Exercise 6(E) Page No: 78

1. The distance by road between two towns A and B is 216 km, and by rail it is 208 km. A car travels at a speed of x km/hr and the train travels at a speed which is 16 km/hr faster than the car. Calculate:
(i) the time taken by the car to reach town B from A, in terms of x;
(ii) the time taken by the train to reach town B from A, in terms of x.
(iii) If the train takes 2 hours less than the car, to reach town B, obtain an equation in x and solve it.
(iv) Hence, find the speed of the train.
Solution:

Given,
Speed of car = x km/hr
Speed of train = (x + 16) km/hr
And, we know that
Time = Distance/ Speed

(i) Time taken by the car to reach town B from town A = 216/x hrs
(ii) Time taken by the train to reach town B from A = 208/(x + 16) hrs
(iii) According to the question, we have

$$\frac{216}{x} - \frac{208}{x+16} = 2$$

$$\frac{216x + 3456 - 208x}{x(x+16)} = 2$$

$$\frac{8x + 3456}{x(x+16)} = 2$$

$4x + 1728 = x^2 + 16x$
$x^2 + 12x - 1728 = 0$
$x^2 + 48x - 36x - 1728 = 0$
$x(x + 48) - 36(x + 48) = 0$
$(x + 48)(x - 36) = 0$
$x = -48, 36$
As speed cannot be negative,
$x = 36$
(iv) Therefore, the speed of the train is (x + 16) = (36 + 16)km/hr = 52 km/h

2. A trader buys x articles for a total cost of Rs 600.
(i) Write down the cost of one article in terms of x.
If the cost per article were Rs 5 more, the number of articles that can be bought for Rs 600 would be four less.
(ii) Write down the equation in x for the above situation and solve it for x.
Solution:

We have,
Number of articles = x

And, the total cost of articles = Rs 600
Then,
(i) Cost of one article = Rs 600/x

(ii) From the question we have,

$$\frac{600}{x-4}-\frac{600}{x}=5$$

$$\frac{600x-600x+2400}{x(x-4)}=5$$

$$\frac{480}{x(x-4)}=1$$

$x^2 – 4x – 480 = 0$
$x^2 – 24x – 20x – 480 = 0$
$x(x - 24) + 20(x - 24) = 0$
$(x - 24) (x + 20) = 0$
$x = 24$ or -20
As the number of articles cannot be negative, $x = 24$.

3. A hotel bill for a number of people for overnight stay is Rs 4800. If there were 4 people more, the bill each person had to pay, would have reduced by Rs 200. Find the number of people staying overnight.
Solution:

Let’s assume the number of people staying overnight as x.
Given, total hotel bill = Rs 4800
So, hotel bill for each person = Rs 4800/x
Then, according to the question

$$\frac{4800}{x}-\frac{4800}{x+4}=200$$

$$\frac{4800x+4800\times4-4800x}{x(x+4)}=200$$

$$\frac{96}{x^2+4x}=1$$

$x^2 + 4x – 96 = 0$
$x^2 + 12x – 8x – 96 = 0$
$x(x +12) – 8(x + 12) = 0$
$(x - 8) (x + 12) = 0$
So, $x = 8$ or -12
As, the number of people cannot be negative. We take $x = 8$.
Therefore, the number of people staying overnight is 8.

4. An aeroplane travelled a distance of 400 km at an average speed of x km/hr. On the return journey, the speed was increased by 40 km/hr. Write down an expression for the time taken for:
(i) the onward journey;
(ii) the return journey.
If the return journey took 30 minutes less than the onward journey, write down an equation in x

and find its value.
Solution:

Given,
Distance = 400 km
Average speed of the aeroplane = x km/hr
And, speed while returning = (x + 40) km/hr
We know that,
Time = Distance/ Speed

(i) Time taken for onward journey = 400/x hrs
(ii) Time take for return journey = 400/(x + 40) hrs
Then according to the question,

$$\frac{400}{x}-\frac{400}{x+40}=\frac{30}{60}$$

$$\frac{400x+16000-400x}{x(x+40)}=\frac{1}{2}$$

$$\frac{16000}{x(x+40)}=\frac{1}{2}$$

$x^2 + 40x – 32000 = 0$
$x^2 + 200x – 160x – 32000 = 0$
$x(x + 200) – 160(x + 200) = 0$
$(x + 200) (x – 160) = 0$
So, x = -200 or 160
As the speed cannot be negative, x = 160 is only valid.

5. Rs 6500 was divided equally among a certain number of persons. Had there been 15 persons more, each would have got Rs 30 less. Find the original number of persons.
Solution:

Let’s take the original number of persons to be x.
Total money which was divided = Rs 6500
Each person’s share = Rs 6500/x
Then, according to the question

$$\frac{6500}{x}-\frac{6500}{x+15}=30$$

$$\frac{6500x+6500\times 15-6500x}{x(x+15)}=30$$

$$\frac{3250}{x(x+15)}=1$$

$x^2 + 15x – 3250 = 0$
$x^2 + 65x – 50x – 3250 = 0$
$x(x + 65) - 50(x + 65) = 0$
$(x + 65) (x – 50) = 0$
So, x = -65 or 50
As, the number of persons cannot be negative. x = 50

Therefore, the original number of persons are 50.

6. A plane left 30 minutes later than the schedule time and in order to reach its destination 1500 km away in time, it has to increase its speed by 250 km/hr from its usual speed. Find its usual speed.
Solution:

Let's consider the usual speed of the plane to be x km/hr
The distance to travel = 1500km
We know that,
Time = Distance/ Speed
Then according to the question, we have

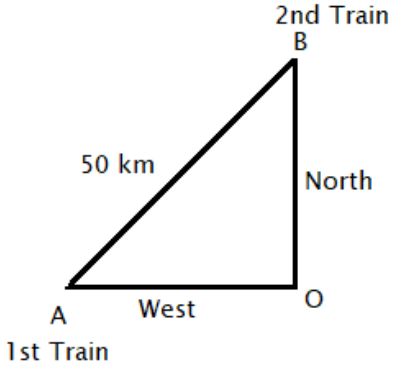
$$\frac{1500}{x} - \frac{1500}{x+250} = \frac{30}{60}$$

$$\frac{1500x + 1500 \times 250 - 1500x}{x(x+250)} = \frac{1}{2}$$

$$\frac{1500 \times 250}{x^2 + 250x} = \frac{1}{2}$$

$x^2 + 250x – 750000 = 0$
$x^2 + 1000x – 750x – 750000 = 0$
$x(x + 1000) – 750(x + 1000) = 0$
$(x + 1000) (x - 750) = 0$
So, x = -1000 or 750
As, speed cannot be negative. We take x = 750 as the solution.
Therefore, the usual speed of the plan is 750km/hr.

7. Two trains leave a railway station at the same time. The first train travels due west and the second train due north. The first train travels 5 km/hr faster than the second train. If after 2 hours, they are 50 km apart, find the speed of each train.
Solution:
Let take the speed of the second train to be x km/hr.
Then, the speed of the first train is (x + 5) km/hr
Let O be the position of the railway station from which the two trains leave.
Distance travelled by the first train in 2 hours = OA = speed x time = 2(x + 5) km
Distance travelled by the second train in 2 hours = OB = speed x time = 2x km



By Pythagoras Theorem, we have
$AB^2 = OA^2 + OB^2$
$(50)^2 = [2(x + 5)]^2 + (2x)^2$
$2500 = 4(x^2 + 10x + 25) + 4x^2$
$2500 = 8x^2 + 40x + 100$
$x^2 + 5x – 300 = 0$
$x^2 + 20x – 15x – 300 = 0$
$(x + 20) (x - 15) = 0$
So, $x = -20$ or $x = 15$
As x cannot be negative, we have $x = 15$
Thus, the speed of the second train is 15 km/hr and the speed of the first train is 20 km/hr.

8. The sum S of first n even natural numbers is given by the relation S = n(n + 1). Find n, if the sum is 420.
Solution:

Given relation, $S = n(n + 1)$
And, $S = 420$
So, $n(n + 1) = 420$
$n^2 + n - 420 = 0$
$n^2 + 21n - 20n - 420 = 0$
$n(n + 21) - 20(n + 21) = 0$
$(n + 21) (n - 20) = 0$
$n = -21, 20$
As, n cannot be negative.
Therefore, $n = 20$.

9. The sum of the ages of a father and his son is 45 years. Five years ago, the product of their ages (in years) was 124. Determine their present ages.
Solution:

Let’s assume the present ages of father and his son to be x years and (45 - x) years respectively.
So five years ago,
Father's age = (x - 5) years
Son's age = (45 - x - 5) years = (40 - x) years
From the question, the below equation can be formed
$(x - 5) (40 - x) = 124$
$40x - x^2 - 200 + 5x = 124$
$x^2 - 45x + 324 = 0$
$x^2 - 36x - 9x + 324 = 0$
$x(x - 36) - 9(x - 36) = 0$
$(x - 36) (x - 9) = 0$
$x = 36, 9$
So, if $x = 9$,
The father's age = 9 years and the son's age = (45 - x) = 36 years
This is not possible.

Hence, x = 36
Therefore,
The father's age = 36 years
The son's age = (45 - 36) years = 9 years