Exercise 5(A)

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1. Find which of the following equations are quadratic:
(i) $(3x - 1)^2 = 5(x + 8)$
(ii) $5x^2 – 8x = -3(7 – 2x)$
(iii) $(x -4) (3x + 1) = (3x – 1) (x + 2)$
(iv) $x^2 + 5x – 5 = (x - 3)^2$
(v) $7x^3 – 2x^2 + 10 = (2x - 5)^2$
(vi) $(x - 1)^2 + (x + 2)^2 + 3(x + 1) = 0$
Solution:

(i) $(3x - 1)^2 = 5(x + 8)$
$\Rightarrow (9x^2 - 6x + 1) = 5x + 40$
$\Rightarrow 9x^2 - 11x - 39 = 0$; which is of the general form $ax^2 + bx + c = 0$.
Thus, the given equation is a quadratic equation.

(ii) $5x^2 - 8x = -3(7 - 2x)$
$\Rightarrow 5x^2 - 8x = 6x - 21$
$\Rightarrow 5x^2 - 14x + 21 = 0$; which is of the general form $ax^2 + bx + c = 0$.
Thus, the given equation is a quadratic equation.

(iii) $(x - 4) (3x + 1) = (3x - 1) (x +2)$
$\Rightarrow 3x^2 + x - 12x - 4 = 3x^2 + 6x - x - 2$
$\Rightarrow 16x + 2 = 0$; which is not of the general form $ax^2 + bx + c = 0$. And it's a linear equation.
Thus, the given equation is not a quadratic equation.

(iv) $x^2 + 5x - 5 = (x - 3)^2$
$\Rightarrow x^2 + 5x - 5 = x^2 - 6x + 9$
$\Rightarrow 11x - 14 = 0$; which is not of the general form $ax^2 + bx + c = 0$. And it's a linear equation.
Thus, the given equation is not a quadratic equation.

(v) $7x^3 - 2x^2 + 10 = (2x - 5)^2$
$\Rightarrow 7x^3 - 2x^2 + 10 = 4x^2 - 20x + 25$
$\Rightarrow 7x^3 - 6x^2 + 20x - 15 = 0$; which is not of the general form $ax^2 + bx + c = 0$. And it's a cubic equation.
Thus, the given equation is not a quadratic equation.

(vi) $(x - 1)^2 + (x + 2)^2 + 3(x +1) = 0$
$\Rightarrow x^2 - 2x + 1 + x^2 + 4x + 4 + 3x + 3 = 0$
$\Rightarrow 2x^2 + 5x + 8 = 0$; which is of the general form $ax^2 + bx + c = 0$.
Thus, the given equation is a quadratic equation.

2. (i) Is x = 5 a solution of the quadratic equation $x^2 - 2x - 15 = 0$?
Solution:

Given quadratic equation, $x^2 - 2x - 15 = 0$

We know that, for x = 5 to be a solution of the given quadratic equation it should satisfy the equation.
Now, on substituting x = 5 in the given equation, we have
L.H.S $= (5)^2 - 2(5) - 15$
$= 25 - 10 - 15$
$= 0$
= R.H.S
Therefore, x = 5 is a solution of the given quadratic equation $x^2 - 2x - 15 = 0$

(ii) Is x = -3 a solution of the quadratic equation $2x^2 - 7x + 9 = 0$?
Solution:

Given quadratic equation, $2x^2 - 7x + 9 = 0$
We know that, for x = -3 to be solution of the given quadratic equation it should satisfy the equation.
Now, on substituting x = 5 in the given equation, we have
L.H.S $= 2(-3)^2 - 7(-3) + 9$
$= 18 + 21 + 9$
$= 48$
$\neq$ R.H.S
Therefore, x = -3 is not a solution of the given quadratic equation $2x^2 - 7x + 9 = 0$.

Exercise 5(B) Page No: 56

1. Without solving, comment upon the nature of roots of each of the following equations:

(i) $7x^2 - 9x + 2 = 0$ **(ii) $6x^2 - 13x + 4 = 0$**

(iii) $25x^2 - 10x + 1 = 0$ **(iv) $x^2 + 2\sqrt{3}x - 9 = 0$**

(v) $x^2 - ax - b^2 = 0$ **(vi) $2x^2 + 8x + 9 = 0$**

Solution:

(i) Given quadratic equation, $7x^2 - 9x + 2 = 0$

Here, a = 7, b = -9 and c = 2

So, the Discriminant (D) = $b^2 - 4ac$

$D = (-9)^2 - 4(7)(2)$

$= 81 - 56$

$= 25$

As D > 0, the roots of the equation is real and unequal.

(ii) Given quadratic equation, $6x^2 - 13x + 4 = 0$

Here, a = 6, b = -13 and c = 4

So, the Discriminant (D) = $b^2 - 4ac$

$D = (-13)^2 - 4(6)(4)$

$= 169 - 48$

$= 121$

As D > 0, the roots of the equation is real and unequal.

(iii) Given quadratic equation, $25x^2 - 10x + 1 = 0$

Here, a = 25, b = -10 and c = 1

So, the Discriminant (D) = $b^2 - 4ac$

$D = (-10)^2 - 4(25)(1)$

$= 100 - 100$

$= 0$

As D = 0, the roots of the equation is real and equal.

(iv) Given quadratic equation, $x^2 + 2\sqrt{3}x - 9 = 0$

Here, a = 1, b = $2\sqrt{3}$ and c = -9

So, the Discriminant (D) = $b^2 - 4ac$

$D = (2\sqrt{3})^2 - 4(1)(-9)$

$= 12 + 36$

$= 48$

As D > 0, the roots of the equation is real and unequal.

(v) Given quadratic equation, $x^2 - ax - b^2 = 0$

Here, a = 1, b = -a and c = $-b^2$

So, the Discriminant (D) = $b^2 - 4ac$

$D = (a)^2 - 4(1)(-b^2)$

$= a^2 + 4b^2$

$a^2 + 4b^2$ is always positive value.

Thus D > 0, and the roots of the equation is real and unequal

(vi) Given quadratic equation, $2x^2 + 8x + 9 = 0$
Here, a = 2, b = 8 and c = 9
So, the Discriminant (D) = $b^2 – 4ac$
$D = (8)^2 – 4(2)(9)$
$= 64 – 72$
$= -8$
As D < 0, the equation has no roots.

2. Find the value of 'p', if the following quadratic equations has equal roots:
(i) $4x^2 - (p - 2)x + 1 = 0$
(ii) $x^2 + (p - 3)x + p = 0$
Solution:

(i) $4x^2 - (p - 2)x + 1 = 0$
Here, a = 4, b = -(p - 2), c = 1
Given that the roots are equal,
So, Discriminant = $0 \Rightarrow b^2 - 4ac = 0$
$D = (-(p - 2))^2 - 4(4)(1) = 0$
$\Rightarrow p^2 + 4 - 4p - 16 = 0$
$\Rightarrow p^2 - 4p - 12 = 0$
$\Rightarrow p^2 - 6p + 2p - 12 = 0$
$\Rightarrow p(p – 6) + 2(p – 6) = 0$
$\Rightarrow (p + 2)(p - 6) = 0$
$\Rightarrow p + 2 = 0$ or $p - 6 = 0$
Hence, p = -2 or p = 6

(ii) $x^2 + (p - 3)x + p = 0$
Here, a = 1, b = (p - 3), c = p
Given that the roots are equal,
So, Discriminant = $0 \Rightarrow b^2 - 4ac = 0$
$D = (p - 3)^2 - 4(1)(p) = 0$
$\Rightarrow p^2 + 9 - 6p - 4p = 0$
$\Rightarrow p^2 - 10p + 9 = 0$
$\Rightarrow p^2 - 9p - p + 9 = 0$
$\Rightarrow p(p - 9) - 1(p - 9) = 0$
$\Rightarrow (p -9)(p - 1) = 0$
$\Rightarrow p - 9 = 0$ or $p - 1 = 0$
Hence, p = 9 or p = 1

Exercise 5(C)

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Solve equations, number 1 to 20, given below, using factorization method:

1. $x^2 – 10x – 24 = 0$

Solution:

Given equation, $x^2 – 10x – 24 = 0$
$x^2 – 12x + 2x – 24 = 0$
$x(x - 12) + 2(x - 12) = 0$
$(x + 2)(x - 12) = 0$
So, $x + 2 = 0$ or $x - 12 = 0$
Hence,
$x = -2$ or $x = 12$

2. $x^2 – 16 = 0$

Solution:

Given equation, $x^2 – 16 = 0$
$x^2 + 4x – 4x + 16 = 0$
$x(x + 4) -4(x + 4) = 0$
$(x - 4) (x + 4) = 0$
So, $(x - 4) = 0$ or $(x + 4) = 0$
Hence,
$x = 4$ or $x = -4$

3. $2x^2 – ½ x = 0$

Solution:

Given equation, $2x^2 – ½ x = 0$
$4x^2 – x = 0$
$x(4x - 1) = 0$
So, either $x = 0$ or $4x – 1 = 0$
Hence,
$x = 0$ or $x = ¼$

4. $x(x - 5) = 24$

Solution:

Given equation, $x(x - 5) = 24$
$x^2 – 5x = 24$
$x^2 – 5x – 24 = 0$
$x^2 – 8x + 3x – 24 = 0$
$x(x - 8) + 3(x - 8) = 0$
$(x + 3)(x - 8) = 0$
So, $x + 3 = 0$ or $x - 8 = 0$
Hence,

x = -3 or x = 8

5. 9/2 x = 5 + x^2
Solution:

Given equation, 9/2 x = 5 + x^2
On multiplying by 2 both sides, we have
9x = 2(5 + x^2)
9x = 10 + $2x^2$
$2x^2$ - 9x + 10 = 0
$2x^2$ - 4x – 5x + 10 = 0
2x(x - 2) – 5(x - 2) = 0
(2x - 5)(x -2) = 0
So, 2x – 5 = 0 or x – 2 = 0
Hence,
x = 5/2 or x = 2

6. 6/x = 1 + x
Solution:

Given equation, 6/x = 1 + x
On multiplying by x both sides, we have
6 = x(1 + x)
6 = x + x^2
x^2 + x – 6 = 0
x^2 + 3x – 2x – 6 = 0
x(x + 3) – 2(x + 3) = 0
(x - 2) (x + 3) = 0
So, x – 2 = 0 or x + 3 = 0
Hence,
x = 2 or x = -3

7. x = (3x + 1)/ 4x
Solution:

Given equation, x = (3x + 1)/ 4x
On multiplying by 4x both sides, we have
4x(x) = 3x + 1
$4x^2$ = 3x + 1
$4x^2$ – 3x – 1 = 0
$4x^2$ – 4x + x – 1 = 0
4x(x - 1) + 1(x - 1) = 0
(4x + 1) (x - 1) = 0
So, 4x + 1 = 0 or x – 1 = 0
Hence,
x = -1/4 or x = 1

8. $x + 1/x = 2.5$
Solution:

Given equation, $x + 1/x = 2.5$
$x + 1/x = 5/2$
Taking LCM on L.H.S, we have
$(x^2 + 1)/ x = 5/2$
$2(x^2 + 1) = 5x$
$2x^2 + 2 = 5x$
$2x^2 - 5x + 2 = 0$
$2x^2 - 4x - x + 2 = 0$
$2x(x - 2) -1(x - 2) = 0$
$(2x - 1)(x - 2) = 0$
So, $2x - 1 = 0$ or $x – 2 = 0$
Hence,
$x = ½$ or $x = 2$

9. $(2x - 3)^2 = 49$
Solution:

Given equation, $(2x - 3)^2 = 49$
Expanding the L.H.S, we have
$4x^2 – 12x + 9 = 49$
$4x^2 – 12x – 40 = 0$
Dividing by 4 on both side
$x^2 – 3x – 10 = 0$
$x^2 – 5x + 2x – 10 = 0$
$x(x - 5) + 2(x - 5) = 0$
$(x + 2) (x - 5) = 0$
So, $x + 2 = 0$ or $x – 5 = 0$
Hence,
$x = -2$ or 5

10. $2(x^2 – 6) = 3(x - 4)$
Solution:

Given equation, $2(x^2 - 6) = 3(x - 4)$
$2x^2 – 12 = 3x – 12$
$2x^2 = 3x$
$x(2x - 3) = 0$
So, $x = 0$ or $(2x - 3) = 0$
Hence,
$x = 0$ or $x = 3/2$

11. $(x + 1) (2x + 8) = (x + 7) (x + 3)$

Solution:

Given equation, $(x + 1)(2x + 8) = (x + 7)(x + 3)$
$2x^2 + 2x + 8x + 8 = x^2 + 7x + 3x + 21$
$2x^2 + 10x + 8 = x^2 + 10x + 21$
$x^2 = 21 – 8$
$x^2 – 13 = 0$
$(x - \sqrt{13})(x + \sqrt{13}) = 0$
So, $x - \sqrt{13} = 0$ or $x + \sqrt{13} = 0$
Hence,
$x = -\sqrt{13}$ or $x = \sqrt{13}$

12. $x^2 – (a + b)x + ab = 0$
Solution:

Given equation, $x^2 – (a + b)x + ab = 0$
$x^2 – ax - bx + ab = 0$
$x(x - a) – b(x - a) = 0$
$(x – b)(x - a) = 0$
So, $x – b = 0$ or $x – a = 0$
Hence,
$x = b$ or $x = a$

13. $(x + 3)^2 – 4(x + 3) – 5 = 0$
Solution:

Given equation, $(x + 3)^2 – 4(x + 3) – 5 = 0$
$(x^2 + 9 + 6x) – 4x – 12 – 5 = 0$
$x^2 + 2x – 8 = 0$
$x^2 + 4x – 2x – 8 = 0$
$x(x + 4) – 2(x - 4) = 0$
$(x - 2)(x + 4) = 0$
So, $x – 2 = 0$ or $x + 4 = 0$
Hence,
$x = 2$ or $x = -4$

14. $4(2x - 3)^2 – (2x - 3) – 14 = 0$
Solution:

Given equation, $4(2x - 3)^2 – (2x - 3) – 14 = 0$
Let substitute $2x – 3 = y$
Then the equation becomes,
$4y^2 – y – 14 = 0$
$4y^2 - 8y + 7y – 14 = 0$
$4y(y - 2) + 7(y – 2) = 0$
$(4y + 7)(y - 2) = 0$

So, 4y + 7 = 0 or y - 2 = 0
Hence,
y = -7/4 or y = 2
But we have taken y = 2x – 3
Thus,
2x – 3 = -7/4 or 2x – 3 = 2
2x = 5/ 4 or 2x = 5
x = 5/8 or x = 5/2

15. 3x – 2/ 2x- 3 = 3x – 8/ x + 4
Solution:

Given equation, 3x – 2/ 2x- 3 = 3x – 8/ x + 4
On cross-multiplying we have,
(3x - 2)(x + 4) = (3x – 8)(2x - 3)
$3x^2 – 2x + 12x – 8 = 6x^2 – 16x – 9x + 24$
$3x^2 + 10x – 8 = 6x^2 – 25x + 24$
$3x^2 – 35x + 32 = 0$
$3x^2 – 3x – 32x + 32 = 0$
3x(x - 1) – 32(x - 1) = 0
(3x - 32)(x - 1) = 0
So, 3x – 32 = 0 or x – 1 = 0
Hence,
x = 32/3 or x = 1

16. $2x^2 – 9x + 10 = 0$, when:
(i) x ∈ N (ii) x ∈ Q
Solution:

Given equation, $2x^2 – 9x + 10 = 0$
$2x^2 – 4x – 5x + 10 = 0$
2x(x - 2) – 5(x - 2) = 0
(2x - 5)(x - 2) = 0
So, 2x – 5 = 0 or x - 2 = 0
Hence,
x = 5/2 or x = 2
(i) When x ∈ N
x = 2 is the solution.
(ii) When x ∈ Q
x = 2, 5/2 are the solutions

17. $\frac{x-3}{x+3}+\frac{x+3}{x-3}=2\frac{1}{2}$

Solution:

$$\frac{x-3}{x+3}+\frac{x+3}{x-3}=2\frac{1}{2}$$

$$\Rightarrow \frac{(x-3)^2+(x+3)^2}{(x+3)(x-3)}=\frac{5}{2}$$

$$\Rightarrow \frac{x^2-6x+9+x^2+6x+9}{x^2-9}=\frac{5}{2}$$

$2(2x^2 + 18) = 5(x^2 – 9)$

$4x^2 + 36 = 5x^2 – 45$

$x^2 – 81 = 0$

$(x - 9)(x + 9) = 0$

So, $x – 9 = 0$ or $x + 9 = 0$

Hence,

$x = 9$ or $x = -9$

Exercise 5(D)

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1. Solve, each of the following equations, using the formula:

(i) $x^2 - 6x = 27$

Solution:

Given equation, $x^2 - 6x = 27$

$x^2 - 6x - 27 = 0$

Here, a = 1 , b = -6 and c = -27

By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-27)}}{2(1)}$$

$$x = \frac{6 \pm \sqrt{36 - -108}}{2}$$

$$x = \frac{6 \pm \sqrt{144}}{2}$$

$$x = \frac{6 \pm 12}{2}$$

$$x = \frac{18}{2} \quad x = -\frac{6}{2}$$

$$x = 9$$

$$x = -3$$

Therefore, x = 9 or -3

(ii) $x^2 – 10x + 21 = 0$

Solution:

Given equation, $x^2 – 10x + 21 = 0$

Here, a = 1, b = -10 and c = 21

By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(21)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 - 84}}{2}$$

$$x = \frac{10 \pm \sqrt{16}}{2}$$

$$x = \frac{10 \pm 4}{2}$$

$$x = \frac{14}{2} \quad x = \frac{6}{2}$$

$$x = 7 \quad x = 3$$

Therefore, x = 7 or x = 3

(iii) $x^2 + 6x – 10 = 0$
Solution:

Given equation, $x^2 + 6x – 10 = 0$
Here, a = 1, b = 6 and c = -10
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - -40}}{2}$$

$$x = \frac{-6 \pm \sqrt{76}}{2}$$

$$x = \frac{-6 \pm 2\sqrt{19}}{2}$$

$$x = \frac{-6}{2} \pm \frac{2\sqrt{19}}{2}$$

$$x = -3 \pm \sqrt{19}$$

Therefore, x = -3 + √19 or x = -3 - √19

(iv) $x^2 + 2x – 6 = 0$
Solution:

Given equation, $x^2 + 2x – 6 = 0$
Here, a = 1, b = 2 and c = -6
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{-2 \pm \sqrt{4 - -24}}{2}$$

$$x = \frac{-2 \pm \sqrt{28}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{7}}{2}$$

$$x = \frac{-2}{2} \pm \frac{2\sqrt{7}}{2}$$

$$x = -1 \pm \sqrt{7}$$

Therefore, $x = -1 + \sqrt{7}$ or $x = -1 - \sqrt{7}$

(v) $3x^2 + 2x – 1 = 0$
Solution:

Given equation, $3x^2 + 2x – 1 = 0$
Here, a = 3, b = 2 and c = -1
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{-2 \pm \sqrt{4 - -12}}{6}$$

$$x = \frac{-2 \pm \sqrt{16}}{6}$$

$$x = \frac{-2 \pm 4}{6}$$

$$x = \frac{2}{6} \quad x = -\frac{6}{6}$$

$$x = \frac{1}{3} \quad x = -1$$

Therefore, x = 1/3 or x = -1

(vi) $2x^2 + 7x + 5 = 0$
Solution:

Given equation, $2x^2 + 7x + 5 = 0$
Here, a = 2, b = 7 and c = 5
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-7 \pm \sqrt{7^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{-7 \pm \sqrt{49 - 40}}{4}$$

$$x = \frac{-7 \pm \sqrt{9}}{4}$$

$$x = \frac{-7 \pm 3}{4}$$

$$x = -\frac{4}{4} \quad x = -\frac{10}{4}$$

$$x = -1 \quad x = -\frac{5}{2}$$

Therefore, x = -1 or x = -5/2

(vii) 2/3 x = -1/6 x^2 – 1/3
Solution:

Given equation, 2/3 x = -1/6 x^2 – 1/3
1/6 x^2 + 2/3 x + 1/3 = 0
Multiplying by 6 on both sides
x^2 + 4x + 2 = 0
Here, a = 1, b = 4 and c = 2
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(2)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{16 - 8}}{2}$$

$$x = \frac{-4 \pm \sqrt{8}}{2}$$

$$x = \frac{-4 \pm 2\sqrt{2}}{2}$$

$$x = \frac{-4}{2} \pm \frac{2\sqrt{2}}{2}$$

$$x = -2 \pm \sqrt{2}$$

Therefore, x = -2 + √2 or x = -2 - √2

(viii) 1/15 x^2 + 5/3 = 2/3 x
Solution:

Given equation, 1/15 x^2 + 5/3 = 2/3 x
1/15 x^2 - 2/3 x + 5/3 = 0
Multiplying by 15 on both sides
x^2 - 10x + 25 = 0
Here, a = 1, b = -10 and c = 25
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(25)}}{2(1)}$$

$$x = \frac{10 \pm \sqrt{100 - 100}}{2}$$

$$x = \frac{10 \pm \sqrt{0}}{2}$$

$$x = \frac{10}{2}$$

$$x = 5$$

Therefore, x = 5 (equal roots)

(ix) $x^2 - 6 = 2\sqrt{2}x$
Solution:

Given equation, $x^2 - 6 = 2\sqrt{2}x$
$x^2 - 2\sqrt{2}x - 6 = 0$
Here, a = 1, b = $-2\sqrt{2}$ and c = -6
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-2\sqrt{2}) \pm \sqrt{(-2\sqrt{2})^2 - 4(1)(-6)}}{2(1)}$$

$$= \frac{2\sqrt{2} \pm \sqrt{32}}{2} = \frac{2\sqrt{2} \pm 4\sqrt{2}}{2} = \frac{2\sqrt{2} + 4\sqrt{2}}{2} \text{ and } \frac{2\sqrt{2} - 4\sqrt{2}}{2}$$

$$= \frac{6\sqrt{2}}{2} \text{ and } \frac{-2\sqrt{2}}{2} = 3\sqrt{2} \text{ and } -\sqrt{2}$$

Therefore, $x = 3\sqrt{2}$ or $x = -\sqrt{2}$

(x) 4/x – 3 = 5/ (2x + 3)
Solution:

Given equation, 4/x – 3 = 5/ (2x + 3)
(4 – 3x)/ x = 5/ (2x + 3)
On cross multiplying, we have
(4 – 3x)(2x + 3) = 5x
$8x - 6x^2 + 12 - 9x = 5x$
$6x^2 + 6x - 12 = 0$

Dividing by 6, we get
$x^2 + x – 2 = 0$
Here, a = 1, b = 1 and c = -2
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-1 \pm \sqrt{1^2 - 4(1)(-2)}}{2(1)}$$

$$x = \frac{-1 \pm \sqrt{1 - -8}}{2}$$

$$x = \frac{-1 \pm \sqrt{9}}{2}$$

$$x = \frac{-1 \pm 3}{2}$$

$$x = \frac{2}{2} \quad x = -\frac{4}{2}$$

$$x = 1$$

$$x = -2$$

Therefore, x = 1 or x = -2

(xi) 2x + 3/ x + 3 = x + 4/ x + 2
Solution:

Given equation, 2x + 3/ x + 3 = x + 4/ x + 2
On cross-multiplying, we have
(2x + 3) (x + 2) = (x + 4) (x + 3)
$2x^2 + 4x + 3x + 6 = x^2 + 3x + 4x + 12$
$2x^2 + 7x + 6 = x^2 + 7x + 12$
$x^2 + 0x – 6 = 0$
Here, a = 1, b = 0 and c = -6
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{0 \pm \sqrt{0^2 - 4(1)(-6)}}{2(1)}$$

$$x = \frac{0 \pm \sqrt{0 - -24}}{2}$$

$$x = \frac{0 \pm \sqrt{24}}{2}$$

$$x = \frac{0 \pm 2\sqrt{6}}{2}$$

$$x = \frac{0}{2} \pm \frac{2\sqrt{6}}{2}$$

$$x = 0 \pm \sqrt{6}$$

Therefore, x = √6 or x = -√6

(xii) $\sqrt{6}x^2 – 4x - 2\sqrt{6} = 0$
Solution:

Given equation, $\sqrt{6}x^2 – 4x - 2\sqrt{6} = 0$
Here, a = √6, b = -4 and c = $-2\sqrt{6}$
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(\sqrt{6})(-2\sqrt{6})}}{2(\sqrt{6})}$$

$$= \frac{4 \pm \sqrt{64}}{2\sqrt{6}} = \frac{4 \pm 8}{2\sqrt{6}} = \frac{4+8}{2\sqrt{6}} \text{ and } \frac{4-8}{2\sqrt{6}}$$

$$= \frac{6}{\sqrt{6}} \text{ and } \frac{-2}{\sqrt{6}} = \sqrt{6} \text{ and } \frac{-\sqrt{6}}{3}$$

Therefore, x = √6 or -√6/3

(xiii) 2x/ x – 4 + (2x - 5)/(x - 3) = $8\frac{1}{3}$
Solution:

Given equation, 2x/ x – 4 + (2x - 5)/(x - 3) = $8\frac{1}{3}$

$$\Rightarrow \frac{2x(x-3)+(x-4)(2x-5)}{(x-4)(x-3)} = \frac{25}{3}$$

$$\Rightarrow \frac{2x^2-6x+2x^2-5x-8x+20}{x^2-3x-4x+12} = \frac{25}{3}$$

$$\Rightarrow \frac{4x^2-19x+20}{x^2-7x+12} = \frac{25}{3}$$

$25x^2 – 175x + 300 = 12x^2 – 57x + 60$

$13x^2 – 118x + 240 = 0$

Here, a = 13, b = -118 and c = 240

By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-118) \pm \sqrt{(-118)^2-4(13)(240)}}{2(13)}$$

$$x = \frac{118 \pm \sqrt{13924-12480}}{26}$$

$$x = \frac{118 \pm \sqrt{1444}}{26}$$

$$x = \frac{118 \pm 38}{26}$$

$$x = \frac{156}{26} \quad x = \frac{80}{26}$$

$$x = 6 \quad x = \frac{40}{13}$$

Therefore, x = 6 or x = 40/13

(ix) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = 3\frac{1}{3}$

Solution:

From the given equation,

$$\Rightarrow \frac{(x-1)(x-4)+(x-2)(x-3)}{(x-2)(x-4)} = \frac{10}{3}$$

$$\Rightarrow \frac{x^2-4x-x+4+x^2-3x-2x+6}{x^2-4x-2x+8} = \frac{10}{3}$$

$$\Rightarrow \frac{2x^2-10x+10}{x^2-6x+8} = \frac{10}{3}$$

$10x^2 – 60x + 80 = 6x^2 – 30x + 30$
$4x^2 – 30x + 50 = 0$
$2x^2 – 15x + 25 = 0$
Here, a = 2, b = -15 and c = 25

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$x = \frac{-(-15) \pm \sqrt{(-15)^2-4(2)(25)}}{2(2)}$$

$$x = \frac{15 \pm \sqrt{225-200}}{4}$$

$$x = \frac{15 \pm \sqrt{25}}{4}$$

$$x = \frac{15 \pm 5}{4}$$

$$x = \frac{20}{4} \quad x = \frac{10}{4}$$

$$x = 5 \quad x = \frac{5}{2}$$

Therefore, x = 5 or x = 5/2

2. Solve each of the following equations for x and give, in each case, your answer correct to one decimal place:
(i) x^2 - 8x +5 = 0
(ii) $5x^2$ + 10x - 3 = 0
Solution:

(i) x^2 - 8x + 5 = 0
Here, a = 1, b = -8 and c = 5
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(5)}}{2(1)}$$

$$x = \frac{8 \pm \sqrt{64 - 20}}{2}$$

$$x = \frac{8 \pm \sqrt{44}}{2}$$

$$x = \frac{8 \pm 2\sqrt{11}}{2}$$

$$x = \frac{8}{2} \pm \frac{2\sqrt{11}}{2}$$

$$x = 4 \pm \sqrt{11}$$

x = 4 ± 3.3
Thus, x = 7.7 or x = 0.7

(ii) $5x^2 + 10x - 3 = 0$
Here, a = 5, b = 10 and c = -3
By quadratic formula, we have

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-10 \pm \sqrt{10^2 - 4(5)(-3)}}{2(5)}$$

$$x = \frac{-10 \pm \sqrt{100 - -60}}{10}$$

$$x = \frac{-10 \pm \sqrt{160}}{10}$$

$$x = \frac{-10 \pm 4\sqrt{10}}{10}$$

$$x = \frac{-10}{10} \pm \frac{4\sqrt{10}}{10}$$

$$x = -1 \pm \frac{2\sqrt{10}}{5}$$

Thus, x = 0.3 or x = -2.3

3. Solve each of the following equations for x and give, in each case, your answer correct to 2 decimal places:
(i) $2x^2 – 10x + 5 = 0$
Solution:

Given equation, $2x^2 – 10x + 5 = 0$
Here, a = 2, b = -10 and c = 5

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{10 \pm \sqrt{100 - 40}}{4}$$

$$x = \frac{10 \pm \sqrt{60}}{4}$$

$$x = \frac{10 \pm 2\sqrt{15}}{4}$$

$$x = \frac{10}{4} \pm \frac{2\sqrt{15}}{4}$$

$$x = \frac{5}{2} \pm \frac{\sqrt{15}}{2}$$

$$x = 4.43649$$

$$x = 0.563508$$

Therefore, x = 4.44 or x = 0.56

(ii) 4x + 6/x + 13 = 0
Solution:

Given equation, 4x + 6/x + 13 = 0
Multiplying by x both sides, we get
$4x^2 + 13x + 6 = 0$
Here, a = 4, b = 13 and c = 6

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-13 \pm \sqrt{13^2 - 4(4)(6)}}{2(4)}$$

$$x = \frac{-13 \pm \sqrt{169 - 96}}{8}$$

$$x = \frac{-13 \pm \sqrt{73}}{8}$$

$$x = \frac{-13 \pm \sqrt{73}}{8}$$

$$x = \frac{-13}{8} \pm \frac{\sqrt{73}}{8}$$

$$x = -\frac{13}{8} \pm \frac{\sqrt{73}}{8}$$

$$x = -0.557$$

$$x = -2.693$$

Therefore, x = -0.56 or x = -2.70

(iii) $4x^2 – 5x – 3 = 0$
Solution:

Given equation, $4x^2 – 5x – 3 = 0$
Here, a = 4, b = -5 and c = -3

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(4)(-3)}}{2(4)}$$

$$x = \frac{5 \pm \sqrt{25 - -48}}{8}$$

$$x = \frac{5 \pm \sqrt{73}}{8}$$

$$x = \frac{5 \pm \sqrt{73}}{8}$$

$$x = \frac{5}{8} \pm \frac{\sqrt{73}}{8}$$

$$x = 1.693$$

$$x = -0.443$$

Therefore, x = 1.70 or x = -0.44

(iv) $x^2 – 3x – 9 = 0$
Solution:

Given equation, $x^2 – 3x – 9 = 0$
Here, a = 1, b = -3 and c = -9

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-9)}}{2(1)}$$

$$x = \frac{3 \pm \sqrt{9 - -36}}{2}$$

$$x = \frac{3 \pm \sqrt{45}}{2}$$

$$x = \frac{3 \pm 3\sqrt{5}}{2}$$

$$x = \frac{3}{2} \pm \frac{3\sqrt{5}}{2}$$

$$x = 4.8541$$

$$x = -1.8541$$

Therefore, x = 4.85 or x = -1.85

(v) $x^2 – 5x – 10 = 0$
Solution:

Given equation, $x^2 – 5x – 10 = 0$
Here, a = 1, b = -5 and c = -10

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-10)}}{2(1)}$$

$$x = \frac{5 \pm \sqrt{25 - -40}}{2}$$

$$x = \frac{5 \pm \sqrt{65}}{2}$$

$$x = \frac{5 \pm \sqrt{65}}{2}$$

$$x = \frac{5}{2} \pm \frac{\sqrt{65}}{2}$$

$$x = 6.53113$$

$$x = -1.53113$$

Therefore, x = 6.53 or x = -1.53

4. Solve each of the following equations for x and give, in each case, your answer correct to 3 decimal places:
(i) $3x^2 - 12x - 1 = 0$
(ii) $x^2 - 16x + 6 = 0$
(iii) $2x^2 + 11x + 4 = 0$
Solution:

(i) Given equation, $3x^2 - 12x - 1 = 0$
Here, a = 3, b = -12 and c = -1

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(3)(-1)}}{2(3)}$$

$$x = \frac{12 \pm \sqrt{144 - -12}}{6}$$

$$x = \frac{12 \pm \sqrt{156}}{6}$$

$$x = \frac{12 \pm 2\sqrt{39}}{6}$$

$$x = \frac{12}{6} \pm \frac{2\sqrt{39}}{6}$$

$$x = 2 \pm \frac{\sqrt{39}}{3}$$

$$x = 4.08167$$

$$x = -0.081666$$

Therefore, x = 4.082 or x = -0.082

(ii) Given equation, x^2 - 16 x + 6 = 0
Here, a = 1, b = -16 and c = 6

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-16) \pm \sqrt{(-16)^2 - 4(1)(6)}}{2(1)}$$

$$x = \frac{16 \pm \sqrt{256 - 24}}{2}$$

$$x = \frac{16 \pm \sqrt{232}}{2}$$

$$x = \frac{16 \pm 2\sqrt{58}}{2}$$

$$x = \frac{16}{2} \pm \frac{2\sqrt{58}}{2}$$

$$x = 8 \pm \sqrt{58}$$

$$x = 15.6158$$

$$x = 0.384227$$

Therefore, x = 15.616 or x = 0.384

(iii) Given equation, $2x^2 + 11x + 4 = 0$
Here, a = 2, b = 11 and c = 4

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-11 \pm \sqrt{11^2 - 4(2)(4)}}{2(2)}$$

$$x = \frac{-11 \pm \sqrt{121 - 32}}{4}$$

$$x = \frac{-11 \pm \sqrt{89}}{4}$$

$$x = \frac{-11 \pm \sqrt{89}}{4}$$

$$x = \frac{-11}{4} \pm \frac{\sqrt{89}}{4}$$

$$x = -\frac{11}{4} \pm \frac{\sqrt{89}}{4}$$

$$x = -0.391505$$

$$x = -5.1085$$

Therefore, x = -0.392 or x = -5.110

5. Solve:
(i) $x^4 – 2x^2 – 3 = 0$
Solution:

Given equation, $x^4 – 2x^2 – 3 = 0$
$x^4 – 3x^2 + x^2 – 3 = 0$
$x^2(x^2 - 3) + 1(x^2 - 3) = 0$
$(x^2 + 1) (x^2 - 3) = 0$
So, $x^2 + 1 = 0$ (which is not possible) or $x^2 – 3 = 0$
Hence,
$x^2 – 3 = 0$
$x = \pm \sqrt{3}$

(ii) $x^4 – 10x^2 + 9 = 0$
Solution:

Given equation, $x^4 – 10x^2 + 9 = 0$
$x^4 – x^2 – 9x^2 + 9 = 0$
$x^2(x^2 - 1) – 9(x^2 - 1) = 0$
$(x^2 - 9)(x^2 - 1) = 0$

So, we have
$x^2 – 9 = 0$ or $x^2 – 1 = 0$
Hence,
$x = \pm 3$ or $x = \pm 1$

Exercise 5(E)

1. Solve each of the following equations:

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0;\ x\neq 3,\ x\neq -\frac{3}{2}$$

Solution:

Given equation,

$$\frac{2x}{x-3}+\frac{1}{2x+3}+\frac{3x+9}{(x-3)(2x+3)}=0$$

$$\Rightarrow \frac{2x(2x+3)+1(x-3)+3x+9}{(x-3)(2x+3)}=0$$

$4x^2 + 6x + x – 3 + 3x + 9 = 0$
$4x^2 + 10x + 6 = 0$
$4x^2 + 4x + 6x + 6 = 0$
$4x(x + 1) + 6(x + 1) = 0$
$(4x + 6) (x + 1) = 0$
So, $4x + 6 = 0$ or $x + 1 = 0$
$x = -1$ or $x = -6/4 = -3/2$ (rejected as this value is excluded in the domain)
Therefore,
$x = -1$ is the only solution

2. $(2x + 3)^2 = 81$
Solution:

Given, $(2x + 3)^2 = 81$
Taking square root on both sides we have,
$2x + 3 = \pm 9$
$2x = \pm 9 – 3$
$x = (\pm 9 – 3)/ 2$
So,
$x = (9 – 3)/ 2$ or $(-9 – 3)/ 2$
Therefore,
$x = 3$ or $x = -6$

3. $a^2x^2 – b^2 = 0$
Solution:

Given equation, $a^2x^2 – b^2 = 0$
$(ax)^2 – b^2 = 0$
$(ax + b)(ax - b) = 0$
So,
$ax + b = 0$ or $ax – b = 0$
Therefore,
$x = -b/a$ or b/a

4. $x^2 – 11/4\ x + 15/8 = 0$
Solution:

Given equation, $x^2 – 11/4\ x + 15/8 = 0$
Taking L.C.M we have,
$(8x^2 – 22x + 15)/ 8 = 0$
$8x^2 – 22x + 15 = 0$
$8x^2 – 12x – 10x + 15 = 0$
$4x(2x - 3) – 5(2x - 3) = 0$
$(4x - 5)(2x - 3) = 0$
So, $4x – 5 = 0$ or $2x – 3 = 0$
Therefore,
$x = 5/4$ or $x = 3/2$

5. $x + 4/x = -4; x \neq 0$
Solution:

Given equation, $x + 4/x = -4$
$(x^2 + 4)/ x = -4$
$x^2 + 4 = -4x$
$x^2 + 4x + 4 = 0$
$x^2 + 2x + 2x + 4 = 0$
$x(x + 2) + 2(x + 2) = 0$
$(x + 2)(x + 2) = 0$
$(x + 2)^2 = 0$
Taking square - root we have,
$x + 2 = 0$
Therefore, $x = -2$

6. $2x^4 – 5x^2 + 3 = 0$
Solution:

Given equation, $2x^4 – 5x^2 + 3 = 0$
Let's take $x^2 = y$
Then, the equation becomes
$2y^2 – 5y + 3 = 0$
$2y^2 – 2y – 3y + 3 = 0$
$2y(y - 1) - 3(y - 1) = 0$
$(2y - 3)\ (y - 1) = 0$
So, $2y – 3 = 0$ or $y – 1 = 0$
$y = 3/2$ or $y = 1$
And, we have taken $y = x^2$
Thus,
$x^2 = 3/2$ or $x^2 = 1$
$x = \pm \sqrt{(3/2)}$ or $x = \pm 1$

7. $x^4 – 2x^2 – 3 = 0$
Solution:

Given equation, $x^4 – 2x^2 – 3 = 0$
Let's take $x^2 = y$
Then, the equation becomes
$y^2 – 2y – 3 = 0$
$y^2 – 3y + y – 3 = 0$
$y(y - 3) + 1(y - 3) = 0$
$(y + 1)(y - 3) = 0$
So, $y + 1 = 0$ or $y – 3 = 0$
$y = -1$ or $y = 3$
And, we have taken $y = x^2$
Thus,
$x^2 = - 1$(impossible, no real solution)
$x^2 = 3$
$x = \pm \sqrt{3}$

8. $9\left(x^2+\frac{1}{x^2}\right)-9\left(x+\frac{1}{x}\right)-52=0$
Solution:

Let us take $(x + 1/x) = y$ …. (1)
Now, squaring it on both sides
$(x + 1/x)^2 = y^2$
$x^2 + 1/x^2 + 2 = y^2$
So,
$x^2 + 1/x^2 = y^2 – 2$ ….. (2)
Using (1) and (2) in the given equation, we have
$9(y^2 – 2) – 9(y) – 52 = 0$
$9y^2 – 18 – 9y – 52 = 0$
$9y^2 – 9y – 70 = 0$
$9y^2 – 30y + 21y – 70 = 0$
$3y(3y - 10) + 7(3y - 10) = 0$
$(3y + 7)(3y - 10) = 0$
So, $3y + 7 = 0$ or $3y – 10 = 0$
$y = -7/3$ or $y = 10/3$
Now,

$x + 1/x = -7/3$	or	$x + 1/x = 10/3$
$(x^2 + 1)/x = -7/3$	or	$(x^2 + 1)/x = 10/3$
$3x^2 – 10x + 3 = 0$	or	$3x^2 + 7x + 3 = 0$

$3x^2 – 9x – x + 3 = 0$ or
$3x(x - 3) – 1(x - 3) = 0$
$(3x - 1)(x - 3) = 0$
So, x = 1/3 or 3

$$x=\frac{-7\pm\sqrt{(-7)^2-4(3)(3)}}{2(3)}$$
$$x=\frac{-7\pm\sqrt{13}}{6}$$

9. $2\left(x^2+\frac{1}{x^2}\right)-\left(x+\frac{1}{x}\right)=11$

Solution:

Let us take $(x + 1/x) = y$ …. (1)
Now, squaring it on both sides
$(x + 1/x)^2 = y^2$
$x^2 + 1/x^2 + 2 = y^2$
So,
$x^2 + 1/x^2 = y^2 – 2$ ….. (2)
Using (1) and (2) in the given equation, we have
$2(y^2 – 2) – (y) = 11$
$2y^2 – 4 – y = 11$
$2y^2 – y – 15 = 0$
$2y^2 – 6y + 5y – 15 = 0$
$2y(y - 3) + 5(y - 3) = 0$
$(2y + 5) (y - 3) = 0$
So,
$2y + 5 = 0$ or $y - 3 = 0$
$y = -5/2$ or $y = 3$
Now,

$x + 1/x = -5/2$	or	$x + 1/x = 3$
$(x^2 + 1)/x = -5/2$	or	$(x^2 + 1)/x = 3$
$2(x^2 + 1) = -5x$	or	$x^2 + 1 = 3x$
$2x^2 + 5x + 2 = 0$	or	$x^2 - 3x + 1 = 0$

$2x^2 + 4x + x + 2 = 0$ or
$2x(x + 2) + 1(x + 2) = 0$
$(2x + 1)(x + 2) = 0$
Hence, x = -1/2 or -2

$$x=\frac{-3\pm\sqrt{(-3)^2-4(1)(1)}}{2(1)}$$
$$x=\frac{-3\pm\sqrt{5}}{2}$$

10. $\left(x^2+\frac{1}{x^2}\right)-3\left(x-\frac{1}{x}\right)-2=0$

Solution:

Let us take $(x - 1/x) = y$ …. (1)
Now, squaring it on both sides

$(x - 1/x)^2 = y^2$
$x^2 + 1/x^2 - 2 = y^2$
So,
$x^2 + 1/x^2 = y^2 + 2$ ….. (2)
Using (1) and (2) in the given equation, we have
$(y^2 + 2) – 3(y) – 2 = 0$
$y^2 - 3y = 0$
$y(y - 3) = 0$
So, $y = 0$ or $y – 3 = 0$
Now,

$(x - 1/x) = 0$	or	$(x - 1/x) = 3$
$x^2 – 1 = 0$	or	$x^2 – 1 = 3x$
$x^2 = 1$	or	$x^2 – 3x – 1 = 0$

Therefore,
$x = \pm 1$ or $x=\frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$

$x=\frac{3 \pm \sqrt{13}}{2}$

Exercise 5(F)

1. Solve:

(i) (x + 5) (x - 5) = 24

Solution:

Given equation, (x + 5) (x - 5) = 24
$x^2 – 25 = 24$
$x^2 = 49$
Thus,
$x = \pm 7$

(ii) $3x^2 - 2\sqrt{6}x + 2 = 0$

Solution:

Given equation, $3x^2 - 2\sqrt{6}x + 2 = 0$
$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$
$\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$
$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$
So, $\sqrt{3}x - \sqrt{2} = 0$ or $\sqrt{3}x - \sqrt{2} = 0$
Therefore,
$x = \sqrt{(2/3)}, \sqrt{(2/3)}$ (equal roots)

(iii) $3\sqrt{2}x^2 – 5x - \sqrt{2} = 0$

Solution:

Given equation, $3\sqrt{2}x^2 – 5x - \sqrt{2} = 0$
$3\sqrt{2}x^2 – 6x + x - \sqrt{2} = 0$
$3\sqrt{2}x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$
$(3\sqrt{2}x + 1)(x - \sqrt{2}) = 0$
So, $3\sqrt{2}x + 1 = 0$ or $x - \sqrt{2} = 0$
Therefore,
$x = -1/ 3\sqrt{2}$ or $x = \sqrt{2}$

(iv) $2x – 3 = \sqrt{(2x^2 – 2x + 21)}$

Solution:

Given equation, $2x – 3 = \sqrt{(2x^2 – 2x + 21)}$
On squaring on both sides, we have
$(2x – 3)^2 = 2x^2 – 2x + 21$
$4x^2 + 9 – 12x = 2x^2 – 2x + 21$
$2x^2 – 10x – 12 = 0$
Dividing by 2, we get
$x^2 – 5x – 6 = 0$
$x^2 – 6x + x – 6 = 0$
$x(x - 6) – 1(x - 6) = 0$

(x - 1) (x - 6) = 0
So, x – 1 = 0 or x – 6 = 0
Thus, we get
x = 1 or x = 6
But, putting x = 1 the L.H.S become negative. And we know that the square root function always gives a positive vale.
Therefore,
x = 6 is the only solution.

2. One root of the quadratic equation $8x^2 + mx + 15 = 0$ is ¾. Find the value of m. Also, find the other root of the equation.
Solution:

Given equation, $8x^2 + mx + 15 = 0$
One of the roots is ¾, and hence it satisfies the given equation
So,
$8(3/4)^2 + m(3/4) + 15 = 0$
$8(9/16) + m(3/4) + 15 = 0$
$18/4 + 3m/4 + 15 = 0$
Taking L.C.M, we have
$(18 + 3m + 60)/4 = 0$
$18 + 3m + 60 = 0$
$3m = - 78$
$m = -26$
Now, putting the value of m in the given equation, we get
$8x^2 + (-26)x + 15 = 0$
$8x^2 - 26x + 15 = 0$
$8x^2 – 20x – 6x + 15 = 0$
$4x(2x - 5) – 3(2x - 5) = 0$
$(4x – 3) (2x - 5) = 0$
So, 4x – 3 = 0 or 2x – 5 = 0
Therefore.
x = ¾ or x = 5/2

3. Show that one root of the quadratic equation $x^2 + (3 – 2a)x – 6a = 0$ is -3. Hence, find its other root.
Solution:

Given quadratic equation, $x^2 + (3 – 2a)x – 6a = 0$
Now, putting x = -3 we have
$(-3)^2 + (3 – 2a)(-3) – 6a = 0$
$9 – 9 + 6a – 6a = 0$
$0 = 0$
Since, x = -3 satisfies the given equation -3 is one of the root of the quadratic equation.
$x^2 + (3 – 2a)x – 6a = 0$
$x^2 + 3x – 2ax – 6a = 0$

$x(x + 3) – 2a(x + 3) = 0$
$(x – 2a) (x + 3) = 0$
So, $x – 2a = 0$ or $x + 3 = 0$
$x = 2a$ or $x = -3$
Hence, the other root is 2a.

4. If $p – 15 = 0$ and $2x^2 + px + 25 = 0$: find the values of x.
Solution:

Given equations, $p – 15 = 0$ and $2x^2 + px + 25 = 0$
Thus, $p = 15$
Now, using p in the quadratic equation, we get
$2x^2 + (15)x + 25 = 0$
$2x^2 + 10x + 5x + 25 = 0$
$2x(x + 5) + 5(x + 5) = 0$
$(2x + 5) (x + 5) = 0$
So, $2x + 5 = 0$ or $x + 5 = 0$
Hence,
$x = -5/2$ or $x = -5$

5. Find the solution of the quadratic equation $2x^2 – mx – 25n = 0$; if $m + 5 = 0$ and $n – 1 = 0$.
Solution:

Given,
$m + 5 = 0$ and $n – 1 = 0$
so,
$m = -5$ and $n = 1$
Now, putting these values in the given quadratic equation $2x^2 – mx – 25n = 0$, we get
$2x^2 – (-5)x – 25(1) = 0$
$2x^2 + 5x – 25 = 0$
$2x^2 + 10x – 5x – 25 = 0$
$2x(x + 5) -5(x + 5) = 0$
$(2x - 5) (x + 5) = 0$
So, $2x – 5 = 0$ or $x + 5 = 0$
Hence,
$x = 5/2$ or $x = -5$

6. If m and n are roots of the equation: $1/x – 1/(x-2) = 3$: where $x \neq 0$ and $x \neq 2$; find m x n.
Solution:

Given equation, $1/x – 1/(x-2) = 3$
$(x - 2 - x)/ (x(x - 2)) = 3$
$-2 = 3(x^2 – 2x)$
$3x^2 – 6x + 2 = 0$
Solving by using quadratic formula, we get

$$\Rightarrow x = \frac{6 \pm \sqrt{6^2 - 4(3)(2)}}{2 \times 3}$$

$$\Rightarrow x = \frac{6 \pm \sqrt{12}}{2 \times 3}$$

$$\Rightarrow x = \frac{\sqrt{3} \pm 1}{\sqrt{3}}$$

And, since m and n are roots of the equation, we have
m = (√3 + 1)/ √3 n = (√3 - 1)/ √3
So,
m x n = (√3 + 1)/ √3 x (√3 - 1)/ √3 = [(√3)^2 – 1]/ (√3)^2
Thus,
m x n = 2/3