

### Exercise 4(A)

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**1. State, true or false:**

- (i)  $x < -y \Rightarrow -x > y$   
 (ii)  $-5x \geq 15 \Rightarrow x \geq -3$   
 (iii)  $2x \leq -7 \Rightarrow 2x/-4 \geq -7/-4$   
 (iv)  $7 > 4 \Rightarrow 1/7 < 1/5$

**Solution:**

- (i) Given statement is true. (according to Rule 5)  
 (ii) Given statement is false. (according to Rule 4)  
 (iii) Given statement is true. (according to Rule 4)  
 (iv) Given statement is true. (according to Rule 6)

**2. State whether the following statements are true or false.**

- (i)  $a < b$ , then  $a - c < b - c$   
 (ii) If  $a > b$ , then  $a + c > b + c$   
 (iii) If  $a < b$ , then  $ac > bc$   
 (iv) If  $a > b$ , then  $a/c < b/c$   
 (v) If  $a - c > b - d$ , then  $a + d > b + c$   
 (vi) If  $a < b$ , and  $c > 0$ , then  $a - c > b - c$

Where  $a, b, c$  and  $d$  are real numbers and  $c \neq 0$ .

**Solution:**

- (i) Given statement is true. (Subtracting equals on both sides will not change the inequality)  
 (ii) Given statement is true. (Adding equals on both sides will not change the inequality)  
 (iii) Given statement is false. (According to rule 3)  
 (iv) Given statement is false. (According to rule 3)  
 (v) Given statement is true. As  $a - c > b - d \Rightarrow a + d > b + c$   
 (vi) Given statement is false. As  $a < b$ ,  $a - c < b - c$  (since  $c > 0$ )

**3. If  $x \in \mathbb{N}$ , find the solution set of inequations.**

- (i)  $5x + 3 \leq 2x + 18$   
 (ii)  $3x - 2 < 19 - 4x$

**Solution:**

(i)  $5x + 3 \leq 2x + 18$   
 $5x - 2x \leq 18 - 3$   
 $3x \leq 15$   
 $x \leq 5$

As,  $x \in \mathbb{N}$ , thus the solution set is  $\{1, 2, 3, 4, 5\}$ .

(ii)  $3x - 2 < 19 - 4x$   
 $3x + 4x < 19 + 2$   
 $7x < 21$   
 $x < 3$

As,  $x \in \mathbb{N}$ , thus the solution set is  $\{1, 2\}$ .

**4. If the replacement set is the set of whole numbers, solve:**

(i)  $x + 7 \leq 11$

(ii)  $3x - 1 > 8$

(iii)  $8 - x > 5$

(iv)  $7 - 3x \geq -1/2$

(v)  $x - 3/2 < 3/2 - x$

(vi)  $18 \leq 3x - 2$

**Solution:**

(i)  $x + 7 \leq 11$

$$x \leq 11 - 7$$

$$x \leq 4$$

As the replacement set =  $W$  (set of whole numbers)

Therefore, the solution set =  $\{0, 1, 2, 3, 4\}$

(ii)  $3x - 1 > 8$

$$3x > 8 + 1$$

$$x > 3$$

As the replacement set =  $W$  (set of whole numbers)

Therefore, the solution set =  $\{4, 5, 6, \dots\}$

(iii)  $8 - x > 5$

$$-x > 5 - 8$$

$$-x > -3$$

$$x < 3$$

As the replacement set =  $W$  (set of whole numbers)

Therefore, the solution set =  $\{0, 1, 2\}$

(iv)  $7 - 3x \geq -1/2$

$$-3x \geq -1/2 - 7$$

$$-3x \geq -15/2$$

$$x \geq 5/2$$

As the replacement set =  $W$  (set of whole numbers)

Therefore, the solution set =  $\{0, 1, 2\}$

(v)  $x - 3/2 < 3/2 - x$

$$x + x < \frac{3}{2} + \frac{3}{2}$$

$$2x < 3$$

$$x < \frac{3}{2}$$

As the replacement set =  $W$  (set of whole numbers)

Therefore, the solution set =  $\{0, 1\}$

- (vi)  $18 \leq 3x - 2$   
 $18 + 2 \leq 3x$   
 $20 \leq 3x$   
 $x \geq 20/3$   
As the replacement set = W (set of whole numbers)  
Therefore, the solution set =  $\{7, 8, 9, \dots\}$

**5. Solve the inequation:** **$3 - 2x \geq x - 12$  given that  $x \in \mathbb{N}$ .****Solution:**

$$3 - 2x \geq x - 12$$
$$-2x - x \geq -12 - 3$$
$$-3x \geq -15$$
$$x \leq 5$$

As,  $x \in \mathbb{N}$ ,Thus, the solution set =  $\{1, 2, 3, 4, 5\}$ **6. If  $25 - 4x \leq 16$ , find:****(i) the smallest value of  $x$ , when  $x$  is a real number,****(ii) the smallest value of  $x$ , when  $x$  is an integer.****Solution:**

$$25 - 4x \leq 16$$
$$-4x \leq 16 - 25$$
$$-4x \leq -9$$
$$x \geq 9/4$$
$$x \geq 2.25$$

Now,

(i) The smallest value of  $x$ , when  $x$  is a real number is 2.25.(ii) The smallest value of  $x$ , when  $x$  is an integer is 3.

**Exercise 4(B)**

1. Represent the following inequalities on real number line:

- (i)  $2x - 1 < 5$                       (ii)  $3x + 1 \geq -5$   
 (iii)  $2(2x - 3) \leq 6$               (iv)  $-4 < x < 4$   
 (v)  $-2 \leq x < 5$                     (vi)  $8 \geq x > -3$   
 (vii)  $-5 < x \leq -1$

**Solution:**

- (i)  $2x - 1 < 5$   
 $2x < 6$   
 $x < 3$

Solution on the number line is as below



- (ii)  $3x + 1 \geq -5$   
 $3x \geq -5 - 1$   
 $3x \geq -6$   
 $x \geq -2$

Solution on the number line is as below



- (iii)  $2(2x - 3) \leq 6$   
 $4x - 6 \leq 6$   
 $4x \leq 12$   
 $x \leq 3$

Solution on the number line is as below



- (iv)  $-4 < x < 4$

Solution on the number line is as below

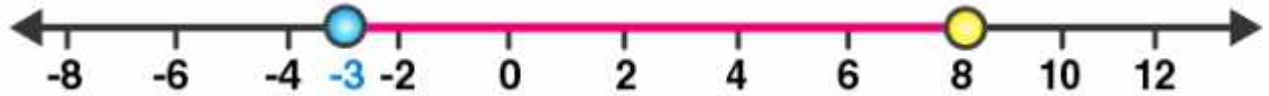


- (v)  $-2 \leq x < 5$

Solution on the number line is as below



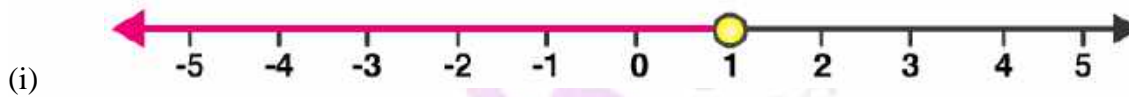
- (vi)  $8 \geq x > -3$   
Solution on the number line is as below



- (vii)  $-5 < x \leq -1$   
Solution on the number line is as below



2. For each graph given alongside, write an inequation taking  $x$  as the variable:  
Solution:



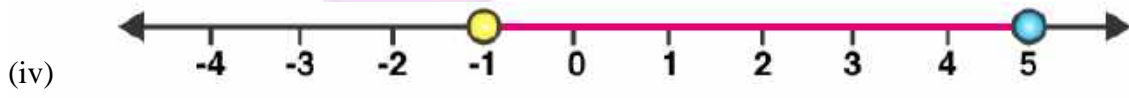
$x \leq -1, x \in \mathbb{R}$



$x \geq 2, x \in \mathbb{R}$



$-4 \leq x < 3, x \in \mathbb{R}$



$-1 < x \leq 5, x \in \mathbb{R}$

3. For the following inequation, graph the solution set on the real number line:

(i)  $-4 \leq 3x - 1 < 8$

(ii)  $x - 1 < 3 - x \leq 5$

Solution:

- (i)  $-4 \leq 3x - 1 < 8$   
 $-4 \leq 3x - 1$  and  $3x - 1 < 8$   
 $-3 \leq 3x$  and  $3x < 9$   
 $-1 \leq x$  and  $x < 3$

The solution on the number is as below



- (ii)  $x - 1 < 3 - x \leq 5$   
 $x - 1 < 3 - x$  and  $3 - x \leq 5$   
 $2x < 4$  and  $-x \leq 2$   
 $x < 2$  and  $x \geq -2$   
 The solution on the number is as below

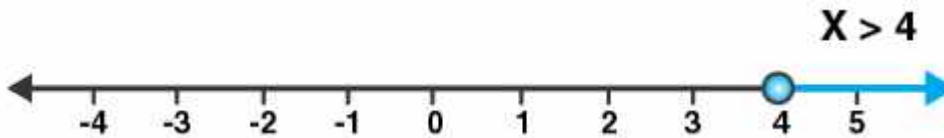


4. Represent the solution of each of the following inequations on the real number line:

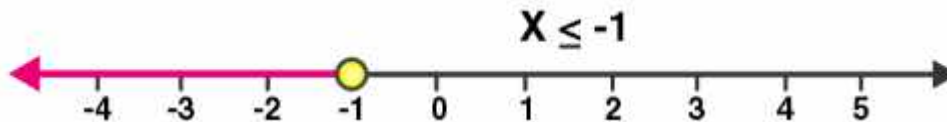
- (i)  $4x - 1 > x + 11$   
 (ii)  $7 - x \leq 2 - 6x$   
 (iii)  $x + 3 \leq 2x + 9$   
 (iv)  $2 - 3x > 7 - 5x$   
 (v)  $1 + x \geq 5x - 11$   
 (vi)  $(2x + 5)/3 > 3x - 3$

**Solution:**

- (i)  $4x - 1 > x + 11$   
 $4x - x > 1 + 11$   
 $3x > 12$   
 $x > 4$   
 The solution on number line is as below



- (ii)  $7 - x \leq 2 - 6x$   
 $6x - x \leq 2 - 7$   
 $5x \leq -5$   
 $x \leq -1$   
 The solution on number line is as below

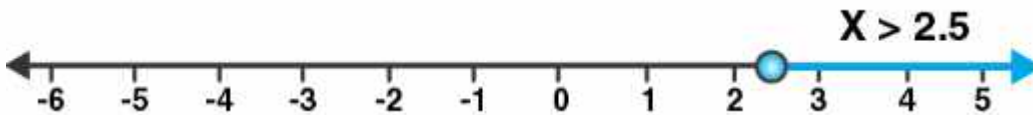


- (iii)  $x + 3 \leq 2x + 9$   
 $x - 2x \leq -3 + 9$   
 $-x \leq 6$   
 $x \geq -6$   
 The solution on number line is as below



(iv)  $2 - 3x > 7 - 5x$   
 $5x - 3x > 7 - 2$   
 $2x > 5$   
 $x > 5/2$   
 $x > 2.5$

The solution on number line is as below



(v)  $1 + x \geq 5x - 11$   
 $12 \geq 4x$   
 $x \leq 3$

The solution on number line is as below



(vi)  $(2x + 5)/3 > 3x - 3$   
 $2x + 5 > 3(3x - 3)$   
 $2x + 5 > 9x - 9$   
 $9 + 5 > 9x - 2x$   
 $7x < 14$   
 $x < 2$

The solution on number line is as below



5.  $x \in \{\text{real numbers}\}$  and  $-1 < 3 - 2x \leq 7$ , evaluate  $x$  and represent it on a number line.

**Solution:**

$$-1 < 3 - 2x \leq 7$$

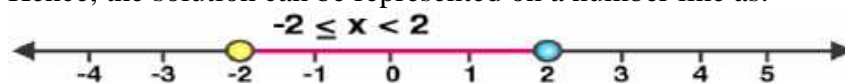
$$-1 < 3 - 2x \quad \text{and} \quad 3 - 2x \leq 7$$

$$2x < 4 \quad \text{and} \quad -2x \leq 4$$

$$x < 2 \quad \text{and} \quad x \geq -2$$

The solution set =  $\{-2 \leq x < 2, x \in \mathbb{R}\}$

Hence, the solution can be represented on a number line as:



6. List the elements of the solution set of the inequation

$$-3 < x - 2 \leq 9 - 2x; x \in \mathbb{N}.$$

**Solution:**

$$-3 < x - 2 \leq 9 - 2x$$

$$-3 < x - 2 \text{ and } x - 2 \leq 9 - 2x$$

$$-1 < x \text{ and } 3x \leq 11$$

$$-1 < x \leq 11/3$$

As,  $x \in \mathbb{N}$

Hence, the solution set =  $\{1, 2, 3\}$

7. Find the range of values of  $x$  which satisfies

$$-2 \frac{2}{3} \leq x + \frac{1}{3} < 3 \frac{1}{3}; x \in \mathbb{R}.$$

**Graph these values of  $x$  on the number line.**

**Solution:**

$$-2 \frac{2}{3} \leq x + \frac{1}{3} \text{ and } x + \frac{1}{3} < 3 \frac{1}{3}$$

$$\Rightarrow -\frac{8}{3} \leq x + \frac{1}{3} \text{ and } x + \frac{1}{3} < \frac{10}{3}$$

$$\Rightarrow -\frac{8}{3} - \frac{1}{3} \leq x \text{ and } x < \frac{10}{3} - \frac{1}{3}$$

$$\Rightarrow -\frac{9}{3} \leq x \text{ and } x < \frac{9}{3}$$

$$\Rightarrow -3 \leq x \text{ and } x < 3$$

Therefore,  $-3 \leq x < 3$

And the required graph of the solution set is as below:



8. Find the values of  $x$ , which satisfy the inequation:

$$-2 \leq \frac{1}{2} - \frac{2x}{3} \leq 1 \frac{5}{6}, x \in \mathbb{N}.$$

**Graph the solution on the number line.**

**Solution:**

Given inequation,



$$\begin{aligned}
 -2 &\leq \frac{1}{2} - \frac{2x}{3} \leq 1\frac{5}{6}, \quad x \in \mathbb{N} \\
 \Rightarrow -2 - \frac{1}{2} &\leq \frac{1}{2} - \frac{2x}{3} - \frac{1}{2} \leq \frac{11}{6} - \frac{1}{2} \\
 \Rightarrow -\frac{5}{2} &\leq -\frac{2x}{3} \leq \frac{8}{6} \\
 \Rightarrow -\frac{5}{2} &\leq -\frac{2x}{3} \quad \text{and} \quad -\frac{2x}{3} \leq \frac{8}{6} \\
 \Rightarrow -15 &\leq -4x \quad \text{and} \quad -2x \leq 4 \\
 \Rightarrow 15 &\geq 4x \quad \text{and} \quad 2x \geq -4 \\
 \Rightarrow \frac{15}{4} &\geq x \quad \text{and} \quad x \geq -2 \\
 \Rightarrow 3.75 &\geq x \quad \text{and} \quad x \geq -2
 \end{aligned}$$

Hence, the solution set is  $\{x \in \mathbb{N} : -2 \leq x \leq 3.75\}$

And as  $x \in \mathbb{N}$ , the values of  $x$  are 1, 2, 3

The required graph of the solution on the number line is as below:



9. Given  $x \in \{\text{real numbers}\}$ , find the range of values of  $x$  for which  $-5 \leq 2x - 3 < x + 2$  and represent it on a real number line.

**Solution:**

Given inequation,

$$\begin{aligned}
 -5 &\leq 2x - 3 < x + 2 \\
 -5 &\leq 2x - 3 \quad \text{and} \quad 2x - 3 < x + 2 \\
 -2 &\leq 2x \quad \text{and} \quad x < 5 \\
 -1 &\leq x \quad \text{and} \quad x < 5
 \end{aligned}$$

Thus, the required range is  $-1 \leq x < 5$ .

And the required graph is as below:



10. If  $5x - 3 \leq 5 + 3x \leq 4x + 2$ , express it as  $a \leq x \leq b$  and then state the values of  $a$  and  $b$ .

**Solution:**

Given inequation,

$$\begin{aligned}
 5x - 3 &\leq 5 + 3x \leq 4x + 2 \\
 5x - 3 &\leq 5 + 3x \quad \text{and} \quad 5 + 3x \leq 4x + 2 \\
 2x &\leq 8 \quad \text{and} \quad -x \leq -3 \\
 x &\leq 4 \quad \text{and} \quad x \leq 3
 \end{aligned}$$

Hence,  $3 \leq x \leq 4$ .

Therefore, we have  $a = 3$  and  $b = 4$ .

**11. Solve the following inequation and graph the solution set on the number line:**

$$2x - 3 < x + 2 \leq 3x + 5, x \in \mathbf{R}.$$

**Solution:**

Given inequation,

$$2x - 3 < x + 2 \leq 3x + 5$$

$$2x - 3 < x + 2 \quad \text{and} \quad x + 2 \leq 3x + 5$$

$$x < 5 \quad \text{and} \quad -3 \leq 2x$$

$$x < 5 \quad \text{and} \quad -1.5 \leq x$$

So, the solution set =  $\{-1.5 \leq x < 5\}$

And the solution set be graphed on the number line as below:



**12. Solve and graph the solution set of:**

(i)  $2x - 9 < 7$  and  $3x + 9 \leq 25, x \in \mathbf{R}$

(ii)  $2x - 9 \leq 7$  and  $3x + 9 > 25, x \in \mathbf{I}$

(iii)  $x + 5 \geq 4(x - 1)$  and  $3 - 2x < -7, x \in \mathbf{R}$

**Solution:**

(i)  $2x - 9 < 7$  and  $3x + 9 \leq 25$

$$2x < 16 \quad \text{and} \quad 3x \leq 16$$

$$x < 8 \quad \text{and} \quad x \leq 16/3$$

Thus, the solution set =  $\{x \leq 16/3, x \in \mathbf{R}\}$

And the required graph on number line is as below:



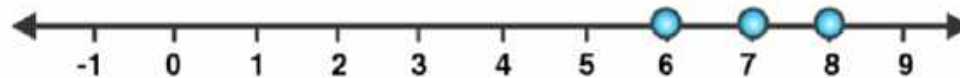
(ii)  $2x - 9 \leq 7$  and  $3x + 9 > 25$

$$2x \leq 16 \quad \text{and} \quad 3x > 16$$

$$x \leq 8 \quad \text{and} \quad x > 16/3$$

Thus, the solution set =  $\{16/3 < x \leq 8, x \in \mathbf{I}\} = \{6, 7, 8\}$

And the required graph on number line is as below:



(iii)  $x + 5 \geq 4(x - 1)$  and  $3 - 2x < -7$

$$9 \geq 3x \quad \text{and} \quad -2x < -10$$

$$3 \geq x \quad \text{and} \quad x > 5$$

Hence, the solution set = Empty set (as there is no cross section)

**13. Solve and graph the solution set of:**

(i)  $3x - 2 > 19$  or  $3 - 2x \geq -7, x \in \mathbf{R}$

(ii)  $5 > p - 1 > 2$  or  $7 \leq 2p - 1 \leq 17$ ,  $p \in \mathbb{R}$

**Solution:**

$$\begin{aligned} \text{(i)} \quad 3x - 2 > 19 & \quad \text{or} \quad 3 - 2x \geq -7 \\ 3x > 21 & \quad \text{or} \quad -2x \geq -10 \\ x > 7 & \quad \text{or} \quad x \leq 5 \end{aligned}$$

The graph of solution set of  $x > 7$  or  $x \leq 5$  is equal to the graph of points which belong to  $x > 7$  or  $x \leq 5$  or both.

Thus, the graph of the solution set is as below:



$$\begin{aligned} \text{(ii)} \quad 5 > p - 1 > 2 & \quad \text{or} \quad 7 \leq 2p - 1 \leq 17 \\ 6 > p > 3 & \quad \text{or} \quad 8 \leq 2p \leq 18 \\ 6 > p > 3 & \quad \text{or} \quad 4 \leq p \leq 9 \end{aligned}$$

Now, we have to understand that:

Graph of solution set of  $6 > p > 3$  or  $4 \leq p \leq 9$

= Graph of points which belong to  $6 > p > 3$  or  $4 \leq p \leq 9$  or both

= Graph of points which belong to  $3 < p \leq 9$

Thus, the graph of the solution set is as below:



14. The diagram represents two inequations A and B on real number lines:



(i) Write down A and B in set builder notation.

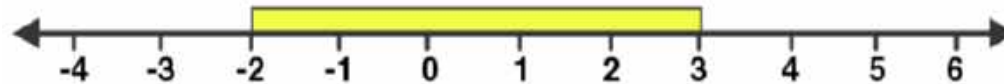
(ii) Represent  $A \cap B$  and  $A \cap B'$  on two different number lines.

**Solution:**

$$\begin{aligned} \text{(i)} \quad A &= \{x \in \mathbb{R} : -2 \leq x < 5\} \\ B &= \{x \in \mathbb{R} : -4 \leq x < 3\} \end{aligned}$$

$$\text{(ii)} \quad A \cap B = \{x \in \mathbb{R} : -2 \leq x < 3\}$$

And this can be represented on number line as:



$$B' = \{x \in \mathbb{R} : 3 < x \leq -4\}$$

$$A \cap B' = \{x \in \mathbb{R} : 3 \leq x < 5\}$$

And this can be represented on number line as:



15. Use real number line to find the range of values of  $x$  for which:

(i)  $x > 3$  and  $0 < x < 6$

(ii)  $x < 0$  and  $-3 \leq x < 1$

(iii)  $-1 < x \leq 6$  and  $-2 \leq x \leq 3$

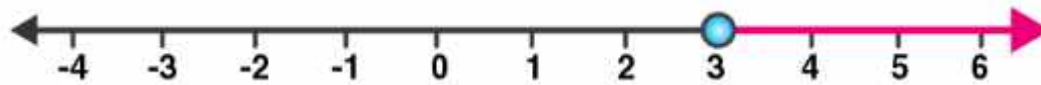
**Solution:**

(i)  $x > 3$  and  $0 < x < 6$

Both the given inequations are true in the range where their graphs on the real number lines overlap.

And, the graphs of these inequations can be drawn as below:

$x > 3$



$0 < x < 6$



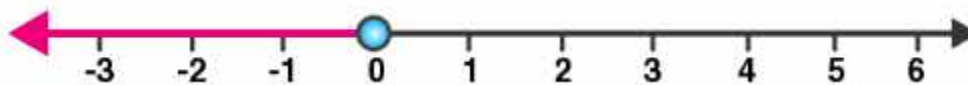
Thus, from both the graphs, it is clear that their common range is  $3 < x < 6$

(ii)  $x < 0$  and  $-3 \leq x < 1$

Both the given inequations are true in the range where their graphs on the real number lines overlap.

And, the graphs of these inequations can be drawn as below:

$x < 0$



$-3 \leq x < 1$



Thus, from both the graphs, it is clear that their common range is  $-3 \leq x < 0$

(iii)  $-1 < x \leq 6$  and  $-2 \leq x \leq 3$

Both the given inequations are true in the range where their graphs on the real number lines overlap.

And, the graphs of these inequations can be drawn as below:

$-1 < x \leq 6$



$-2 \leq x \leq 3$



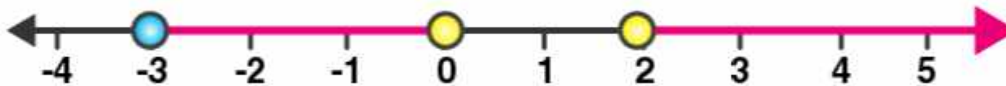
Thus, from both the graphs, it is clear that their common range is  $-1 < x \leq 3$

**16. Illustrate the set  $\{x: -3 \leq x < 0 \text{ or } x > 2, x \in \mathbf{R}\}$  on the real number line.**

**Solution:**

We have to understand that:

Graph of solution set of  $-3 \leq x < 0$  or  $x > 2$  = Graph of points which belong to  $-3 \leq x < 0$  or  $x > 2$  or both  
Therefore, the required graph is as below:



**17. Given  $A = \{x: -1 < x \leq 5, x \in \mathbf{R}\}$  and  $B = \{x: -4 \leq x < 3, x \in \mathbf{R}\}$**

**Represent on different number lines:**

- (i)  $A \cap B$
- (ii)  $A' \cap B$
- (iii)  $A - B$

**Solution:**

- (i)  $A \cap B = \{x: -1 < x < 3, x \in \mathbf{R}\}$

And, it can be represented on a number line as:



- (ii) Numbers which belong to B but do not belong to A =  $B - A$

$$A' \cap B = \{x: -4 \leq x \leq -1, x \in \mathbf{R}\}$$

And, it can be represented on a number line as:



- (iii)  $A - B = \{x: 3 \leq x \leq 5, x \in \mathbf{R}\}$

And, it can be represented on a number line as:



**18. P is the solution set of  $7x - 2 > 4x + 1$  and Q is the solution set of  $9x - 45 \geq 5(x - 5)$ ; where  $x \in \mathbf{R}$ .**

**Represent:**

- (i)  $P \cap Q$
- (ii)  $P - Q$
- (iii)  $P \cap Q'$  on different number lines.

**Solution:**

$$P = \{x: 7x - 2 > 4x + 1, x \in \mathbb{R}\}$$

$$7x - 2 > 4x + 1$$

$$7x - 4x > 1 + 2$$

$$3x > 3$$

$$x > 1$$

And,

$$Q = \{x: 9x - 45 \geq 5(x - 5), x \in \mathbb{R}\}$$

$$9x - 45 \geq 5x - 25$$

$$9x - 5x \geq 45 - 25$$

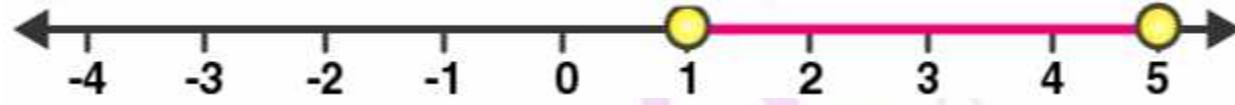
$$4x \geq 20$$

$$x \geq 5$$

(i)  $P \cap Q = \{x: x \geq 5, x \in \mathbb{R}\}$



(ii)  $P - Q = \{x: 1 < x < 5, x \in \mathbb{R}\}$



(iii)  $P \cap Q' = \{x: 1 < x < 5, x \in \mathbb{R}\}$

