

## EXERCISE 3.3

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1. Find the identity element in the set  $I^+$  of all positive integers defined by  $a * b = a + b$  for all  $a, b \in I^+$ .

**Solution:**

Let  $e$  be the identity element in  $I^+$  with respect to  $*$  such that

$$a * e = a = e * a, \forall a \in I^+$$

$$a * e = a \text{ and } e * a = a, \forall a \in I^+$$

$$a + e = a \text{ and } e + a = a, \forall a \in I^+$$

$$e = 0, \forall a \in I^+$$

Thus, 0 is the identity element in  $I^+$  with respect to  $*$ .

2. Find the identity element in the set of all rational numbers except  $-1$  with respect to  $*$  defined by  $a * b = a + b + ab$

**Solution:**

Let  $e$  be the identity element in  $I^+$  with respect to  $*$  such that

$$a * e = a = e * a, \forall a \in Q - \{-1\}$$

$$a * e = a \text{ and } e * a = a, \forall a \in Q - \{-1\}$$

$$a + e + ae = a \text{ and } e + a + ea = a, \forall a \in Q - \{-1\}$$

$$e + ae = 0 \text{ and } e + ea = 0, \forall a \in Q - \{-1\}$$

$$e(1 + a) = 0 \text{ and } e(1 + a) = 0, \forall a \in Q - \{-1\}$$

$$e = 0, \forall a \in Q - \{-1\} \text{ [because } a \text{ not equal to } -1]$$

Thus, 0 is the identity element in  $Q - \{-1\}$  with respect to  $*$ .