$$n = 28$$

Therefore, 28 such cones can be formed.

23. A solid cuboid of iron with dimensions 53 cm x 40 cm x 15 cm is melted and recast into a cylindrical pipe. The outer and inner diameters of pipe are 8 cm and 7 cm respectively. Find the length of pipe.

Solution:

Let the length of the pipe be h cm.

$$\text{Then, Volume of cuboid} = (53 \times 40 \times 15) \text{ cm}^3$$

$$\text{Internal radius of the pipe} = \frac{7}{2} \text{ cm} = r$$

$$\text{External radius of the pipe} = \frac{8}{2} = 4 \text{ cm} = R$$

$$\text{So, the volume of iron in the pipe} = (\text{External Volume}) - (\text{Internal Volume})$$

$$= \pi R^2 h - \pi r^2 h$$

$$= \pi h (R^2 - r^2)$$

$$= \pi h (R - r) (R + r)$$

$$= \pi (4 - \frac{7}{2}) (4 + \frac{7}{2}) \times h$$

$$= \pi(1/2) (15/2) \times h$$

Then from the question it's understood that,

The volume of iron in the pipe = volume of iron in cuboid

$$\pi(1/2) (15/2) \times h = 53 \times 40 \times 15$$

$$h = (53 \times 40 \times 15 \times 7/22 \times 2/15 \times 2) \text{ cm}$$

$$h = 2698 \text{ cm}$$

Therefore, the length of the pipe is 2698 cm.

24. The diameters of the internal and external surfaces of a hollow spherical shell are 6 cm and 10 cm respectively. If it is melted and recast into a solid cylinder of diameter 14 cm, find the height of the cylinder.

Solution:

Given,

Internal diameter of hollow spherical shell = 6 cm

So, the internal radius of hollow spherical shell = $6/2 = 3 \text{ cm} = r$

External diameter of hollow spherical shell = 10 cm

So, the external radius of hollow spherical shell = $10/2 = 5 \text{ cm} = R$

Diameter of the cylinder = 14 cm

So, the radius of cylinder = $14/2 = 7 \text{ cm}$

Let the height of cylinder be taken as $h \text{ cm}$

Then, according to the question we have

Volume of cylinder = Volume of spherical shell

$$\pi \times r^2 \times h = \frac{4}{3} \pi \times (R^3 - r^3)$$

$$\pi \times 7^2 \times h = \frac{4}{3} \pi \times (5^3 - 3^3)$$

$$h = \frac{4}{3} \times 2$$

$$h = \frac{8}{3} \text{ cm}$$

Therefore, the height of the cylinder = $\frac{8}{3} \text{ cm}$

25. A hollow sphere of internal and external diameters 4 cm and 8 cm respectively is melted into a cone of base diameter 8 cm. Calculate the height of the cone?

Solution:

Given,

Internal diameter of hollow sphere = 4 cm

So, the internal radius of hollow sphere = 2 cm

External diameter of hollow sphere = 8 cm

So, the external radius of hollow sphere = 4 cm

We know that,

$$\text{Volume of the hollow sphere } \frac{4}{3} \pi \times (4^3 - 2^3) \quad \dots \text{ (i)}$$

Also given,

Diameter of the cone = 8 cm

So, the radius of the cone = 4 cm

Let the height of the cone be $x \text{ cm}$

$$\text{Volume of the cone } \frac{1}{3} \pi \times 4^2 \times h \quad \dots \text{ (ii)}$$

As the volume of the hollow sphere and cone are equal. We can equate equations (i) and (ii)

So, we get

$$\frac{4}{3} \pi \times (4^3 - 2^3) = \frac{1}{3} \pi \times 4^2 \times h$$

$$4 \times (64 - 8) = 16 \times h$$

$$h = 14$$

Therefore, the height of the cone so obtained will have a height of 14 cm

26. A hollow sphere of internal and external radii 2 cm and 4 cm respectively is melted into a cone of base radius 4 cm. Find the height and slant height of the cone.

Solution:

Given,

The internal radius of hollow sphere = 2 cm

The external radius of hollow sphere = 4 cm

We know that,

$$\text{Volume of the hollow sphere } \frac{4}{3} \pi \times (4^3 - 2^3) \quad \dots (i)$$

Also given,

The base radius of the cone = 4 cm

Let the height of the cone be x cm

$$\text{Volume of the cone } \frac{1}{3} \pi \times 4^2 \times h \quad \dots (ii)$$

As the volume of the hollow sphere and cone are equal. We can equate equations (i) and (ii)

So, we get

$$\frac{4}{3} \pi \times (4^3 - 2^3) = \frac{1}{3} \pi \times 4^2 \times h$$

$$4 \times (64 - 8) = 16 \times h$$

$$h = 14$$

Now,

Slant height of the cone (l) is given by

$$l = \sqrt{h^2 + r^2}$$

$$l = \sqrt{14^2 + 4^2} = \sqrt{212}$$

$$l = 14.56 \text{ cm}$$

Therefore, the height and slant height of the conical heap are 14 cm and 14.56 cm respectively.

27. A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of the balls are 1.5 cm and 2 cm. Find the diameter of the third ball.

Solution:

Given,

Radius of the spherical ball = 3 cm

We know that,

The volume of the sphere = $\frac{4}{3} \pi r^3$

So, it's volume (V) = $\frac{4}{3} \pi r^3$

That the ball is melted and recast into 3 spherical balls.

Volume (V_1) of first ball = $\frac{4}{3} \pi 1.5^3$

Volume (V_2) of second ball = $\frac{4}{3} \pi 2^3$

Let the radius of the third ball = r cm

Volume of third ball (V_3) = $\frac{4}{3} \pi r^3$

Volume of the spherical ball is equal to the volume of the 3 small spherical balls.

$$V = V_1 + V_2 + V_3$$

$$\frac{4}{3}\pi r^3 = \frac{4}{3}\pi 1.5^3 + \frac{4}{3}\pi 2^3 + \frac{4}{3}\pi r^3$$

Now,

Cancelling out the common part from both sides of the equation we get,

$$(3)^3 = (2)^3 + (1.5)^3 + r^3$$

$$r^3 = 3^3 - 2^3 - 1.5^3 \text{ cm}^3$$

$$r^3 = 15.6 \text{ cm}^3$$

$$r = (15.6)^{1/3} \text{ cm}$$

$$r = 2.5 \text{ cm}$$

$$\begin{aligned} \text{As diameter} &= 2 \times \text{radius} = 2 \times 2.5 \text{ cm} \\ &= 5.0 \text{ cm.} \end{aligned}$$

Thus, the diameter of the third ball is 5 cm

28. A path 2 m wide surrounds a circular pond of diameter 40 m. How many cubic meters of gravel are required to grave the path to a depth of 20 cm?

Solution:

Given,

Diameter of the circular pond = 40 m

So, the radius of the pond = $40/2 = 20 \text{ m} = r$

Thickness (width of the path) = 2 m

As the whole view of the pond looks like a hollow cylinder.

And the height will be 20 cm = 0.2 m

So,

$$\text{Thickness (t)} = R - r$$

$$2 = R - 20$$

$$R = 22 \text{ m}$$

$$\begin{aligned} \text{Volume of the hollow cylinder} &= \pi (R^2 - r^2) \times h \\ &= \pi (22^2 - 20^2) \times 0.2 \\ &= 52.8 \text{ m}^3 \end{aligned}$$

Therefore, the volume of the hollow cylinder is the required amount of sand needed to spread across to a depth of 20 m.

29. A 16 m deep well with diameter 3.5 m is dug up and the earth from it is spread evenly to form a platform 27.5 m by 7m. Find the height of the platform?

Solution:

Let us assume the well to be a solid right circular cylinder

Radius(r) of the cylinder = $3.5/2 \text{ m} = 1.75 \text{ m}$

Depth of the well or height of the cylinder (h) = 16 m

We know that,

$$\text{Volume of the cylinder (V}_1) = \pi r^2 h$$

$$= \pi \times 1.75^2 \times 16$$

Given,

The length of the platform (l) = 27.5 m

Breadth of the platform (b) = 7 m

Now, let the height of the platform be x m

We know that,

Volume of the rectangle = l*b*h

$$V_2 = 27.5 * 7 * x$$

As the earth dug up is spread evenly to form the platform

Volumes of both, the well and the platform should be the same.

$$V_1 = V_2$$

$$\pi \times 1.75 \times 1.75 \times 16 = 27.5 \times 7 \times x$$

$$x = 0.8 \text{ m} = 80 \text{ cm}$$

Therefore, the height of the platform is 80 cm.

30. A well of diameter 2 m is dug 14 m deep. The earth taken out of it is evenly spread all around it to form an embankment of height 40 cm. Find the width of the embankment?

Solution:

Given,

Radius of the circular cylinder (r) = 2/2 m = 1 m

Height of the well (h) = 14 m

We know that,

$$\begin{aligned} \text{Volume of the solid circular cylinder} &= \pi r^2 h \\ &= \pi \times 1^2 \times 14 \dots (i) \end{aligned}$$

And,

The height of the embankment (h) = 40 cm = 0.4 m

Let the width of the embankment be (x) m.

The embankment is a hollow cylinder with external radius = 1 + x and internal radius = 1

$$\begin{aligned} \text{Volume of the embankment} &= \pi \times r^2 \times h \\ &= \pi \times [(1 + x)^2 - (1)^2] \times 0.4 \dots (ii) \end{aligned}$$

As the well is spread evenly to form embankment then the volumes will be same.

So, on equating equations (i) and (ii), we get

$$\pi \times 1^2 \times 14 = \pi \times [(1 + x)^2 - (1)^2] \times 0.4$$

$$14/0.4 = 1 + x^2 + 2x - 1$$

$$35 = x^2 + 2x$$

$$x^2 + 2x - 35 = 0$$

Solving by factorization method, we have

$$(x + 7)(x - 5) = 0$$

So, x = 5 m can only be the solution as it's a positive value.

Therefore, the width of the embankment is 5 m.

31. A well with inner radius 4 m is dug up and 14 m deep. Earth taken out of it has spread evenly all around a width of 3 m it to form an embankment. Find the height of the embankment?

Solution:

Given,

Inner radius of the well = 4 m

Depth of the well = 14 m

We know that,

$$\begin{aligned} \text{Volume of the cylinder} &= \pi r^2 h \\ &= \pi \times 4^2 \times 14 \dots (i) \end{aligned}$$

From the question, it's told that

The earth taken out from the well is evenly spread all around it to form an embankment

And, the width of the embankment = 3 m

So, the outer radius of the well = 3 + 4 m = 7 m

We know that,

$$\begin{aligned} \text{Volume of the hollow embankment} &= \pi (R^2 - r^2) \times h \\ &= \pi \times (7^2 - 4^2) \times h \dots (ii) \end{aligned}$$

On equating both the equations (i) and (ii), we get

$$\pi \times 4^2 \times 14 = \pi \times (7^2 - 4^2) \times h$$

$$h = 4^2 \times 14 / (33)$$

$$h = 6.78 \text{ m}$$

Therefore, the height of the embankment so formed is 6.78 m.

32. A well of diameter 3 m is dug up to 14 m deep. The earth taken out of it has been spread evenly all around it to a width of 4 m to form an embankment. Find the height of the embankment.

Solution:

Given,

Diameter of the well = 3 m

So, the radius of the well = 3/2 m = 1.5 m

Depth of the well (h) = 14 m

Width of the embankment (thickness) = 4 m

So, the radius of the outer surface of the embankment = (4 + 1.5) m = 5.5 m

Let the height of the embankment be taken as h m

We know that the embankment is a hollow cylinder

$$\begin{aligned} \text{Volume of the embankment} &= \pi (R^2 - r^2) \times h \\ &= \pi (5.5^2 - 1.5^2) \times h \dots (i) \end{aligned}$$

$$\begin{aligned} \text{Volume of earth dug out} &= \pi \times r^2 \times h \\ &= \pi \times 2^2 \times 14 \dots (ii) \end{aligned}$$

On equating both (i) and (ii) we get,

$$\pi (5.5^2 - 1.5^2) \times h = \pi \times (3/2)^2 \times 14$$

$$(30.25 - 2.25) \times h = 9 \times 14 / 4$$

$$h = 9 \times 14 / (4 \times 28)$$

$$h = 9/8 \text{ m}$$

Therefore, the height of the embankment is 9/8 m

33. Find the volume largest right circular cone that can be cut out of a cube whose edge is 9 cm.

Solution:

Given,

The side of the cube = 9 cm

The largest cone that can be cut from cube will have the base diameter = side of the cube

$$2r = 9$$

$$r = 9/2 \text{ cm} = 4.5 \text{ cm}$$

And,

Height of cone = side of cube

So, height of cone (h) = 9 cm

Thus,

$$\begin{aligned} \text{Volume of the largest cone to fit in} &= \frac{1}{3} \pi \times r^2 \times h \\ &= \frac{1}{3} \pi \times 4.5^2 \times 9 \\ &= 190.93 \text{ cm}^3 \end{aligned}$$

Therefore, the volume of the largest cone to fit in the cube has a volume of 190.93 cm³

34. A cylindrical bucket, 32 cm high and 18 cm of radius of the base, is filled with sand. This bucket is emptied on the ground and a conical heap of sand is formed. If the height of the conical heap is 24 cm, find the radius and slant height of the heap.

Solution:

Given,

Height of the cylindrical bucket = 32 cm

Radius of the cylindrical bucket = 18 cm

Height of conical heap = 24 cm

We know that,

$$\text{Volume of cylinder} = \pi \times r^2 \times h$$

$$\text{And, volume of cone} = \frac{1}{3} \pi \times r^2 \times h$$

Then, from the question

Volume of the conical heap = Volume of the cylindrical bucket

$$\frac{1}{3} \pi \times r^2 \times 24 = \pi \times 18^2 \times 32$$

$$r^2 = 18^2 \times 4$$

$$r = 18 \times 2 = 36 \text{ cm}$$

Now,

Slant height of the conical heap (l) is given by

$$l = \sqrt{(h^2 + r^2)}$$

$$l = \sqrt{(24^2 + 36^2)} = \sqrt{1872}$$

$$l = 43.26 \text{ cm}$$

Therefore, the radius and slant height of the conical heap are 36 cm and 43.26 cm respectively.

35. Rain water, which falls on a flat rectangular surface of length 6 m and breadth 4 m is transferred into a cylindrical vessel of internal radius 20 cm .What will be the height of water in the cylindrical vessel if a rainfall of 1 cm has fallen?

Solution:

Given,

Length of the rectangular surface = 6 m = 600 cm

Breadth of the rectangular surface = 4 m = 400 cm

Height of the perceived rain = 1 cm

So,

$$\begin{aligned}\text{Volume of the rectangular surface} &= \text{length} * \text{breadth} * \text{height} \\ &= 600 * 400 * 1 \text{ cm}^3 \\ &= 240000 \text{ cm}^3 \dots\dots\dots (i)\end{aligned}$$

Also given,

Radius of the cylindrical vessel = 20 cm

Let the height of the cylindrical vessel be taken as h cm

We know that,

$$\begin{aligned}\text{Volume of the cylindrical vessel} &= \pi \times r^2 \times h \\ &= \pi \times 20^2 \times h \dots\dots\dots (ii)\end{aligned}$$

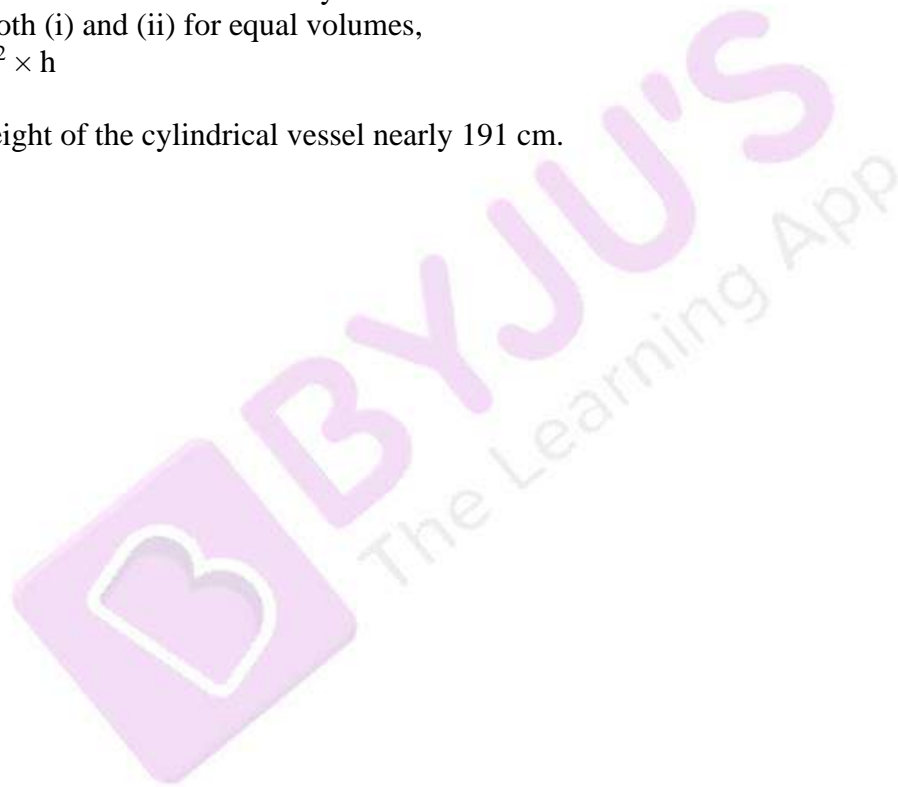
As all the rain water is transferred to the cylindrical vessel

We can equate both (i) and (ii) for equal volumes,

$$240000 = \pi \times 20^2 \times h$$

$$h = 190.9 \text{ cm}$$

Therefore, the height of the cylindrical vessel nearly 191 cm.

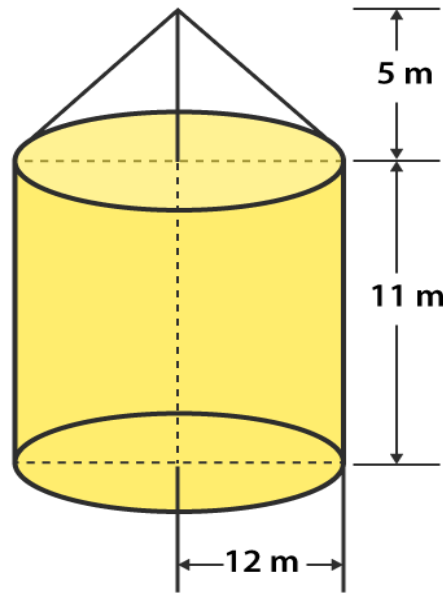


Exercise 16.2

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1. A tent is in the form of a right circular cylinder surmounted by a cone. The diameter of cylinder is 24 m. The height of the cylindrical portion is 11 m while the vertex of the cone is 16 m above the ground. Find the area of canvas required for the tent.

Solution:



Given,

The diameter of the cylinder (also the same for cone) = 24 m.

So, its radius (R) = $24/2 = 12$ m

The height of the Cylindrical part (H_1) = 11m

So, Height of the cone part (H_2) = $16 - 11 = 5$ m

Now,

Vertex of the cone above the ground = $11 + 5 = 16$ m

Curved Surface area of the Cone (S_1) = $\pi RL = 22/7 \times 12 \times L$

The slant height (L) is given by,

$$L = \sqrt{(R^2 + H_2^2)} = \sqrt{(12^2 + 5^2)} = \sqrt{169}$$

$$L = 13 \text{ m}$$

So,

$$\text{Curved Surface Area of Cone } (S_1) = 22/7 \times 12 \times 13$$

And,

$$\text{Curved Surface Area of Cylinder } (S_2) = 2\pi RH_1$$

$$S_2 = 2\pi(12)(11) \text{ m}^2$$

Thus, the area of Canvas required for tent

$$S = S_1 + S_2 = (22/7 \times 12 \times 13) + (2 \times 22/7 \times 12 \times 11)$$

$$S = 490 + 829.38$$

$$S = 1319.8 \text{ m}^2$$

$$S = 1320 \text{ m}^2$$

Therefore, the area of canvas required for the tent is 1320 m^2

2. A rocket is in the form of a circular cylinder closed at the lower end with a cone of the same radius attached to the top. The cylinder is of radius 2.5 m and height 21 m and the cone has the slant height 8 m. Calculate the total surface area and the volume of the rocket.

Solution:

Given,

Radius of the cylindrical portion of the rocket (R) = 2.5 m

Height of the cylindrical portion of the rocket (H) = 21 m

Slant Height of the Conical surface of the rocket (L) = 8 m

Curved Surface Area of the Cone (S_1) = $\pi RL = \pi(2.5)(8) = 20\pi$

And,

Curved Surface Area of the Cone (S_2) = $2\pi RH + \pi R^2$

$$S_2 = (2\pi \times 2.5 \times 21) + \pi (2.5)^2$$

$$S_2 = (\pi \times 105) + (\pi \times 6.25)$$

Thus, the total curved surface area S is

$$S = S_1 + S_2$$

$$S = (\pi 20) + (\pi 105) + (\pi 6.25)$$

$$S = (22/7)(20 + 105 + 6.25) = 22/7 \times 131.25$$

$$S = 412.5 \text{ m}^2$$

Therefore, the total Surface Area of the Conical Surface = 412.5 m^2

Now, calculating the volume of the rocket

Volume of the conical part of the rocket (V_1) = $1/3 \times 22/7 \times R^2 \times h$

$$V_1 = 1/3 \times 22/7 \times (2.5)^2 \times h$$

Let, h be the height of the conical portion in the rocket.

We know that,

$$L^2 = R^2 + h^2$$

$$h^2 = L^2 - R^2 = 8^2 - 2.5^2$$

$$h = 7.6 \text{ m}$$

Using the value of h, we will get

$$\text{Volume of the conical part } (V_1) = 1/3 \times 22/7 \times 2.5^2 \times 7.6 \text{ m}^2 = 49.67 \text{ m}^2$$

Next,

Volume of the Cylindrical Portion (V_2) = $\pi R^2 h$

$$V_2 = 22/7 \times 2.5^2 \times 21 = 412.5 \text{ m}^2$$

Thus, the total volume of the rocket = $V_1 + V_2$

$$V = 412.5 + 49.67 = 462.17 \text{ m}^2$$

Hence, the total volume of the Rocket is 462.17 m^2

3. A tent of height 77 dm is in the form of a right circular cylinder of diameter 36 m and height 44 dm surmounted by a right circular cone. Find the cost of the canvas at Rs. 3.50 per m^2

Solution:

Given,

Height of the tent = 77 dm

Height of a surmounted cone = 44 dm

Height of the Cylindrical Portion = Height of the tent – Height of the surmounted Cone

$$= 77 - 44$$

$$= 33 \text{ dm} = 3.3 \text{ m}$$

And, given diameter of the cylinder (d) = 36 m

So, its radius (r) of the cylinder = $36/2 = 18 \text{ m}$

Let's consider L as the slant height of the cone.

Then, we know that

$$L^2 = r^2 + h^2$$

$$L^2 = 18^2 + 3.3^2$$

$$L^2 = 324 + 10.89$$

$$L^2 = 334.89$$

$$L = 18.3 \text{ m}$$

Thus, slant height of the cone (L) = 18.3 m

Now, the Curved Surface area of the Cylinder (S_1) = $2\pi rh$

$$S_1 = 2\pi (18 \times 4.4) \text{ m}^2$$

And, the Curved Surface area of the cone (S_2) = πrL

$$S_2 = \pi \times 18 \times 18.3 \text{ m}^2$$

So, the total curved surface of the tent (S) = $S_1 + S_2$

$$S = S_1 + S_2$$

$$S = (2\pi \times 18 \times 4.4) + (\pi \times 18 \times 18.3)$$

$$S = 1533.08 \text{ m}^2$$

Hence, the total Curved Surface Area (S) = 1533.08 m²

Next,

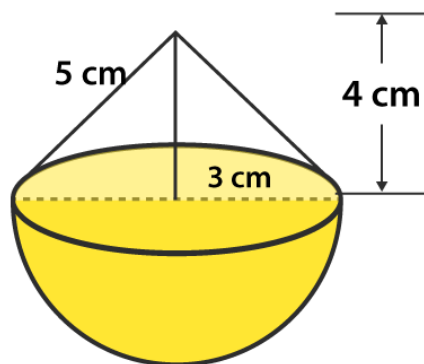
The cost of 1 m² canvas = Rs 3.50

$$\text{So, } 1533.08 \text{ m}^2 \text{ of canvas will cost} = \text{Rs } (3.50 \times 1533.08)$$

$$= \text{Rs } 5365.8$$

4. A toy is in the form of a cone surmounted on a hemisphere. The diameter of the base and the height of the cone are 6 cm and 4 cm, respectively. Determine the surface area of the toy.

Solution:



Given that,

The height of the cone (h) = 4 cm

Diameter of the cone (d) = 6 cm

So, its radius (r) = 3

Let, 'l' be the slant height of cone.

Then, we know that

$$l^2 = r^2 + h^2$$

$$l^2 = 3^2 + 4^2 = 9 + 16 = 25$$

$$l = 5 \text{ cm}$$

Hence, the slant height of the cone (l) = 5 cm

So, the curved surface area of the cone (S_1) = $\pi r l$

$$S_1 = \pi(3)(5)$$

$$S_1 = 47.1 \text{ cm}^2$$

And, the curved surface area of the hemisphere (S_2) = $2\pi r^2$

$$S_2 = 2\pi(3)^2$$

$$S_2 = 56.23 \text{ cm}^2$$

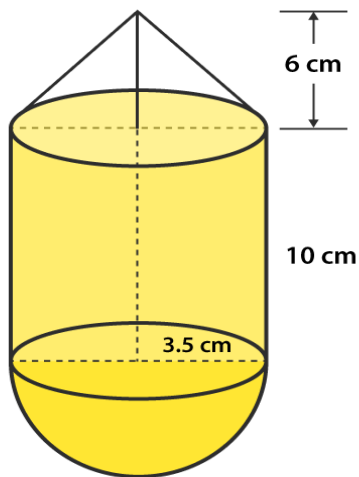
So, the total surface area (S) = $S_1 + S_2$

$$S = 47.1 + 56.23$$

$$S = 103.62 \text{ cm}^2$$

Therefore, the curved surface area of the toy is 103.62 cm^2

**5. A solid is in the form of a right circular cylinder, with a hemisphere at one end and a cone at the other end. The radius of the common base is 3.5 cm and the height of the cylindrical and conical portions are 10 cm and 6 cm, respectively. Find the total surface area of the solid. (Use $\pi = 22/7$).
Solution:**



Given,

Radius of the common base (r) = 3.5 cm

Height of the cylindrical part (h) = 10 cm

Height of the conical part (H) = 6 cm

Let, ' l ' be the slant height of the cone

Then, we know that

$$l^2 = r^2 + H^2$$

$$l^2 = 3.5^2 + 6^2 = 12.25 + 36 = 48.25$$

$$l = 6.95 \text{ cm}$$

So, the curved surface area of the cone (S_1) = $\pi r l$

$$S_1 = \pi(3.5)(6.95)$$

$$S_1 = 76.38 \text{ cm}^2$$

And, the curved surface area of the hemisphere (S_2) = $2\pi r^2$

$$S_2 = 2\pi(3.5)^2$$

$$S_2 = 77 \text{ cm}^2$$

Next, the curved surface area of the cylinder (S_3) = $2\pi rh$

$$S_2 = 2\pi(3.5)(10)$$

$$S_2 = 220 \text{ cm}^2$$

Thus, the total surface area (S) = $S_1 + S_2 + S_3$

$$S = 76.38 + 77 + 220 = 373.38 \text{ cm}^2$$

Therefore, the total surface area of the solid is 373.38 cm^2

6. A toy is in the shape of a right circular cylinder with a hemisphere on one end and a cone on the other. The radius and height of the cylindrical parts are 5cm and 13 cm, respectively. The radii of the hemispherical and conical parts are the same as that of the cylindrical part. Find the surface area of the toy if the total height of the toy is 30 cm.

Solution:

Given,

Height of the Cylindrical portion (H) = 13 cm

Radius of the Cylindrical portion (r) = 5 cm

Height of the whole solid = 30 cm

Then,

The curved surface area of the cylinder (S_1) = $2\pi rh$

$$S_1 = 2\pi(5)(13)$$

$$S_1 = 408.2 \text{ cm}^2$$

Let, 'L' be the slant height of the cone

And, the curved surface area of the cone (S_2) = πrL

$$S_2 = \pi(5)L$$

For conical part, we have

$$h = 30 - 13 - 5 = 12 \text{ cm}$$

Then, we know that

$$L^2 = r^2 + h^2$$

$$L^2 = 5^2 + 12^2$$

$$L^2 = 25 + 144$$

$$L^2 = 169$$

$$L = 13 \text{ m}$$

So,

$$S_2 = \pi(5)(13) \text{ cm}^2$$

$$S_2 = 204.28 \text{ cm}^2$$

Now, the curved surface area of the hemisphere (S_3) = $2\pi r^2$

$$S_3 = 2\pi(5)^2$$

$$S_3 = 157.14 \text{ cm}^2$$

Thus, the total curved surface area (S) = $S_1 + S_2 + S_3$

$$S = (408.2 + 204.28 + 157.14)$$

$$S = 769.62 \text{ cm}^2$$

Therefore, the surface area of the toy is 770 cm^2

7. Consider a cylindrical tub having radius as 5 cm and its length 9.8 cm. It is full of water. A solid in the form of a right circular cone mounted on a hemisphere is immersed in tub. If the radius of the hemisphere is 3.5 cm and the height of the cone outside the hemisphere is 5 cm, find the volume of water left in the tub.

Solution:

Given,

The radius of the Cylindrical tub (r) = 5 cm

Height of the Cylindrical tub (H) = 9.8 cm

Height of the cone outside the hemisphere (h) = 5 cm

Radius of the hemisphere = 3.5 cm

Now, we know that

The volume of the Cylindrical tub (V_1) = $\pi r^2 H$

$$V_1 = \pi(5)^2 9.8$$

$$V_1 = 770 \text{ cm}^3$$

And, the volume of the Hemisphere (V_2) = $\frac{2}{3} \times \pi \times r^3$

$$V_2 = \frac{2}{3} \times \frac{22}{7} \times 3.5^3$$

$$V_2 = 89.79 \text{ cm}^3$$

And, the volume of the Hemisphere (V_3) = $\frac{2}{3} \times \pi \times r^2 \times h$

$$V_3 = \frac{2}{3} \times \frac{22}{7} \times 3.5^2 \times 5$$

$$V_3 = 64.14 \text{ cm}^3$$

Thus, total volume (V) = Volume of the cone + Volume of the hemisphere

$$= V_2 + V_3$$

$$V = 89.79 + 64.14 \text{ cm}^3$$

$$= 154 \text{ cm}^3$$

So, the total volume of the solid = 154 cm^3

In order to find the volume of the water left in the tube, we have to subtract the volume of the hemisphere and the cone from the volume of the cylinder.

Hence, the volume of water left in the tube = $V_1 - V_2$

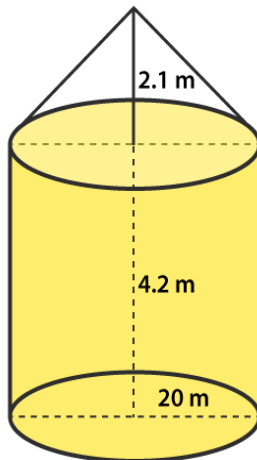
$$= 770 - 154$$

$$= 616 \text{ cm}^3$$

Therefore, the volume of water left in the tube is 616 cm^3 .

8. A circus tent has a cylindrical shape surmounted by a conical roof. The radius of the cylindrical base is 20 cm. The heights of the cylindrical and conical portions is 4.2 cm and 2.1 cm respectively. Find the volume of that tent.

Solution:



Given,

Radius of the cylindrical portion (R) = 20 m

Height of the cylindrical portion (h_1) = 4.2 m

Height of the conical portion (h_2) = 2.1 m

Now, we know that

Volume of the Cylindrical portion (V_1) = $\pi r^2 h_1$

$$V_1 = \pi(20)^2 4.2$$

$$V_1 = 5280 \text{ m}^3$$

And, the volume of the conical part (V_2) = $\frac{1}{3} \times \frac{22}{7} \times r^2 \times h^2$

$$V_2 = \frac{1}{3} \times \frac{22}{7} \times 20^2 \times 2.1$$

$$V_2 = 880 \text{ m}^3$$

Thus, the total volume of the tent (V) = volume of the conical portion + volume of the Cylindrical portion

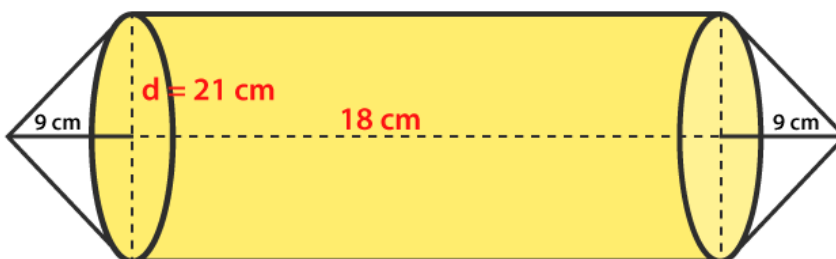
$$V = V_1 + V_2$$

$$V = 6160 \text{ m}^3$$

Therefore, volume of the tent is 6160 m^3

9. A petrol tank is a cylinder of base diameter 21 cm and length 18 cm fitted with the conical ends each of axis length 9 cm. Determine the capacity of the tank.

Solution:



Given,

Base diameter of the cylindrical base of the petrol tank = 21 cm

So, its radius (r) = diameter/2 = 21/2 = 10.5 cm

Height of the Cylindrical portion of the tank (h_1) = 18 cm

Height of the Conical portion of the tank (h_2) = 9 cm

Now, we know that

The volume of the Cylindrical portion (V_1) = $\pi r^2 h_1$

$$V_1 = \pi(10.5)^2 18$$

$$V_1 = 6237 \text{ cm}^3$$

The volume of the Conical portion (V_2) = $\frac{1}{3} \times \frac{22}{7} \times r^2 \times h_2$

$$V_2 = \frac{1}{3} \times \frac{22}{7} \times 10.5^2 \times 9$$

$$V_2 = 1039.5 \text{ cm}^3$$

Therefore, the total volume of the tank (V) = 2 x volume of a conical portion + volume of the Cylindrical portion

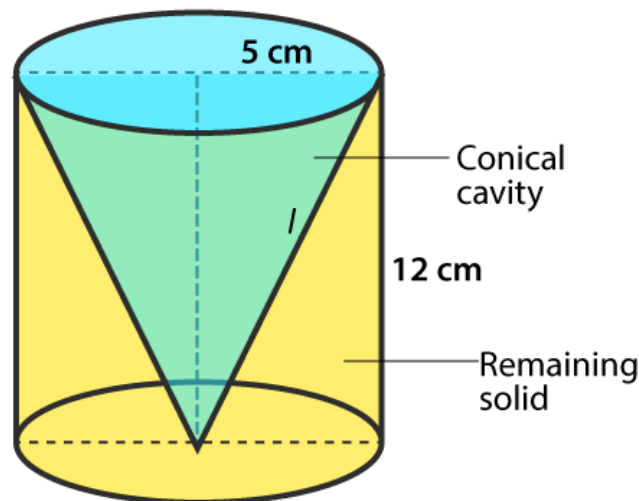
$$V = V_1 + V_2 = 2 \times 1039.5 + 6237$$

$$V = 8316 \text{ cm}^3$$

So, the capacity of the tank = $V = 8316 \text{ cm}^3$

10. A conical hole is drilled in a circular cylinder of height 12 cm and base radius 5 cm. The height and base radius of the cone are also the same. Find the whole surface and volume of the remaining Cylinder.

Solution:



Given,

Height of the circular Cylinder (h_1) = 12 cm

Base radius of the circular Cylinder (r) = 5 cm

Height of the conical hole = Height of the circular cylinder, i.e., $h_1 = h_2 = 12$ cm

And, Base radius of the conical hole = Base radius of the circular Cylinder = 5 cm

Let's consider, L as the slant height of the conical hole.

Then, we know that

$$L = \sqrt{r^2 + h^2}$$

$$L = 5^2 + 12^2$$

$$L = \sqrt{25 + 144}$$

$$L = 13 \text{ cm}$$

Now,

The total surface area of the remaining portion in the circular cylinder (V_1) = $\pi r^2 + 2\pi rh + \pi rl$

$$V_1 = \pi(5)^2 + 2\pi(5)(12) + \pi(5)(13)$$

$$V_1 = 210 \pi \text{ cm}^2$$

And, the volume of the remaining portion of the circular cylinder = Volume of the cylinder – Volume of the conical hole

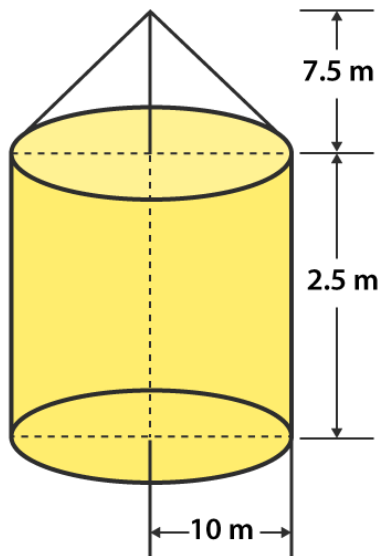
$$V = \pi r^2 h - \frac{1}{3} \times \frac{22}{7} \times r^2 \times h$$

$$V = \pi(5)^2(12) - \frac{1}{3} \times \frac{22}{7} \times 5^2 \times 12$$

$$V = 200 \pi \text{ cm}^2$$

11. A tent is in the form of a cylinder of diameter 20 m and height 2.5 m, surmounted by a cone of equal base and height 7.5 m. Find the capacity of tent and the cost of the canvas at Rs 100 per square meter.

Solution:



Given,

Diameter of the cylinder = 20 m

So, its radius of the cylinder (R) = 10 m

Height of the cylinder (h_1) = 2.5 m

Radius of the cone = Radius of the cylinder (r) = 10 m

Height of the Cone (h_2) = 7.5 m

Let us consider L as the slant height of the Cone, then we know that

$$L^2 = r^2 + h_2^2$$

$$L^2 = 15^2 + 7.5^2$$

$$L^2 = 225 + 56.25$$

$$L^2 = 281.25$$

$$L = 12.5 \text{ m}$$

Now,

$$\text{Volume of the cylinder} = \pi R^2 h_1 = V_1$$

$$V_1 = \pi(10)^2 2.5$$

$$V_1 = 250 \pi \text{ m}^3$$

$$\text{Volume of the Cone} = \frac{1}{3} \times \frac{22}{7} \times r^2 \times h_2^2 = V_2$$

$$V_2 = \frac{1}{3} \times \frac{22}{7} \times 10^2 \times 7.5$$

$$V_2 = 250 \pi \text{ m}^3$$

So, the total capacity of the tent = volume of the cylinder + volume of the cone = $V_1 + V_2$

$$V = 250 \pi + 250 \pi$$

$$V = 500 \pi \text{ m}^3$$

Hence, the total capacity of the tent is $500 \pi \text{ m}^3$

And, the total area of canvas required for the tent is $S = 2 \pi R h_1 + \pi r L$

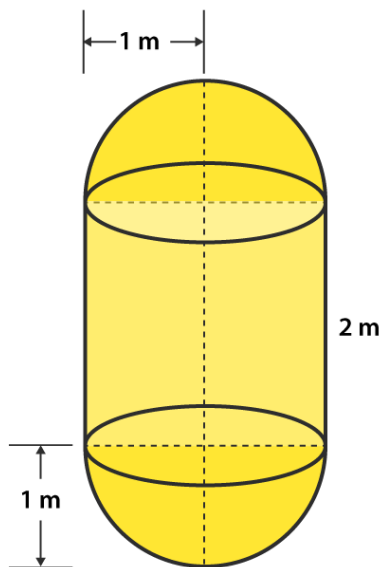
$$S = 2(\pi)(10)(2.5) + \pi(10)(12.5)$$

$$S = 550 \pi \text{ m}^2$$

Therefore, the total cost for canvas is $(100) (550) = \text{Rs. } 55000$

12. A boiler which is in the form of a cylinder 2 m long with hemispherical ends each of 2 m diameter. Find the volume of the boiler.

Solution:



Given,

Diameter of the hemisphere = 2 m

So, the radius of the hemisphere (r) = 1 m

Height of the cylinder (h_1) = 2 m

And, the volume of the Cylinder = $\pi r^2 h_1 = V_1$

$$V_1 = \pi(1)^2 \cdot 2$$

$$V_1 = 22/7 \times 2 = 44/7 \text{ m}^3$$

As at each of the ends of the cylinder, hemispheres are attached.

So, totally there are 2 hemispheres.

Then the volume of two hemispheres = $2 \times 2/3 \times 22/7 \times r^3 = V_2$

$$V_2 = 2 \times 2/3 \times 22/7 \times 1^3$$

$$V_2 = 22/7 \times 4/3 = 88/21 \text{ m}^3$$

Thus,

The volume of the boiler (V) = volume of the cylindrical portion + volume of the two hemispheres

$$V = V_1 + V_2$$

$$V = 44/7 + 88/21$$

$$V = 220/21 \text{ m}^3$$

Therefore, the volume of the boiler $220/21 \text{ m}^3$

13. A vessel is a hollow cylinder fitted with a hemispherical bottom of the same base. The depth of the cylinder is $14/3$ and the diameter of the hemisphere is 3.5 m. Calculate the volume and the internal surface area of the solid.

Solution:

Given,

Diameter of the hemisphere = 3.5 m

So, the radius of the hemisphere (r) = 1.75 m

Height of the cylinder (h) = $14/3$ m

We know that, volume of the Cylinder = $\pi r^2 h_1 = V_1$

$$V_1 = \pi(1.75)^2 \times 14/3 \text{ m}^3$$

The volume of the hemispherical bottom = $2 \times 2/3 \times 22/7 \times r^3 = V_2$

$$V_2 = 2/3 \times 22/7 \times 1.75^3 \text{ m}^3$$

Therefore,

The total volume of the vessel (V) = volume of the cylinder + volume of the hemisphere

$$V = V_1 + V_2$$

$$V = \pi(1.75)^2 \times 14/3 + 2/3 \times 22/7 \times 1.75^3$$

$$V = \pi(1.75)^2 (14/3 + 2/3 \times 1.75)$$

$$V = 56.15 \text{ m}^3$$

Hence, the volume of the vessel = $V = 56.15 \text{ m}^3$

Now,

Internal surface area of solid (S) = Surface area of the cylinder + Surface area of the hemisphere

$$S = 2 \pi r h_1 + 2 \pi r^2$$

$$S = 2 \pi(1.75)(14/3) + 2 \pi(1.75)^2$$

$$S = 70.51 \text{ m}^2$$

Therefore, the internal surface area of the solid is 70.51 m^2

14. A solid is composed of a cylinder with hemispherical ends. If the complete length of the solid is 104 cm and the radius of each of the hemispherical ends is 7 cm, find the cost of polishing its surface at the rate of Rs.10 per dm^2 .

Solution:

Given,

Radius of the hemispherical end (r) = 7 cm

Height of the solid = ($h + 2r$) = 104 cm

$$\Rightarrow h + 2r = 104$$

$$\Rightarrow h = 104 - (2 \times 7)$$

So, $h = 90$ cm

We know that,

The curved surface area of the cylinder (S_1) = $2 \pi r h$

$$S_1 = 2 \pi (7)(90)$$

$$S_1 = 3960 \text{ cm}^2$$

Next,

Curved surface area of the two hemisphere (S_2) = $2 (2\pi r^2)$

$$S_2 = 2 \times 2\pi (7)^2$$

$$S_2 = 616 \text{ cm}^2$$

Therefore,

The total curved surface area of the solid (S) = Curved surface area of the cylinder + Curved surface area of the two hemispheres

$$S = S_1 + S_2$$

$$S = 3960 + 616$$

$$S = 4576 \text{ cm}^2 = 45.76 \text{ dm}^2$$

Given that the cost of polishing the 1 dm^2 surface of the solid is Rs. 10

So, the cost of polishing the 45.76 dm^2 surface of the solid = Rs (10×45.76) = Rs. 457.6

Therefore,

The cost of polishing the whole surface of the solid is Rs. 457.60

15. A cylindrical vessel of diameter 14 cm and height 42 cm is fixed symmetrically inside a similar vessel of diameter 16cm and height of 42 cm. The total space between the two vessels is filled with cork dust for heat insulation purposes. How many cubic cms of the cork dust will be required?

Solution:

Given,

Depth of the cylindrical vessel = Height of the cylindrical vessel = $h = 42$ cm (common for both)

Inner diameter of the cylindrical vessel = 14 cm

So, the inner radius of the cylindrical vessel = $r_1 = 14/2 = 7$ cm

Outer diameter of the cylindrical vessel = 16 cm

So, the outer radius of the cylindrical vessel = $r_2 = 16/2 = 8$ cm

Now,

The volume of the cylindrical vessel

$$V = \pi(r_2^2 - r_1^2) \times h$$

$$= \pi(8^2 - 7^2) \times 42$$

$$= 22/7 \times 15 \times 42$$

$$V = 1980 \text{ cm}^3$$

Therefore, the total space between the two vessels is 1980 cm^3 , which is also the amount of cork dust required.

16. A cylindrical road roller made of iron is 1 m long. Its internal diameter is 54 cm and the thickness of the iron sheet used in making roller is 9 cm. Find the mass of the road roller, if 1

cm^3 of the iron has 7.8 gm mass.

Solution:

Given,

Height/length of the cylindrical road roller = $h = 1 \text{ m} = 100 \text{ cm}$

Internal Diameter of the cylindrical road roller = 54 cm

So, the internal radius of the cylindrical road roller = 27 cm = r

Also given, the thickness of the road roller (T) = 9 cm

Let us assume that the outer radii of the cylindrical road roller be R .

$$T = R - r$$

$$9 = R - 27$$

$$R = 27 + 9$$

$$R = 36 \text{ cm}$$

Now,

The volume of the iron sheet (V) = $\pi \times (R^2 - r^2) \times h$

$$V = \pi \times (36^2 - 27^2) \times 100$$

$$V = 1780.38 \text{ cm}^3$$

Hence, the volume of the iron sheet = 1780.38 cm^3

It's given that, mass of 1 cm^3 of the iron sheet = 7.8 gm

So, the mass of 1780.38 cm^3 of the iron sheet = 1388696.4gm = 1388.7 kg

Therefore, the mass of the road roller is 1388.7 kg

17. A vessel in the form of a hollow hemisphere mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13cm. Find the inner surface area of the vessel.

Solution:

Given,

Diameter of the hemisphere = 14 cm

So, the radius of the hemisphere = 7 cm

Total height of the vessel = 13 cm = $h + r$

Now,

Inner surface area of the vessel = $2 \pi r (h + r)$

$$= 2 (22/7)(7) (13)$$

$$= 572 \text{ cm}^2$$

Therefore, the inner surface area of the vessel is 572 cm^2

18. A toy is in the form of a cone of radius 3.5 cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Solution:

Given,

Radius of the conical portion of the toy = 3.5 cm = r

Total height of the toy = 15.5 cm = H

If H is the length of the conical portion

Then,

Length of the cone (h) = $H - r = 15.5 - 3.5 = 12$ cm

Now, we know that

The curved surface area of the cone (S_1) = πrL , where L is the slant height of the cone.

$$L^2 = r^2 + h^2$$

$$L^2 = 3.5^2 + 12^2$$

$$L^2 = 12.25 + 144 = 156.25$$

$$L = 12.5$$

So,

$$S_1 = \pi (3.5)(12.5)$$

$$S_1 = 137.5 \text{ cm}^2$$

Next, the curved surface area of the hemisphere (S_2) = $2\pi r^2$

$$S_2 = 2\pi (3.5)^2$$

$$S_2 = 77 \text{ cm}^2$$

Therefore,

The total surface area of the toy (S) = Curved surface area of the cone + curved surface area of the hemisphere

$$S = S_1 + S_2$$

$$S = 137.5 + 77$$

$$S = 214.5 \text{ cm}^2$$

Hence, the total surface area of the children's toy is 214.5 cm^2

Exercise 16.3

Page No: 16.60

1. A bucket has top and bottom diameters of 40 cm and 20 cm respectively. Find the volume of the bucket if its depth is 12 cm. Also, find the cost of tin sheet used for making the bucket at the rate of Rs 1.20 per dm^2 .

Solution:

Given,

Diameter to top of bucket = 40 cm

So, the radius (r_1) = $40/2 = 20$ cm

Diameter of bottom part of the bucket = 20 cm

So, the radius (r_2) = $20/2 = 10$ cm

Depth of the bucket (h) = 12 cm

$$\begin{aligned} \text{Volume of the bucket} &= \frac{1}{3} \pi (r_2^2 + r_1^2 + r_1 r_2) h \\ &= \frac{\pi}{3} (20^2 + 10^2 + 20 \times 10) 12 \\ &= 8800 \text{ cm}^3 \end{aligned}$$

Now,

$$\text{Let 'L' be slant height of the bucket} \Rightarrow L = \sqrt{(r_2 - r_1)^2 + h^2}$$

$$\Rightarrow L = \sqrt{(20 - 10)^2 + 12^2}$$

$$\Rightarrow L = 15.620 \text{ cm}$$

$$\text{Total surface area of bucket} = \pi (r_1 + r_2) \times L + \pi \times r_2^2$$

$$= \pi (20 + 10) \times 15.620 + \pi \times 10^2$$

$$= \frac{1320\sqrt{61} + 2200}{7} = 17.87 \text{ dm}^2$$

Given that the cost of tin sheet used for making bucket per $\text{dm}^2 = \text{Rs } 1.20$

So, the total cost for $17.87 \text{ dm}^2 = 1.20 \times 17.87 = \text{Rs } 21.40$

Therefore, the cost of tin sheet used for making the bucket is Rs 21.40

2. A frustum of a right circular cone has a diameter of base 20 cm, of top 12 cm and height 3 cm. Find the area of its whole surface and volume.

Solution:

Given,

Base diameter of cone (d_1) = 20 cm

So the radius (r_1) = $20/2$ cm = 10 cm

Top diameter of Cone (d_2) = 12 cm

So, the radius (r_2) = $12/2$ cm = 6 cm

Height of the cone (h) = 3 cm

$$\text{Volume of the frustum of a right circular cone} = \frac{1}{3} \pi (r_2^2 + r_1^2 + r_1 r_2) h$$

$$= \pi/3(10^2 + 6^2 + 10 \times 6)^3$$

$$= 616 \text{ cm}^3$$

Let 'L' be the slant height of cone, then we know that

$$L = \sqrt{(r_1 - r_2)^2 + h^2}$$

$$L = \sqrt{(10 - 6)^2 + 3^2}$$

$$L = \sqrt{16 + 9}$$

$$L = 5 \text{ cm}$$

So, the slant height of cone (L) = 5 cm

Thus,

$$\begin{aligned} \text{Total surface area of the frustum} &= \pi(r_1 + r_2) \times L + \pi r_1^2 + \pi r_2^2 \\ &= \pi(10 + 6) \times 5 + \pi \times 10^2 + \pi \times 6^2 \\ &= \pi(80 + 100 + 36) \\ &= \pi(216) \\ &= 678.85 \text{ cm}^2 \end{aligned}$$

3. The slant height of the frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface of the frustum.

Solution:

Given,

Slant height of frustum of cone (l) = 4 cm

Let ratio of the top and bottom circles be r_1 and r_2

And given perimeters of its circular ends as 18 cm and 6 cm

$$\Rightarrow 2\pi r_1 = 18 \text{ cm}; 2\pi r_2 = 6 \text{ cm}$$

$$\Rightarrow \pi r_1 = 9 \text{ cm and } \pi r_2 = 3 \text{ cm}$$

We know that,

$$\begin{aligned} \text{Curved surface area of frustum of a cone} &= \pi(r_1 + r_2)l \\ &= \pi(r_1 + r_2)l \\ &= (\pi r_1 + \pi r_2)l = (9 + 3) \times 4 = (12) \times 4 = 48 \text{ cm}^2 \end{aligned}$$

Therefore, the curved surface area of the frustum = 48 cm²

4. The perimeters of the ends of a frustum of a right circular cone are 44 cm and 33 cm. If the height of the frustum be 16 cm, find its volume, the slant surface and the total surface.

Solution:

Given,

Perimeter of the upper end = 44 cm

$$2\pi r_1 = 44$$

$$2(22/7) r_1 = 44$$

$$r_1 = 7 \text{ cm}$$

Perimeter of the lower end = 33 cm

$$2\pi r_2 = 33$$

$$2(22/7) r_2 = 33$$

$$r_2 = 21/4 \text{ cm}$$

Now,

Let the slant height of the frustum of a right circular cone be L

$$L = \sqrt{(r_1 - r_2)^2 + h^2}$$

$$L = \sqrt{(7 - 5.25)^2 + 16^2}$$

$$L = 16.1 \text{ cm}$$

$$\begin{aligned} \text{So, the curved surface area of the frustum cone} &= \pi(r_1 + r_2)l \\ &= \pi(7 + 5.25)16.1 \end{aligned}$$

$$\text{Curved surface area of the frustum cone} = 619.65 \text{ cm}^3$$

Next,

$$\begin{aligned} \text{The volume of the frustum cone} &= \frac{1}{3} \pi(r_2^2 + r_1^2 + r_1 r_2)h \\ &= \frac{1}{3} \pi(7^2 + 5.25^2 + (7)(5.25)) \times 16 \\ &= 1898.56 \text{ cm}^3 \end{aligned}$$

$$\text{Thus, volume of the cone} = 1898.56 \text{ cm}^3$$

Finally, the total surface area of the frustum cone

$$\begin{aligned} &= \pi(r_1 + r_2) \times L + \pi r_1^2 + \pi r_2^2 \\ &= \pi(7 + 5.25) \times 16.1 + \pi 7^2 + \pi 5.25^2 \\ &= \pi(7 + 5.25) \times 16.1 + \pi(7^2 + 5.25^2) = 860.27 \text{ cm}^2 \end{aligned}$$

Therefore, the total surface area of the frustum cone is 860.27 cm^2

5. If the radii of the circular ends of a conical bucket which is 45 cm high be 28 cm and 7 cm, find the capacity of the bucket.

Solution:

Given,

Height of the conical bucket = 45 cm

Radii of the 2 circular ends of the conical bucket are 28 cm and 7 cm

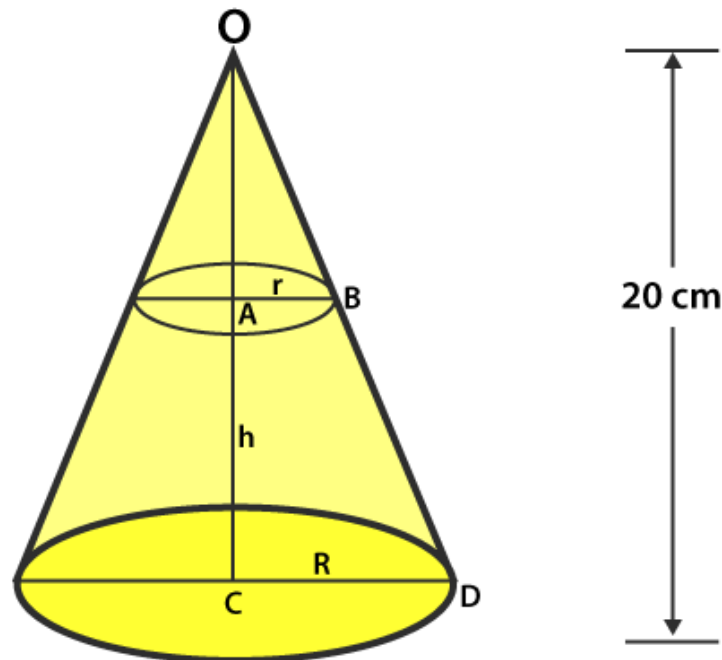
So, $r_1 = 28 \text{ cm}$ $r_2 = 7 \text{ cm}$

$$\begin{aligned} \text{Volume of the conical bucket} &= \frac{1}{3} \pi(r_1^2 + r_2^2 + r_1 r_2)h \\ &= \frac{1}{3} \pi(28^2 + 7^2 + 28 \times 7)45 = 15435\pi \end{aligned}$$

Therefore, the volume/ capacity of the bucket is 48510 cm^3 .

6. The height of a cone is 20 cm. A small cone is cut off from the top by a plane parallel to the base. If its volume be $\frac{1}{125}$ of the volume of the original cone, determine at what height above the base the section is made.

Solution:



Let the radius of the small cone be r cm
 Radius of the big cone = R cm
 Given, height of the big cone = 20 cm
 Let the height of section made = h cm
 Then, the height of small cone will be = $(20 - h)$ cm
 Now,

In $\triangle OAB$ and $\triangle OCD$

$$\angle AOB = \angle COD \quad [\text{common}]$$

$$\angle OAB = \angle OCD \quad [\text{each } 90^\circ]$$

Then, $\triangle OAB \sim \triangle OCD$ [by AA similarity]

So, by C.P.S.T we have

$$OA/OC = AB/CD$$

$$(20 - h)/20 = r/R \dots\dots (i)$$

Also given,

Volume of small cone = $1/125$ x volume of big cone

$$1/3 \pi r^2(20 - h) = 1/125 \times 1/3 \pi R^2 \times 20$$

$$r^2/R^2 = 1/125 \times 20/(20 - h) \quad [\text{From (i)}]$$

$$(20 - h)^2/20^2 = 1/125 \times 20/20 - h$$

$$(20 - h)^3 = 20^3/125$$

$$20 - h = 20/5$$

$$20 - h = 4$$

$$h = 20 - 4 = 16 \text{ cm}$$

Therefore, it's found that the section was made at a height of 16 cm above the base.

7. If the radii of the circular ends of a bucket 24 cm high are 5 and 15 cm respectively, find the surface area of the bucket.

Solution:

Given,

Height of the bucket (h) = 24 cm

Radius of the circular ends of the bucket 5 cm and 15 cm

So, $r_1 = 5$ cm; $r_2 = 15$ cm

Let 'L' be the slant height of the bucket

Then, we know that

$$L = \sqrt{(r_1 - r_2)^2 + h^2}$$

$$\Rightarrow L = \sqrt{(5 - 15)^2 + 24^2}$$

$$\Rightarrow L = \sqrt{(100 + 576)}$$

$$\Rightarrow L = 26 \text{ cm}$$

Now,

Curved surface area of the bucket

$$= \pi(r_1 + r_2)l + \pi r_1^2$$

$$= \pi(5 + 15)26 + \pi 5^2 = \pi(520 + 25) = 545\pi \text{ cm}^2$$

Therefore, the curved surface area of the bucket = $545\pi \text{ cm}^2$

8. The radii of circular bases of a frustum of a right circular cone are 12 cm and 3 cm and the height is 12 cm. Find the total surface area and volume of frustum.

Solution:

Given the height of frustum cone = 12 cm

Radii of a frustum cone are 12 cm and 3 cm

So, $r_1 = 12$ cm; $r_2 = 3$ cm

Let slant height of the frustum cone be 'L'

Then, we know that

$$L = \sqrt{(r_1 - r_2)^2 + h^2}$$

$$L = \sqrt{81 + 144} = 15 \text{ cm}$$

$$L = 15 \text{ cm}$$

$$\begin{aligned} \text{Now, the total surface area of frustum of a cone} &= \pi(r_1 + r_2) \times L + \pi r_1^2 + \pi r_2^2 \\ &= \pi(12 + 3)15 + \pi 12^2 + \pi 3^2 \end{aligned}$$

Thus, total surface area of the frustum = $378\pi \text{ cm}^2$

Next,

Volume of frustum cone = $\frac{1}{3} \pi(r_2^2 + r_1^2 + r_1 r_2)h$

$$= \frac{1}{3} \pi(12^2 + 3^2 + 12 \times 3) \times 12 = 756\pi \text{ cm}^3$$

Therefore, the volume of the frustum cone = $756 \pi \text{ cm}^3$

9. A tent consists of a frustum of a cone capped by a cone. If radii of the ends of the frustum be 13 m and 7 m, the height of frustum be 8 m and the slant height of the conical cap be 12 m, find the canvas required for the tent.

Solution:

Given,

Height of frustum (h) = 8 m

Radii of the frustum cone are 13 cm and 7 cm

So, $r_1 = 13 \text{ cm}$ and $r_2 = 7 \text{ cm}$

Let 'L' be slant height of the frustum cone

Then, we know that

$$\Rightarrow L = \sqrt{(r_1 - r_2)^2 + h^2}$$

$$\Rightarrow L = \sqrt{(13 - 7)^2 + 8^2}$$

$$\Rightarrow L = \sqrt{36 + 64}$$

$$\Rightarrow L = 10 \text{ m}$$

Curved surface area of the frustum (s_1) = $\pi(r_1 + r_2) \times L = \pi(13 + 7) \times 10 = 200 \pi \text{ m}^2$

Then, given slant height of conical cap = 12 m

Base radius of upper cap cone = 7 m

So, the curved surface area of upper cap cone (s_2) = $\pi r l = \pi \times 7 \times 12 = 264 \text{ m}^2$

Thus, the total canvas required for tent (S) = $s_1 + s_2$

$$S = 200\pi + 264 = 892.57 \text{ m}^2$$

Therefore, the canvas required for the tent is 892.57 m^2 .

10. A milk container of height 16 cm is made of metal sheet in the form of frustum of a cone with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk at the rate of Rs.44 per litre which the container can hold.

Solution:

Given,

A milk container of the form of frustum of a cone

The radius of the lower end (r_1) = 8 cm

The radius of the upper end (r_2) = 20 cm

Let h be its height, $h = 16 \text{ cm}$

Then, the capacity of the container = Volume of frustum of the cone

$$= \frac{1}{3} \pi(r_2^2 + r_1^2 + r_1 r_2)h$$

$$= \frac{1}{3} \pi(20^2 + 8^2 + (20)(8)) \times 16$$

$$= 10459.42 \text{ cm}^3$$

$$= 10.46 \text{ litres}$$

Now, given that cost of 1 litre of milk = Rs 44

Then the cost of 10.46 litres of milk = Rs (44 x 10.46) = Rs 460.24

11. A bucket is in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the capacity and surface area of the bucket. Also, find the cost of milk which can completely fill the container, at the rate of Rs.25 per litre.

Solution:

Let R and r be the radii of the top and base of the bucket respectively,
Let h be its height.

Then, we have R = 20 cm, r = 10 cm, h = 30 cm

$$\begin{aligned}\text{Capacity of the bucket} &= \text{Volume of the frustum of the cone} \\ &= \frac{1}{3} \pi (R^2 + r^2 + Rr)h \\ &= \frac{1}{3} \pi (20^2 + 10^2 + 20 \times 10) \times 30 \\ &= 3.14 \times 10 (400 + 100 + 200) \\ &= 21980 \text{ cm}^3 = 21.98 \text{ litres}\end{aligned}$$

Now,

$$\begin{aligned}\text{Surface area of the bucket} &= \text{CSA of the bucket} + \text{Surface area of the bottom} \\ &= \pi l (R + r) + \pi r^2\end{aligned}$$

We know that,

$$\begin{aligned}l &= \sqrt{h^2 + (R - r)^2} \\ &= \sqrt{[30^2 + (20 - 10)^2]} = \sqrt{(900 + 100)} \\ &= \sqrt{1000} = 31.62 \text{ cm}\end{aligned}$$

So,

$$\begin{aligned}\text{The Surface area of the bucket} &= (3.14) \times 31.62 \times (20 + 10) + (3.14) \times 10^2 \\ &= 2978.60 + 314 \\ &= 3292.60 \text{ cm}^2\end{aligned}$$

Next, given that the cost of 1 litre milk = Rs 25

Thus, the cost of 21.98 litres of milk = Rs (25 x 21.98) = Rs 549.50