EXERCISE 1A

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Very-Short-Answer Questions

1. Find the domain and range of the relation
R = {(-1, 1), (1, 1), (-2, 4), (2, 4)}.
Solution:

The domain of the relation
dom (R) = {-1, 1, -2, 2}

The range of the relation
range (R) = {1, 4}

2. Let R = {(a, a^3): a is a prime number less than 5}.
Find the range of R.
Solution:

By substituting a as 2 and 3 we get
R = {(2, 8), (3, 27)
We know that range of R
range (R) = {8, 27}

3. Let R = {(a, a^3): a is a prime number less than 10}.
Find (i) R (ii) dom (R) (iii) range (R).
Solution:

(i) R = {(2, 8), (3, 27), (5, 125), (7, 343)}

(ii) dom (R) = {2, 3, 5, 7}

(iii) range (R) = {8, 27, 125, 343}

4. Let R = {x, y): x + 2y = 8} be a relation on N.
Write the range of R.
Solution:

By substituting the values of x and y we get
R = {(2, 3), (4, 2), (6, 1)}
The range of R for the relation on N
range (R) = {3, 2, 1}

5. Let R = {(a, b): a, b ∈ N and a + 3b = 12}.
Find the domain and range of R.
Solution:

By substituting values of a and b we get
R = {(3, 3), (6, 2), (9, 1)}
So we get
dom (R) = {3, 6, 9}
range (R) = {3, 2, 1}

6. Let R = {(a, b): b = |a – 1|, a ∈ Z and |a| < 3}.
Find the domain and range of R.
Solution:

We know that a is an integer where – 3 < a < 3
So we get
R = {(-2, 3), (-1, 2), (0, 1), (1, 0), (2, 1)}
dom (R) = {-2, -1, 0, 1, 2}
range (R) = {3, 2, 1, 0}

7. Let R = {(a, 1/a): a ∈ N and 1 < a < 5}.
Find the domain and range of R.
Solution:

By substituting values we get
R = {(2, 1/2), (3, 1/3), (4, 1/4)}
So we get
dom (R) = {2, 3, 4}
range (R) = {1/2, 1/3, 1/4}

8. Let R = {(a, b): a, b ∈ N and b = a + 5, a < 4}.
Find the domain and range of R.
Solution:

By substituting values we get
R = {(1, 6), (2, 7), (3, 8)}
So we get
dom (R) = {1, 2, 3}
range (R) = {6, 7, 8}

9. Let S be the set of all sets and let R = {(A, B): A ⊂ B)}, i.e. A is a proper subset of B. Show that R is (i) transitive (ii) not reflexive (iii) not symmetric.
Solution:

(i) Transitive
Consider A, B and C ∈ S, where (A, B) and (B, C) ∈ R
We get
(A, B) ∈ R => A ⊂ B ……. (1)
(B, C) ∈ R => B ⊂ C …….. (2)
Using both the equations we get
A ⊂ C => (A, C) ∈ R
Hence, R is a transitive relation S.

(ii) Non reflexive
We know that
A ⊄ A where (A, A) ∈ R
Hence, R is non reflexive.

(iii) Non symmetric
Consider A ⊂ B where (A, B) ∈ R

We know that B ⊄ A
So (B, A) ∉ R
We get (A, B) ∈ R and (B, A) ∉ R
Hence, R is non symmetric.

10. Let A be the set of all points in a plane and let O be the origin. Show that the relation R = {(P, Q): P, Q ∈ A and OP = OQ} is an equivalence relation.
Solution:

Consider O as the origin of the plane
So R = {(P, Q): OP = OQ)
By considering properties of relation R

Symmetric:
Consider P and Q as the two points in set A where (P, Q) ∈ R
We can write it as
OP = OQ where (Q, P) ∈ R
So we get (P, Q) ∈ R and (Q, P) ∈ R for P, Q ∈ A
Hence, R is symmetric.

Reflexivity:
Consider P as any point in set A where OP = OP
We know that (P, P) ∈ R for all P ∈ A
Hence, R is reflexive.

Transitivity:
Consider P, Q and S as three points in a set A where (P, Q) ∈ R and (Q, S) ∈ R
We know that OP = OQ and OQ = OS
So we get OP = OS where (P, S) ∈ R
Hence, R is transitive
Therefore, R is an equivalence relation.

Consider P as a fixed point in set A and let Q be a point in set A where (P, Q) ∈ R
We know that OP = OQ where Q moves in the plane that its distance from O.
So we get O (0, 0) = OP
So the locus of Q is a circle having centre at O and OP as the radius
Therefore, the set of all points which is related to P passes through the point P having O as centre.

11. On the set S of all real numbers, define a relation R = {(a, b): a ≤ b}.
Show that R is (i) reflexive, (ii) transitive (iii) not symmetric.
Solution:

(i) Reflexivity
Consider a as an arbitrary element on the set S
So we get a ≤ a where (a, a) ∈ R
Hence, R is reflective.

(ii) Transitivity
Consider a, b and c ∈ S where (a, b) and (b, c) ∈ S
We get

(a, b) ∈ R => $a \leq b$ and (b, c) ∈ R => $b \leq c$
Based on the above equation we get
(a, c) ∈ R => $a \leq c$
Hence, R is transitive.

(iii) Non symmetry
We know that
(5, 6) ∈ R => $5 \leq 6$
In the same way
(6, 5) ∈ R => $6 \nleq 5$
Hence, R is non symmetric.

12. Let A = {1, 2, 3, 4, 5, 6} and let R = {(a, b): a, b ∈ A and b = a + 1}. Show that R is (i) not reflexive, (ii) not symmetric and (iii) not transitive.
Solution:

By using roster form
R = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6)}

(i) Non reflexive
We know that
(1, 1) ∈ R where 1 ∈ A
Hence, R is non reflexive.

(ii) Non symmetric
We know that
(1, 2) ∈ R but (2, 1) ∉ R
Hence, R is non symmetric.

(iii) Non transitive
We know that
(1, 2) ∈ R and (2, 3) ∈ R
In the same way (1, 3) ∉ R
Hence, R is non transitive.