EXERCISE 4.1 PAGE NO: 4.6

1. Find the principal value of the following:

(i) $sin^{-1}(-\sqrt{\frac{3}{2}})$

(ii) $sin^{-1}(\cos\frac{2\pi}{3})$

(iii) $\sin^{-1}(\frac{\sqrt{3}-1}{2\sqrt{2}})$

(iv) $\sin^{-1}(\frac{\sqrt{3}+1}{2\sqrt{2}})$

(v) $\sin^{-1}(\cos\frac{3\pi}{4})$

(vi) $\sin^{-1}(\tan\frac{5\pi}{4})$

Solution:

$(i) Let \sin^{-1}(\frac{-\sqrt{3}}{2}) = y$

$Then\ \ siny = (\frac{-\sqrt{3}}{2})$

$= -\sin(\frac{\pi}{3})$

$= sin(-\frac{\pi}{3})$

$We\ know\ that\ the\ principal\ value\ of\ \sin^{-1} is \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$And - sin\frac{\pi}{3} = sin(\frac{-\pi}{3})$

$Therefore\ principal\ value\ of \sin^{-1}(\frac{-\sqrt{3}}{2}) = \frac{-\pi}{3}$

$(ii) Let \sin^{-1}(\cos\frac{2\pi}{3}) = y$

$Then\ \ siny = \cos(\frac{2\pi}{3})$

$= -\sin(\frac{\pi}{2} + \frac{\pi}{6})$

$= -sin(\frac{\pi}{6})$

$We\ know\ that\ the\ principal\ value\ of\ \sin^{-1} is \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$And\ \ -sin(\frac{\pi}{6}) = cos(\frac{2\pi}{3})$

$Therefore\ principal\ value\ of\ \sin^{-1}(\cos\frac{2\pi}{3})\ is \frac{-\pi}{6}$

(iii) Given functions can be written as

$$\sin^{-1}\left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} - \frac{1}{2\sqrt{2}}\right)$$

Taking 1/√2 as common from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} - \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$$

Taking √3/2 as common, and 1/√2 from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1 - \left(\frac{1}{\sqrt{2}}\right)^2} - \frac{1}{\sqrt{2}} \times \sqrt{1 - \left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

On simplifying, we get

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

By substituting the values,

$$= \frac{\pi}{3} - \frac{\pi}{4}$$

Taking LCM and cross multiplying we get,

$$= \frac{\pi}{12}$$

(iv) The given question can be written as

$$\sin^{-1}\left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}\right)$$

Taking 1/√2 as common from the above equation we get

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}}\right)$$

Taking √3/2 as common, and 1/√2 from the above equation we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2} \times \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2} + \frac{1}{\sqrt{2}} \times \sqrt{1-\left(\frac{\sqrt{3}}{2}\right)^2}\right)$$

On simplifying we get,

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

By substituting the corresponding values we get

$$= \frac{\pi}{3} + \frac{\pi}{4}$$

$$= \frac{7\pi}{12}$$

(v) Let

$\sin^{-1}\left(\cos\frac{3\pi}{4}\right) = y$

Then above equation can be written as

$\sin y = \cos\frac{3\pi}{4} = -\sin\left(\pi - \frac{3\pi}{4}\right) = -\sin(\frac{\pi}{4})$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore above equation becomes,

$-\sin\left(\frac{\pi}{4}\right) = \cos\frac{3\pi}{4}$

Therefore the principal value of $\sin^{-1}\left(\cos\frac{3\pi}{4}\right)$ is $-\frac{\pi}{4}$

(vi) Let

$y = \sin^{-1}\left(\tan\frac{5\pi}{4}\right)$

Therefore above equation can be written as

$\text{Sin } y = \left(\tan\frac{5\pi}{4}\right) = \tan\left(\pi + \frac{\pi}{4}\right) = \tan\frac{\pi}{4} = 1 = \sin\left(\frac{\pi}{2}\right)$

We know that the principal value of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$\sin\left(\frac{\pi}{2}\right) = \tan\frac{5\pi}{4}$

Therefore the principal value of $\sin^{-1}\left(\tan\frac{5\pi}{4}\right)$ is $\frac{\pi}{2}$.

2.

(i) $\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}}$

(ii) $\sin^{-1}\left\{\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)\right\}$

Solution:

(i) The given question can be written as,

$$\sin^{-1}\frac{1}{2} - 2\sin^{-1}\frac{1}{\sqrt{2}} = \sin^{-1}\frac{1}{2} - \sin^{-1}\left(2 \times \frac{1}{\sqrt{2}}\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}\right)$$

On simplifying, we get

$$= \sin^{-1}\frac{1}{2} - \sin^{-1}(1)$$

By substituting the corresponding values, we get

$$= \frac{\pi}{6} - \frac{\pi}{2}$$

$$= -\frac{\pi}{3}$$

(ii) Given question can be written as

We know that $\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \cos(\pi/3)$

$$= \sin^{-1}\left\{\cos\left(\frac{\pi}{3}\right)\right\}$$

Now substituting the values we get,

$$= \sin^{-1}\left\{\frac{\sqrt{3}}{2}\right\}$$

$$= \frac{\pi}{6}$$