Exercise 5(D)

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1. Solve, each of the following equations, using the formula:

(i) $x^2 - 6x = 27$

Solution:

Given equation, $x^2 - 6x = 27$

$x^2 - 6x - 27 = 0$

Here, a = 1 , b = -6 and c = -27

By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-27)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - -108}}{2}$$

$$x = \frac{6 \pm \sqrt{144}}{2}$$

$$x = \frac{6 \pm 12}{2}$$

$$x = \frac{18}{2} \quad x = -\frac{6}{2}$$

$$x = 9$$

$$x = -3$$

Therefore, x = 9 or -3

(ii) $x^2 – 10x + 21 = 0$

Solution:

Given equation, $x^2 – 10x + 21 = 0$

Here, a = 1, b = -10 and c = 21

By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 - 84}}{2}$$

$$x = \frac{10 \pm \sqrt{16}}{2}$$

$$x = \frac{10 \pm 4}{2}$$

$$x = \frac{14}{2} \quad x = \frac{6}{2}$$

$$x = 7 \quad x = 3$$

Therefore, x = 7 or x = 3

(iii) $x^2 + 6x – 10 = 0$
Solution:

Given equation, $x^2 + 6x – 10 = 0$
Here, a = 1, b = 6 and c = -10
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - -40}}{2}$$

$$x = \frac{-6 \pm \sqrt{76}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{19}}{2}$$

$$x = \frac{-6}{2} \pm \frac{2\sqrt{19}}{2}$$

$$x = -3 \pm \sqrt{19}$$

Therefore, x = -3 + √19 or x = -3 - √19

(iv) $x^2 + 2x – 6 = 0$
Solution:

Given equation, $x^2 + 2x – 6 = 0$
Here, a = 1, b = 2 and c = -6
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - -24}}{2}$$

$$x = \frac{-2 \pm \sqrt{28}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{7}}{2}$$

$$x = \frac{-2}{2} \pm \frac{2\sqrt{7}}{2}$$

$$x = -1 \pm \sqrt{7}$$

Therefore, $x = -1 + \sqrt{7}$ or $x = -1 - \sqrt{7}$

(v) $3x^2 + 2x – 1 = 0$
Solution:

Given equation, $3x^2 + 2x – 1 = 0$
Here, a = 3, b = 2 and c = -1
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{4 - -12}}{6}$$

$$x = \frac{-2 \pm \sqrt{16}}{6}$$

$$x = \frac{-2 \pm 4}{6}$$

$$x = \frac{2}{6} \quad x = -\frac{6}{6}$$

$$x = \frac{1}{3} \quad x = -1$$

Therefore, x = 1/3 or x = -1

(vi) $2x^2 + 7x + 5 = 0$
Solution:

Given equation, $2x^2 + 7x + 5 = 0$
Here, a = 2, b = 7 and c = 5
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{-7 \pm \sqrt{49 - 40}}{4}$$

$$x = \frac{-7 \pm \sqrt{9}}{4}$$

$$x = \frac{-7 \pm 3}{4}$$

$$x = -\frac{4}{4} \quad x = -\frac{10}{4}$$

$$x = -1 \quad x = -\frac{5}{2}$$

Therefore, x = -1 or x = -5/2

(vii) 2/3 x = -1/6 x^2 – 1/3
Solution:

Given equation, 2/3 x = -1/6 x^2 – 1/3
1/6 x^2 + 2/3 x + 1/3 = 0
Multiplying by 6 on both sides
x^2 + 4x + 2 = 0
Here, a = 1, b = 4 and c = 2
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 8}}{2}$$

$$x = \frac{-4 \pm \sqrt{8}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{2}}{2}$$

$$x = \frac{-4}{2} \pm \frac{2\sqrt{2}}{2}$$

$$x = -2 \pm \sqrt{2}$$

Therefore, x = -2 + √2 or x = -2 - √2

(viii) 1/15 x^2 + 5/3 = 2/3 x
Solution:

Given equation, 1/15 x^2 + 5/3 = 2/3 x
1/15 x^2 - 2/3 x + 5/3 = 0
Multiplying by 15 on both sides
x^2 - 10x + 25 = 0
Here, a = 1, b = -10 and c = 25
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 - 100}}{2}$$

$$x = \frac{10 \pm \sqrt{0}}{2}$$

$$x = \frac{10}{2}$$

$$x = 5$$

Therefore, x = 5 (equal roots)

(ix) $x^2 - 6 = 2\sqrt{2}x$
Solution:

Given equation, $x^2 - 6 = 2\sqrt{2}x$
$x^2 - 2\sqrt{2}x - 6 = 0$
Here, a = 1, b = $-2\sqrt{2}$ and c = -6
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{2\sqrt{2} \pm \sqrt{32}}{2} = \frac{2\sqrt{2} \pm 4\sqrt{2}}{2} = \frac{2\sqrt{2} + 4\sqrt{2}}{2} \text{ and } \frac{2\sqrt{2} - 4\sqrt{2}}{2}$$

$$= \frac{6\sqrt{2}}{2} \text{ and } \frac{-2\sqrt{2}}{2} = 3\sqrt{2} \text{ and } -\sqrt{2}$$

Therefore, $x = 3\sqrt{2}$ or $x = -\sqrt{2}$

(x) 4/x – 3 = 5/ (2x + 3)
Solution:

Given equation, 4/x – 3 = 5/ (2x + 3)
(4 – 3x)/ x = 5/ (2x + 3)
On cross multiplying, we have
(4 – 3x)(2x + 3) = 5x
$8x - 6x^2 + 12 - 9x = 5x$
$6x^2 + 6x - 12 = 0$

Dividing by 6, we get
$x^2 + x – 2 = 0$
Here, a = 1, b = 1 and c = -2
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 - -8}}{2}$$

$$x = \frac{-1 \pm \sqrt{9}}{2}$$

$$x = \frac{-1 \pm 3}{2}$$

$$x = \frac{2}{2} \quad x = -\frac{4}{2}$$

$$x = 1$$

$$x = -2$$

Therefore, x = 1 or x = -2

(xi) 2x + 3/ x + 3 = x + 4/ x + 2
Solution:

Given equation, 2x + 3/ x + 3 = x + 4/ x + 2
On cross-multiplying, we have
(2x + 3) (x + 2) = (x + 4) (x + 3)
$2x^2 + 4x + 3x + 6 = x^2 + 3x + 4x + 12$
$2x^2 + 7x + 6 = x^2 + 7x + 12$
$x^2 + 0x – 6 = 0$
Here, a = 1, b = 0 and c = -6
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{0 \pm \sqrt{0^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{0 \pm \sqrt{0 - -24}}{2}$$

$$x = \frac{0 \pm \sqrt{24}}{2}$$

$$x = \frac{0 \pm 2\sqrt{6}}{2}$$

$$x = \frac{0}{2} \pm \frac{2\sqrt{6}}{2}$$

$$x = 0 \pm \sqrt{6}$$

Therefore, x = √6 or x = -√6

(xii) $\sqrt{6}x^2 – 4x - 2\sqrt{6} = 0$
Solution:

Given equation, $\sqrt{6}x^2 – 4x - 2\sqrt{6} = 0$
Here, a = √6, b = -4 and c = $-2\sqrt{6}$
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(\sqrt{6})(-2\sqrt{6})}}{2(\sqrt{6})}$$

$$= \frac{4 \pm \sqrt{64}}{2\sqrt{6}} = \frac{4 \pm 8}{2\sqrt{6}} = \frac{4+8}{2\sqrt{6}} \text{ and } \frac{4-8}{2\sqrt{6}}$$

$$= \frac{6}{\sqrt{6}} \text{ and } \frac{-2}{\sqrt{6}} = \sqrt{6} \text{ and } \frac{-\sqrt{6}}{3}$$

Therefore, x = √6 or -√6/3

(xiii) 2x/ x – 4 + (2x - 5)/(x - 3) = $8\frac{1}{3}$
Solution:

Given equation, 2x/ x – 4 + (2x - 5)/(x - 3) = $8\frac{1}{3}$

$$\Rightarrow \frac{2x(x-3)+(x-4)(2x-5)}{(x-4)(x-3)} = \frac{25}{3}$$

$$\Rightarrow \frac{2x^2-6x+2x^2-5x-8x+20}{x^2-3x-4x+12} = \frac{25}{3}$$

$$\Rightarrow \frac{4x^2-19x+20}{x^2-7x+12} = \frac{25}{3}$$

$25x^2 – 175x + 300 = 12x^2 – 57x + 60$

$13x^2 – 118x + 240 = 0$

Here, a = 13, b = -118 and c = 240

By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-118) \pm \sqrt{(-118)^2-4(13)(240)}}{2(13)}$$

$$x = \frac{118 \pm \sqrt{13924-12480}}{26}$$

$$x = \frac{118 \pm \sqrt{1444}}{26}$$

$$x = \frac{118 \pm 38}{26}$$

$$x = \frac{156}{26} \quad x = \frac{80}{26}$$

$$x = 6 \quad x = \frac{40}{13}$$

Therefore, x = 6 or x = 40/13

(ix) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

Solution:

From the given equation,

$$\Rightarrow \frac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)} = \frac{10}{3}$$

$$\Rightarrow \frac{x^2-4x-x+4+x^2-3x-2x+6}{x^2-4x-2x+8} = \frac{10}{3}$$

$$\Rightarrow \frac{2x^2-10x+10}{x^2-6x+8} = \frac{10}{3}$$

$10x^2 – 60x + 80 = 6x^2 – 30x + 30$
$4x^2 – 30x + 50 = 0$
$2x^2 – 15x + 25 = 0$
Here, a = 2, b = -15 and c = 25

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-15) \pm \sqrt{(-15)^2-4(2)(25)}}{2(2)}$$

$$x = \frac{15 \pm \sqrt{225-200}}{4}$$

$$x = \frac{15 \pm \sqrt{25}}{4}$$

$$x = \frac{15 \pm 5}{4}$$

$$x = \frac{20}{4} \quad x = \frac{10}{4}$$

$$x = 5 \quad x = \frac{5}{2}$$

Therefore, x = 5 or x = 5/2

2. Solve each of the following equations for x and give, in each case, your answer correct to one decimal place:
(i) x^2 - 8x +5 = 0
(ii) $5x^2$ + 10x - 3 = 0
Solution:

(i) x^2 - 8x + 5 = 0
Here, a = 1, b = -8 and c = 5
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 20}}{2}$$

$$x = \frac{8 \pm \sqrt{44}}{2}$$

$$x = \frac{8 \pm 2\sqrt{11}}{2}$$

$$x = \frac{8}{2} \pm \frac{2\sqrt{11}}{2}$$

$$x = 4 \pm \sqrt{11}$$

x = 4 ± 3.3
Thus, x = 7.7 or x = 0.7

(ii) $5x^2 + 10x - 3 = 0$
Here, a = 5, b = 10 and c = -3
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(5)(-3)}}{2(5)}$$

$$x = \frac{-10 \pm \sqrt{100 - -60}}{10}$$

$$x = \frac{-10 \pm \sqrt{160}}{10}$$

$$x = \frac{-10 \pm 4\sqrt{10}}{10}$$

$$x = \frac{-10}{10} \pm \frac{4\sqrt{10}}{10}$$

$$x = -1 \pm \frac{2\sqrt{10}}{5}$$

Thus, x = 0.3 or x = -2.3

3. Solve each of the following equations for x and give, in each case, your answer correct to 2 decimal places:
(i) $2x^2 – 10x + 5 = 0$
Solution:

Given equation, $2x^2 – 10x + 5 = 0$
Here, a = 2, b = -10 and c = 5

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{10 \pm \sqrt{100 - 40}}{4}$$

$$x = \frac{10 \pm \sqrt{60}}{4}$$

$$x = \frac{10 \pm 2\sqrt{15}}{4}$$

$$x = \frac{10}{4} \pm \frac{2\sqrt{15}}{4}$$

$$x = \frac{5}{2} \pm \frac{\sqrt{15}}{2}$$

$$x = 4.43649$$

$$x = 0.563508$$

Therefore, x = 4.44 or x = 0.56

(ii) 4x + 6/x + 13 = 0
Solution:

Given equation, 4x + 6/x + 13 = 0
Multiplying by x both sides, we get
$4x^2 + 13x + 6 = 0$
Here, a = 4, b = 13 and c = 6

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-13 \pm \sqrt{13^2 - 4(4)(6)}}{2(4)}$$

$$x = \frac{-13 \pm \sqrt{169 - 96}}{8}$$

$$x = \frac{-13 \pm \sqrt{73}}{8}$$

$$x = \frac{-13 \pm \sqrt{73}}{8}$$

$$x = \frac{-13}{8} \pm \frac{\sqrt{73}}{8}$$

$$x = -\frac{13}{8} \pm \frac{\sqrt{73}}{8}$$

$$x = -0.557$$

$$x = -2.693$$

Therefore, x = -0.56 or x = -2.70

(iii) $4x^2 – 5x – 3 = 0$
Solution:

Given equation, $4x^2 – 5x – 3 = 0$
Here, a = 4, b = -5 and c = -3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{5 \pm \sqrt{25 - -48}}{8}$$

$$x = \frac{5 \pm \sqrt{73}}{8}$$

$$x = \frac{5 \pm \sqrt{73}}{8}$$

$$x = \frac{5}{8} \pm \frac{\sqrt{73}}{8}$$

$$x = 1.693$$

$$x = -0.443$$

Therefore, x = 1.70 or x = -0.44

(iv) $x^2 – 3x – 9 = 0$
Solution:

Given equation, $x^2 – 3x – 9 = 0$
Here, a = 1, b = -3 and c = -9

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 - -36}}{2}$$

$$x = \frac{3 \pm \sqrt{45}}{2}$$

$$x = \frac{3 \pm 3\sqrt{5}}{2}$$

$$x = \frac{3}{2} \pm \frac{3\sqrt{5}}{2}$$

$$x = 4.8541$$

$$x = -1.8541$$

Therefore, x = 4.85 or x = -1.85

(v) $x^2 – 5x – 10 = 0$
Solution:

Given equation, $x^2 – 5x – 10 = 0$
Here, a = 1, b = -5 and c = -10

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - -40}}{2}$$

$$x = \frac{5 \pm \sqrt{65}}{2}$$

$$x = \frac{5 \pm \sqrt{65}}{2}$$

$$x = \frac{5}{2} \pm \frac{\sqrt{65}}{2}$$

$$x = 6.53113$$

$$x = -1.53113$$

Therefore, x = 6.53 or x = -1.53

4. Solve each of the following equations for x and give, in each case, your answer correct to 3 decimal places:
(i) $3x^2 - 12x - 1 = 0$
(ii) $x^2 - 16x + 6 = 0$
(iii) $2x^2 + 11x + 4 = 0$
Solution:

(i) Given equation, $3x^2 - 12x - 1 = 0$
Here, a = 3, b = -12 and c = -1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{12 \pm \sqrt{144 - -12}}{6}$$

$$x = \frac{12 \pm \sqrt{156}}{6}$$

$$x = \frac{12 \pm 2\sqrt{39}}{6}$$

$$x = \frac{12}{6} \pm \frac{2\sqrt{39}}{6}$$

$$x = 2 \pm \frac{\sqrt{39}}{3}$$

$$x = 4.08167$$

$$x = -0.081666$$

Therefore, x = 4.082 or x = -0.082

(ii) Given equation, x^2 - 16 x + 6 = 0
Here, a = 1, b = -16 and c = 6

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{16 \pm \sqrt{256 - 24}}{2}$$

$$x = \frac{16 \pm \sqrt{232}}{2}$$

$$x = \frac{16 \pm 2\sqrt{58}}{2}$$

$$x = \frac{16}{2} \pm \frac{2\sqrt{58}}{2}$$

$$x = 8 \pm \sqrt{58}$$

$$x = 15.6158$$

$$x = 0.384227$$

Therefore, x = 15.616 or x = 0.384

(iii) Given equation, $2x^2 + 11x + 4 = 0$
Here, a = 2, b = 11 and c = 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-11 \pm \sqrt{11^2 - 4(2)(4)}}{2(2)}$$

$$x = \frac{-11 \pm \sqrt{121 - 32}}{4}$$

$$x = \frac{-11 \pm \sqrt{89}}{4}$$

$$x = \frac{-11 \pm \sqrt{89}}{4}$$

$$x = \frac{-11}{4} \pm \frac{\sqrt{89}}{4}$$

$$x = -\frac{11}{4} \pm \frac{\sqrt{89}}{4}$$

$$x = -0.391505$$

$$x = -5.1085$$

Therefore, x = -0.392 or x = -5.110

5. Solve:
(i) $x^4 – 2x^2 – 3 = 0$
Solution:

Given equation, $x^4 – 2x^2 – 3 = 0$
$x^4 – 3x^2 + x^2 – 3 = 0$
$x^2(x^2 - 3) + 1(x^2 - 3) = 0$
$(x^2 + 1) (x^2 - 3) = 0$
So, $x^2 + 1 = 0$ (which is not possible) or $x^2 – 3 = 0$
Hence,
$x^2 – 3 = 0$
$x = \pm \sqrt{3}$

(ii) $x^4 – 10x^2 + 9 = 0$
Solution:

Given equation, $x^4 – 10x^2 + 9 = 0$
$x^4 – x^2 – 9x^2 + 9 = 0$
$x^2(x^2 - 1) – 9(x^2 - 1) = 0$
$(x^2 - 9)(x^2 - 1) = 0$

So, we have
$x^2 – 9 = 0$ or $x^2 – 1 = 0$
Hence,
$x = \pm 3$ or $x = \pm 1$