

EXERCISE 21.1**PAGE NO: 21.8****1. Find the volume of a cuboid whose****(i) length = 12 cm, breadth = 8 cm, height = 6 cm****(ii) length = 1.2 m, breadth = 30 cm, height = 15 cm****(iii) length = 15 cm, breadth = 2.5 dm, height = 8 cm.****Solution:****(i)** The given details are:

Length of a cuboid = 12 cm

Breadth of a cuboid = 8 cm

Height of a cuboid = 6 cm

By using the formula

$$\begin{aligned}\text{Volume of a cuboid} &= \text{length} \times \text{breadth} \times \text{height} \\ &= 12 \times 8 \times 6 \\ &= 576 \text{ cm}^3\end{aligned}$$

(ii) The given details are:

Length of a cuboid = 1.2 m = 120 cm

Breadth of a cuboid = 30 cm

Height of a cuboid = 15 cm

By using the formula

$$\begin{aligned}\text{Volume of a cuboid} &= \text{length} \times \text{breadth} \times \text{height} \\ &= 120 \times 30 \times 15 \\ &= 54000 \text{ cm}^3\end{aligned}$$

(iii) The given details are:

Length of a cuboid = 15 cm

Breadth of a cuboid = 2.5 dm = 25 cm

Height of a cuboid = 8 cm

By using the formula

$$\begin{aligned}\text{Volume of a cuboid} &= \text{length} \times \text{breadth} \times \text{height} \\ &= 15 \times 25 \times 8 \\ &= 3000 \text{ cm}^3\end{aligned}$$

2. Find the volume of a cube whose side is**(i) 4 cm (ii) 8 cm****(iii) 1.5 dm (iv) 1.2 m****(v) 25 mm****Solution:**

(i) Given details are,

Side of cube = 4 cm

$$\begin{aligned}\text{Volume of cube} &= (\text{side})^3 \\ &= 4^3 = 64 \text{ cm}^3\end{aligned}$$

(ii) Given details are,

Side of cube = 8 cm

$$\begin{aligned}\text{Volume of cube} &= (\text{side})^3 \\ &= 8^3 = 512 \text{ cm}^3\end{aligned}$$

(iii) Given details are,

Side of cube = 1.5 dm

$$\begin{aligned}\text{Volume of cube} &= (\text{side})^3 \\ &= 1.5^3 = 3.375 \text{ dm}^3 = 3375 \text{ cm}^3\end{aligned}$$

(iv) Given details are,

Side of cube = 1.2 m

$$\begin{aligned}\text{Volume of cube} &= (\text{side})^3 \\ &= 1.2^3 = 1.728 \text{ m}^3\end{aligned}$$

(v) Given details are,

Side of cube = 25 mm

$$\begin{aligned}\text{Volume of cube} &= (\text{side})^3 \\ &= 25^3 = 15625 \text{ mm}^3 = 15.625 \text{ cm}^3\end{aligned}$$

3. Find the height of a cuboid of volume 100 cm^3 , whose length and breadth are 5 cm and 4 cm respectively.

Solution:

Given details are,

Volume of a cuboid = 100 cm^3

Length of a cuboid = 5 cm

Breadth of a cuboid = 4 cm

Let height of cuboid be 'h' cm

We know that, $l \times b \times h = 100 \text{ cm}^3$

$$h = 100 / (l \times b)$$

$$= 100 / (5 \times 4)$$

$$= 5 \text{ cm}$$

4. A cuboidal vessel is 10 cm long and 5 cm wide. How high it must be made to hold

300 cm³ of a liquid?

Solution:

Given details are,

Volume of a liquid in the vessel = 300 cm³

Length of a cuboidal vessel = 10 cm

Breadth of a cuboidal vessel = 5 cm

Let height of cuboidal vessel be 'h' cm

We know that, $l \times b \times h = 300 \text{ cm}^3$

$$h = 300 / (l \times b)$$

$$= 300 / (10 \times 5)$$

$$= 6 \text{ cm}$$

5. A milk container is 8 cm long and 50 cm wide. What should be its height so that it can hold 4 litres of milk?

Solution:

Given details are,

Volume = 4 litres = 4000 cm³

Length of a milk container = 8 cm

Breadth of a milk container = 50 cm

Let height of milk container be 'h' cm

We know that, $l \times b \times h = 4000 \text{ cm}^3$

$$h = 4000 / (l \times b)$$

$$= 4000 / (50 \times 8)$$

$$= 10 \text{ cm}$$

6. A cuboidal wooden block contains 36 cm³ wood. If it be 4 cm long and 3 cm wide, find its height.

Solution:

Given details are,

Volume of wooden block = 36 cm³

Length of the wooden block = 4 cm

Breadth of a wooden block = 3 cm

Let height of wooden block be 'h' cm

We know that, $l \times b \times h = 36 \text{ cm}^3$

$$h = 36 / (l \times b)$$

$$= 36 / (4 \times 3)$$

$$= 3 \text{ cm}$$

7. What will happen to the volume of a cube, if its edge is

(i) halved (ii) trebled?

Solution:

Let us consider edge of a cube be 'a' cm

Volume of a cube will be 'a³'cm

(i) When halved

Edge = a/2

Volume = (a/2)³ = a³/2³ = a³/8 = 1/8times

(ii) When trebled

Edge = 3a

Volume = (3a)³ = 27a³ = 27times

8. What will happen to the volume of a cuboid if its:

(i) Length is doubled, height is same and breadth is halved?

(ii) Length is doubled, height is doubled and breadth is same?

Solution:

Let us consider,

Length of a cuboid be 'l'

Breadth of a cuboid be 'b'

Height of a cuboid be 'h'

So, Volume of a cuboid = l × b × h

Now,

(i) Length of a cuboid becomes = 2l

Breadth = b/2

Height = h

Volume of cuboid = 2l × b/2 × h = l × b × h (remains same)

(ii) Length of a cuboid becomes = 2l

Breadth = b

Height = 2h

Volume of cuboid = 2l × b × 2h = 4lbh (four times)

9. Three cuboids of dimensions 5 cm × 6cm × 7cm, 4cm × 7cm × 8cm and 2 cm × 3 cm × 13 cm are melted and a cube is made. Find the side of cube.

Solution:

Given details are,

Volume of First cuboid = 5 × 6 × 7 = 210 cm³

Volume of second cuboid = 4 × 7 × 8 = 224 cm³

$$\text{Volume of third cuboid} = 2 \times 3 \times 13 = 78 \text{ cm}^3$$

$$\text{So, Volume of a cube} = 210 + 224 + 78 = 512 \text{ cm}^3$$

Let side of a cube be 'a'

$$a^3 = 512$$

$$\therefore a = 8 \text{ cm}$$

10. Find the weight of solid rectangular iron piece of size 50 cm × 40 cm × 10 cm, if 1 cm³ of iron weights 8 gm.

Solution:

Given details are,

$$\text{Dimension of rectangular iron piece} = 50\text{cm} \times 40\text{cm} \times 10\text{cm}$$

$$\text{Volume of solid rectangular} = 50 \times 40 \times 10 = 20000 \text{ cm}^3$$

$$\text{Weight of 1 cm}^3 \text{ iron} = 8 \text{ gm.}$$

$$\begin{aligned} \therefore \text{Weight of } 20000 \text{ cm}^3 \text{ iron} &= 8 \times 20000 \\ &= 160000 \text{ gm.} \\ &= 160 \text{ kg} \end{aligned}$$

11. How many wooden cubical blocks of side 25 cm can be cut from a log of wood of size 3 m by 75 cm by 50 cm, assuming that there is no wastage?

Solution:

Given details are,

$$\text{Dimensions of log of wood} = 3\text{m} \times 75\text{cm} \times 50\text{cm}$$

$$\text{Side of cubical block} = 25\text{cm}$$

We know that,

$$\text{Number of cubical block that can be made from wooden log} =$$

$$\text{Volume of wooden block} / \text{volume of cubical block}$$

$$= (300 \times 75 \times 50) / (25 \times 25 \times 25)$$

$$= 72 \text{ blocks}$$

12. A cuboidal block of silver is 9 cm long, 4 cm broad and 3.5 cm in height. From it, beads of volume 1.5 cm³ each are to be made. Find the number of beads that can be made from the block.

Solution:

Given details are,

$$\text{Length of a cuboidal block of silver} = 9\text{cm}$$

$$\text{Breadth} = 4\text{cm}$$

$$\text{Height} = 3.5\text{cm}$$

$$\text{Volume of a cuboid} = l \times b \times h$$

$$= 9 \times 4 \times 3.5 = 126\text{cm}^3$$

So, Number of beads of volume 1.5cm^3 that can be made from the block =
Volume of silver block/volume of one bead
 $= 126\text{cm}^3/1.5\text{cm}^3$
 $= 84$ beads

13. Find the number of cuboidal boxes measuring 2 cm by 3 cm by 10 cm which can be stored in a carton whose dimensions are 40 cm, 36 cm, and 24 cm.

Solution:

Given details are,

Dimensions of cuboidal boxes is $= 2\text{cm} \times 3\text{cm} \times 10\text{cm}$

Dimensions of carton is $= 40\text{cm} \times 36\text{cm} \times 24\text{cm}$

So,

Number of boxes that can be stored in carton = volume of carton / volume of one box
 $= (40 \times 36 \times 24) / (2 \times 3 \times 10)$
 $= 576$ cuboidal boxes

14. A cuboidal block of solid iron has dimensions 50 cm, 45 cm, and 34 cm, how many cuboids of size 5 cm by 3 cm by 2 cm can be obtained from this block?

Assume cutting causes no wastage.

Solution:

Given details are,

Dimensions of cuboidal block of iron is $= 50\text{cm} \times 45\text{cm} \times 34\text{cm}$

Size of small cuboids cutting from it is $= 5\text{cm} \times 3\text{cm} \times 2\text{cm}$

So,

Number of small cuboids that can be cut =
Volume of large iron cuboid/ volume of small cuboid
 $= (50 \times 45 \times 34) / (5 \times 3 \times 2)$
 $= 2550$ cuboidal blocks

15. A cube A has side thrice as long as that of cube B. What is the ratio of the volume of cube A to that of cube B?

Solution:

Given details are,

Let side of cube B be 'x' cm

Then, side of cube A = $3x$ cm

So now,

Ratio = volume of cube A / volume of cube B
 $= (3x)^3 / (x)^3$
 $= 27x^3 / x^3 = 27/1 = 27:1$

16. An ice-cream brick measures 20 cm by 10 cm by 7 cm. How many such bricks can be stored in deep fridge whose inner dimensions are 100 cm by 50 cm by 42 cm?

Solution:

Given details are,

Dimensions of ice cream brick = $20 \text{ cm} \times 10 \text{ cm} \times 7 \text{ cm}$

Dimensions of fridge is = $100 \text{ cm} \times 50 \text{ cm} \times 42 \text{ cm}$

So,

Number of bricks that can be put in fridge = volume of fridge / volume of one ice brick
 $= (100 \times 50 \times 42) / (20 \times 10 \times 7)$
 $= 150$ ice cream bricks

17. Suppose that there are two cubes, having edges 2 cm and 4 cm, respectively. Find the volumes V_1 and V_2 of the cubes and compare them.

Solution:

Given details are,

Edge of one cube $a_1 = 2 \text{ cm}$

Edge of second cube $a_2 = 4 \text{ cm}$

So, volume $v_1 = 2^3 = 8 \text{ cm}^3$

Volume $v_2 = 4^3 = 64 \text{ cm}^3$

$$v_2 = 8v_1$$

18. A tea-packet measures 10 cm × 6 cm × 4 cm. How many such tea-packets can be placed in a cardboard box of dimensions 50 cm × 30 cm × 0.2 m?

Solution:

Given details are,

Dimensions of tea packet = $10 \text{ cm} \times 6 \text{ cm} \times 4 \text{ cm}$

Dimension of cardboard box = $50 \text{ cm} \times 30 \text{ cm} \times 0.2 \text{ m}$

So,

Number of tea packets can be put in cardboard box =
Volume of cardboard box / volume of tea packet
 $= (50 \times 30 \times 20) / (10 \times 6 \times 4)$
 $= 125$ tea packets

19. The weight of a metal block of size 5 cm by 4 cm by 3 cm is 1 kg. Find the weight of a block of the same metal of size 15 cm by 8 cm by 3 cm.

Solution:

Given details are,

Dimensions of metal block = $5 \text{ cm} \times 4 \text{ cm} \times 3 \text{ cm}$

Weight of block = 1 kg

Volume of box = $5 \times 4 \times 3 = 60 \text{ cm}^3$

Dimension of new block = $15 \text{ cm} \times 8 \text{ cm} \times 3 \text{ cm}$

Volume of new box = $15 \times 8 \times 3 = 360 \text{ cm}^3$

We know that,

$60 \text{ cm}^3 = 1 \text{ kg}$

$360 \text{ cm}^3 = 6 \times 60 \text{ cm}^3$

$= 6 \times 1$

$= 6 \text{ kg}$

20. How many soap cakes can be placed in a box of size $56 \text{ cm} \times 0.4 \text{ m} \times 0.25 \text{ m}$, if the size of a soap cake is $7 \text{ cm} \times 5 \text{ cm} \times 2.5 \text{ cm}$?

Solution:

Given details are,

Dimensions of box = $56 \text{ cm} \times 0.4 \text{ m} \times 0.25 \text{ m}$

Dimensions of soap cake = $7 \text{ cm} \times 5 \text{ cm} \times 2.5 \text{ cm}$

So,

Number of soap cakes that can be placed in box = volume of box / volume of soap cake
 $= (56 \times 40 \times 25) / (7 \times 5 \times 2.5)$
 $= 640 \text{ soap cakes}$

21. The volume of a cuboidal box is 48 cm^3 . If its height and length are 3 cm and 4 cm respectively, find its breadth.

Solution:

Given details are,

Volume of a cuboidal box = 48 cm^3

Length of a cuboidal box = 4 cm

Height of a cuboidal box = 3 cm

Let breadth of wooden block be 'b' cm

We know that, $l \times b \times h = 48 \text{ cm}^3$

$b = 48 / (l \times h)$

$= 48 / (4 \times 3)$

$= 4 \text{ cm}$

EXERCISE 21.2**PAGE NO: 21.15**

1. Find the volume in cubic metres (cu. m) of each of the cuboids whose dimensions are:

(i) length = 12 m, breadth = 10 m, height = 4.5 m

(ii) length = 4 m, breadth = 2.5m, height = 50 cm

(iii) length = 10m, breadth = 25 dm, height = 25 cm.

Solution:

(i) Given details are,

Length of a cuboid = 12 m

Breadth of a cuboid = 10m

Height of a cuboid = 4.5 m

By using the formula

$$\begin{aligned}\text{Volume of cuboid} &= l \times b \times h \\ &= 12 \times 10 \times 4.5 \\ &= 540 \text{ m}^3\end{aligned}$$

(ii) Given details are,

Length of a cuboid = 4 m

Breadth of a cuboid = 2.5 m

Height of a cuboid = 50 cm = 0.50m

By using the formula

$$\begin{aligned}\text{Volume of a cuboid} &= l \times b \times h \\ &= 4 \times 2.5 \times 0.50 \\ &= 5 \text{ m}^3\end{aligned}$$

(iii) Given details are,

Length of a cuboid = 10m

Breadth of a cuboid = 25 dm = 2.5 m

Height of a cuboid = 25 cm = 0.25 m

By using the formula

$$\begin{aligned}\text{Volume of a cuboid} &= l \times b \times h \\ &= 10 \times 2.5 \times 0.25 \\ &= 6.25 \text{ m}^3\end{aligned}$$

2. Find the volume in cubic decimetre of each of the cubes whose side is

(i) 1.5 m

(ii) 75cm

(iii) 2 dm 5 cm

Solution:

(i) Given details are,

Side of cube = 1.5m = 15 dm

So, Volume of cube = $15^3 = 3375 \text{ dm}^3$

(ii) Given details are,

Side of cube = 75cm = 7.5 dm

So, Volume of cube = $7.5^3 = 421.875 \text{ dm}^3$

(iii) Given details are,

Side of cube = 2dm 5cm = 2.5 dm

So, Volume of cube = $2.5^3 = 15.625 \text{ dm}^3$

3. How much clay is dug out in digging a well measuring 3 m by 2 m by 5 m?**Solution:**

Given details are,

Dimensions of well = $3\text{m} \times 2\text{m} \times 5\text{m}$

So,

Volume of clay dug out from well is = $l \times b \times h$
= $3 \times 2 \times 5$
= 30 m^3

4. What will be the height of a cuboid of volume 168 m^3 , if the area of its base is 28 m^2 ?**Solution:**

Given details are,

Volume of a cuboid = 168 m^3

Area of base = $l \times b = 28\text{m}^2$

Let height of cuboid be 'h' m

We know that,

Volume = $l \times b \times h$

$h = \text{volume} / l \times b$

$$= 168/28$$

$$= 6\text{m}$$

\therefore Height of cuboid is 6 m

5. A tank is 8 m long, 6 m broad and 2 m high. How much water can it contain?**Solution:**

Given details are,

Dimensions of a tank = $8\text{m} \times 6\text{m} \times 2\text{m}$

We know that,

$$\begin{aligned}\text{Volume of tank} &= l \times b \times h \\ &= 8 \times 6 \times 2 \\ &= 96 \text{ m}^3 \\ &= 96000 \text{ litres}\end{aligned}$$

\therefore The tank can contain 96000 litres of water.

6. The capacity of a certain cuboidal tank is 50000 litres of water. Find the breadth of the tank, if its height and length are 10 m and 2.5 m respectively.

Solution:

Given details are,

Capacity (volume) of cuboidal tank is = 50000 litre = 50 m^3

Height of tank = 10 m

Length of tank = 2.5 m

Let breadth of tank be 'b' m

We know that,

$$\text{Volume} = l \times b \times h$$

$$b = \text{volume} / (l \times h)$$

$$= 50 / (10 \times 2.5)$$

$$= 2\text{m}$$

\therefore Breadth of tank is 2m

7. A rectangular diesel tanker is 2m long, 2m wide and 40cm deep. How many litres of diesel can it hold?

Solution:

Given details are,

Length of a tanker = 2m

Breadth of a tanker = 2m

Height of a tanker = 40cm = 0.4m

So, Dimensions of rectangular diesel tank = $2\text{m} \times 2\text{m} \times 0.4\text{m}$

$$\begin{aligned}\text{Volume of tank (amount of diesel it can hold)} &= l \times b \times h \\ &= 2 \times 2 \times 0.4 \\ &= 1.6\text{m}^3 \\ &= 1600 \text{ litres}\end{aligned}$$

\therefore A rectangular diesel tanker can hold 1600 litres of diesel.

8. The length, breadth and height of a room are 5 m, 4.5 m and 3 m respectively. Find the volume of the air it contains.

Solution:

Given details are,

Length of a room = 5m

Breadth of a room = 4.5m

Height of a room = 3m

So, Dimensions of a room are = $5\text{m} \times 4.5\text{m} \times 3\text{m}$

$$\begin{aligned}\text{Volume of air} &= l \times b \times h \\ &= 5 \times 4.5 \times 3 \\ &= 67.5\text{m}^3\end{aligned}$$

\therefore The room contains 67.5m^3 volume of the air.

9. A water tank is 3 m long, 2 m broad and 1 m deep. How many litres of water can it hold?

Solution:

Given details are,

Length of water tank = 3m

Breadth of water tank = 2m

Height of water tank = 1m

So, Dimensions of water tank is = $3\text{m} \times 2\text{m} \times 1\text{m}$

$$\begin{aligned}\text{Volume the water tank can hold} &= l \times b \times h \\ &= 3 \times 2 \times 1 \\ &= 6\text{m}^3 \\ &= 6000 \text{ litres}\end{aligned}$$

\therefore The water tank can hold 6000 litres of water.

10. How many planks each of which is 3 m long, 15 cm broad and 5 cm thick can be prepared from a wooden block 6 m long, 75 cm broad and 45 cm thick?

Solution:

Given details are,

Dimensions of one plank = $3\text{m} \times 15\text{cm} \times 5\text{cm} = 300\text{cm} \times 15\text{cm} \times 5\text{cm}$

Dimensions of wooden block = $6\text{m} \times 75\text{cm} \times 45\text{cm} = 600\text{cm} \times 75\text{cm} \times 45\text{cm}$

We know that,

$$\begin{aligned}\text{Number of planks that can be prepared} &= \text{volume of wooden block} / \text{volume of one plank} \\ &= (600 \times 75 \times 45) / (300 \times 15 \times 5) \\ &= 90 \text{ planks}\end{aligned}$$

\therefore 90 planks are required to prepare the block.

11. How many bricks each of size 25 cm × 10 cm × 8 cm will be required to build a wall 5 m long, 3 m high and 16 cm thick, assuming that the volume of sand and

cement used in the construction is negligible?

Solution:

Given details are,

Size of one brick = $25\text{cm} \times 10\text{cm} \times 8\text{cm}$

Dimensions of wall = $5\text{m} \times 3\text{m} \times 16\text{cm} = 500\text{ cm} \times 300\text{ cm} \times 16\text{cm}$

We know that,

$$\begin{aligned}\text{Number of bricks required to build a wall} &= \text{volume of wall} / \text{volume of one brick} \\ &= (500 \times 300 \times 16) / (25 \times 10 \times 8) \\ &= 1200 \text{ bricks}\end{aligned}$$

\therefore 1200 bricks are required to build the wall.

12. A village, having a population of 4000, required 150 litres water per head per day. It has a tank which is 20 m long, 15 m broad and 6 m high. For how many days will the water of this tank last?

Solution:

Given details are,

Population of village = 4000

Dimensions of water tank = $20\text{m} \times 15\text{m} \times 6\text{m}$

Water required per head per day = 150 litres

Total requirement of water per day = $150 \times 4000 = 600000$ litres

$$\begin{aligned}\text{Volume of water tank} &= l \times b \times h \\ &= 20 \times 15 \times 6 \\ &= 1800\text{m}^3 \\ &= 1800000 \text{ litres}\end{aligned}$$

We know that,

$$\begin{aligned}\text{Number of days water last in the tank} &= \text{volume of tank} / \text{total requirement} \\ &= 1800000/600000 \\ &= 3 \text{ days}\end{aligned}$$

\therefore Water in the tank last for 3 days.

13. A rectangular field is 70 m long and 60 m broad. A well of dimensions 14 m \times 8 m \times 6 m is dug outside the field and the earth dug-out from this well is spread evenly on the field. How much will the earth level rise?

Solution:

Given details are,

Dimensions of rectangular field = $70\text{m} \times 60\text{m}$

Dimensions of well = $14\text{m} \times 8\text{m} \times 6\text{m}$

$$\begin{aligned}\text{Amount of earth dug out from well (volume)} &= l \times b \times h \\ &= 14 \times 8 \times 6 = 672\text{m}^3\end{aligned}$$

We know that,

$$\begin{aligned}\text{Rise in earth level} &= \text{dimensions of rectangular field} / \text{amount of earth dug up} \\ &= (70 \times 60) / 672 \\ &= 0.16\text{m} \\ &= 16\text{cm}\end{aligned}$$

∴ Rise in earth level on a rectangular field is 16cm.

14. A swimming pool is 250 m long and 130 m wide. 3250 cubic metres of water is pumped into it. Find the rise in the level of water.

Solution:

Given details are,

$$\text{Dimensions of swimming pool} = 250 \text{ m} \times 130\text{m}$$

$$\text{Volume of water pumped in it} = 3250 \text{ m}^3$$

We know that,

$$\begin{aligned}\text{Rise in water level in pool} &= \text{volume of water pumped} / \text{dimensions of swimming pool} \\ &= 3250 / (250 \times 130) \\ &= 0.1\text{m}\end{aligned}$$

∴ Rise in level of water is 0.1m

15. A beam 5 m long and 40 cm wide contains 0.6 cubic metre of wood. How thick is the beam?

Solution:

Given details are,

$$\text{Length of beam} = 5 \text{ m}$$

$$\text{Width of beam} = 40 \text{ cm} = 0.4 \text{ m}$$

$$\text{Volume of wood in beam} = 0.6 \text{ m}^3$$

Let thickness of beam be 'h' m

We know that,

$$\text{Volume} = l \times b \times h$$

$$h = \text{volume} / (l \times b)$$

$$= 0.6 / (5 \times 0.4)$$

$$= 0.3\text{m}$$

∴ Thickness of the beam is 0.3m

16. The rainfall on a certain day was 6 cm. How many litres of water fell on 3 hectares of field on that day?

Solution:

Given details are,

$$\text{Area of field} = 3 \text{ hectare} = 3 \times 10000 \text{ m}^2 = 30000 \text{ m}^2$$

Depth of water on the field = 6cm = $6/100 = 0.06$ m

Volume of water = area of field \times depth of water
 $= 30000 \times 0.06$
 $= 1800 \text{ m}^3$

We know that $1\text{m}^3 = 1000$ litre

So, $1800 \text{ m}^3 = 1800 \times 1000$
 $= 18 \times 10^5$ litre

$\therefore 18 \times 10^5$ litres of water fell on 3hectares of field.

17. An 8 m long cuboidal beam of wood when sliced produces four thousand 1 cm cubes and there is no wastage of wood in this process. If one edge of the beam is 0.5 m, find the third edge.

Solution:

Given details are,

Length of cuboidal beam = 8m

One edge of beam = 0.5m

Let the third edge of beam be 'h' m

Number of cubes of side 1cm (.01 m) produced = 4000

We know that,

Volume of beam = volume of each cube \times no. of cubes

$$8 \times 0.5 \times h = 4000 \times (0.01)^3$$

$$h = 0.004/4$$

$$= 0.001\text{m}$$

\therefore Length of third edge is 0.001 m

18. The dimensions of a metal block are 2.25 m by 1.5 m by 27 cm. It is melted and recast into cubes, each of the side 45 cm. How many cubes are formed?

Solution:

Given details are,

Dimensions of metal block = $2.25\text{m} \times 1.5\text{m} \times 27\text{cm} = 2.25\text{m} \times 1.5\text{m} \times 0.27\text{m}$

Side of each cube formed = $45\text{cm} = 0.45 \text{ m}$

We know that,

Number of cubes can formed = volume of metal block / volume of one cube

$$= (2.25 \times 1.5 \times 0.27) / (0.45 \times 0.45 \times 0.45)$$

$$= 0.91125 / 0.091125$$

$$= 10 \text{ cubes}$$

\therefore 10 cubes are formed.

19. A solid rectangular piece of iron measures 6 m by 6 cm by 2 cm. Find the weight

of this piece, if 1 cm^3 of iron weighs 8 gm.

Solution:

Given details are,

Dimensions of solid rectangular piece = $6\text{m} \times 6\text{cm} \times 2\text{cm}$

Volume of rectangular iron = $600\text{cm} \times 6\text{cm} \times 2\text{cm} = 7200\text{cm}^3$

Weight of 1cm^3 iron = 8 gm.

$$\begin{aligned} \text{So, weight of } 7200\text{cm}^3 &= 7200 \times 8 \\ &= 57600 \text{ gm.} \\ &= 57.6 \text{ kg} \end{aligned}$$

\therefore The weight of the piece is 57.6 kg

20. Fill in the blanks in each of the following so as to make the statement true :

(i) $1 \text{ m}^3 = \dots\dots\dots \text{cm}^3$

(ii) 1 litre = $\dots\dots\dots$ cubic decimetre

(iii) 1 kl = $\dots\dots\dots \text{m}^3$

(iv) The volume of a cube of side 8 cm is $\dots\dots\dots$

(v) The volume of a wooden cuboid of length 10 cm and breadth 8 cm is 4000 cm^3 .

The height of the cuboid is $\dots\dots\dots$ cm.

(vi) 1 cu. dm = $\dots\dots\dots$ cu. mm

(vii) 1 cu. km = $\dots\dots\dots$ cu. m

(viii) 1 litre = $\dots\dots\dots$ cu. cm

(ix) 1 ml = $\dots\dots\dots$ cu. cm

(x) 1 kl = $\dots\dots\dots$ cu. dm = $\dots\dots\dots$ cu. cm.

Solution:

(i) $1 \text{ m}^3 = 1 \times (100 \times 100 \times 100) = \underline{10^6 \text{cm}^3}$ (since $1\text{m} = 100\text{cm}$)

(ii) $1 \text{ litre} = 1000\text{cm}^3 = 1000 \times (0.1 \times 0.1 \times 0.1) \text{ dm}^3 = \underline{1\text{dm}^3}$ (since $1\text{cm} = 0.1\text{dm}$)

(iii) $1 \text{ kl} = 1000 \text{ litre} = \underline{1\text{m}^3}$ (since $1\text{m}^3 = 1000 \text{ litre}$)

(iv) The volume of a cube of side 8 cm is $\dots\dots\dots$

We know that, side of a cube = 8cm

So, volume of cube = $8^3 = \underline{512\text{cm}^3}$

(v) The volume of a wooden cuboid of length 10 cm and breadth 8 cm is 4000 cm^3 . The height of the cuboid is $\dots\dots\dots$ cm.

Given, volume of cuboid = 4000 cm^3

Length of cuboid = 10cm

Breadth of cuboid = 8cm

We know that volume = $l \times b \times h$

$$\begin{aligned}h &= \text{volume} / (l \times b) \\ &= 4000 / (10 \times 8) \\ &= \underline{\underline{50\text{cm}}}\end{aligned}$$

$$\begin{aligned}\text{(vi)} \quad 1 \text{ cu. dm} &= 1\text{dm}^3 = 1 \times (10 \times 10 \times 10) = 10^3\text{cm}^3 \text{ (since } 1\text{dm} = 10\text{cm)} \\ &10^3 \times (10 \times 10 \times 10) = \underline{\underline{10^6 \text{mm}^3}} \text{ (since } 1\text{cm} = 10\text{mm)}\end{aligned}$$

$$\text{(vii)} \quad 1 \text{ cu. km} = 1000 \times 1000 \times 1000 = \underline{\underline{10^9\text{m}^3}} \text{ (since } 1\text{km} = 1000\text{m)}$$

$$\text{(viii)} \quad 1 \text{ litre} = 1000 \text{ cm}^3 = \underline{\underline{10^3\text{cm}^3}}$$

$$\text{(ix)} \quad 1 \text{ ml} = 1/1000 \text{ litre} = 1/1000 \times 1000 = \underline{\underline{1\text{cm}^3}} \text{ (since } 1\text{ml} = 1/1000 \text{ litre)}$$

$$\begin{aligned}\text{(x)} \quad 1 \text{ kl} &= 1 \times 1000 \text{ litre} = 1\text{m}^3 = 1 \times (10 \times 10 \times 10) \text{ dm}^3 \text{ (since } 1\text{m} = 10\text{dm)} \\ 1 \text{ kl} &= \underline{\underline{1000 \text{ dm}^3}} = 10^3 \text{ dm}^3 = 1000 \times 1000 = \underline{\underline{10^6 \text{cm}^3}}\end{aligned}$$

EXERCISE 21.3**PAGE NO: 21.22****1. Find the surface area of a cuboid whose****(i) length = 10 cm, breadth = 12 cm, height = 14 cm****(ii) length = 6 dm, breadth = 8 dm, height = 10 dm****(iii) length = 2m, breadth = 4 m, height = 5 m****(iv) length = 3.2 m, breadth = 30 dm, height = 250 cm.****Solution:****(i)** Given details are,

Length of a cuboid = 10 cm

Breadth of a cuboid = 12 cm

Height of a cuboid = 14 cm

We know that,

$$\begin{aligned}\text{Surface area of cuboid} &= 2 (lb + bh + hl) \text{ cm}^2 \\ &= 2 (10 \times 12 + 12 \times 14 + 14 \times 10) \\ &= 2 (120 + 168 + 140) \\ &= 2 (428) \\ &= 856 \text{ cm}^2\end{aligned}$$

(ii) Given details are,

Length of a cuboid = 6 dm

Breadth of a cuboid = 8 dm

Height of a cuboid = 10 dm

We know that,

$$\begin{aligned}\text{Surface area of cuboid} &= 2 (lb + bh + hl) \text{ cm}^2 \\ &= 2 (6 \times 8 + 8 \times 10 + 10 \times 6) \\ &= 2 (48 + 80 + 60) \\ &= 2 (188) \\ &= 376 \text{ dm}^2\end{aligned}$$

(iii) Given details are,

Length of a cuboid = 2m

Breadth of a cuboid = 4m

Height of a cuboid = 5m

We know that,

$$\begin{aligned}\text{Surface area of cuboid} &= 2 (lb + bh + hl) \text{ cm}^2 \\ &= 2 (2 \times 4 + 4 \times 5 + 5 \times 2) \\ &= 2 (8 + 20 + 10) \\ &= 2 (38)\end{aligned}$$

$$= 76 \text{ m}^2$$

(iv) Given details are,

Length of a cuboid = 3.2 m = 32 dm

Breadth of a cuboid = 30 dm

Height of a cuboid = 250 cm = 25 dm

We know that,

$$\begin{aligned} \text{surface area of cuboid} &= 2 (lb + bh + hl) \text{ cm}^2 \\ &= 2 (32 \times 30 + 30 \times 25 + 25 \times 32) \\ &= 2 (960 + 750 + 800) \\ &= 2 (2510) \\ &= 5020 \text{ dm}^2 \end{aligned}$$

2. Find the surface area of a cube whose edge is

(i) 1.2 m

(ii) 27 cm

(iii) 3 cm

(iv) 6 m

(v) 2.1 m

Solution:

(i) Given,

Edge of cube = 1.2 m

We know that,

$$\begin{aligned} \text{Surface area of cube} &= 6 \times \text{side}^2 \\ &= 6 \times 1.2^2 \\ &= 6 \times 1.44 \\ &= 8.64 \text{ m}^2 \end{aligned}$$

(ii) Given,

Edge of cube = 27 cm

We know that,

$$\begin{aligned} \text{Surface area of cube} &= 6 \times \text{side}^2 \\ &= 6 \times 27^2 \\ &= 6 \times 729 \\ &= 4374 \text{ cm}^2 \end{aligned}$$

(iii) Given,

Edge of cube = 3 cm

We know that,

$$\begin{aligned}\text{Surface area of cube} &= 6 \times \text{side}^2 \\ &= 6 \times 3^2 \\ &= 6 \times 9 \\ &= 54 \text{ cm}^2\end{aligned}$$

(iv) Given,

Edge of cube = 6 m

We know that,

$$\begin{aligned}\text{Surface area of cube} &= 6 \times \text{side}^2 \\ &= 6 \times 6^2 \\ &= 6 \times 36 \\ &= 216 \text{ m}^2\end{aligned}$$

(v) Given,

Edge of cube = 2.1 m

We know that,

$$\begin{aligned}\text{Surface area of cube} &= 6 \times \text{side}^2 \\ &= 6 \times 2.1^2 \\ &= 6 \times 4.41 \\ &= 26.46 \text{ m}^2\end{aligned}$$

3. A cuboidal box is 5 cm by 5 cm by 4 cm. Find its surface area.

Solution:

Given details are,

Dimensions of cuboidal box = 5cm × 5cm × 4cm

We know that,

$$\begin{aligned}\text{Surface area of cuboid} &= 2 (lb + bh + hl) \text{ cm}^2 \\ &= 2 (5 \times 5 + 5 \times 4 + 4 \times 5) \\ &= 2 (25 + 20 + 20) \\ &= 2 (65) \\ &= 130 \text{ cm}^2\end{aligned}$$

4. Find the surface area of a cube whose volume is

(i) 343 m^3

(ii) 216 dm^3

Solution:

(i) Given details are,

Volume of cube = 343 m^3

Side of cube, $a = \sqrt[3]{(343)} = 7\text{m}$

We know that,

$$\begin{aligned}\text{Surface area of cube} &= 6 \times \text{side}^2 \\ &= 6 \times 7^2 \\ &= 6 \times 49 \\ &= 294 \text{ m}^2\end{aligned}$$

(ii) Given details are,

$$\text{Volume of cube} = 216 \text{ dm}^3$$

$$\text{Side of cube } a = \sqrt[3]{(216)} = 6 \text{ dm}$$

We know that,

$$\begin{aligned}\text{Surface area of cube} &= 6 \times \text{side}^2 \\ &= 6 \times 6^2 \\ &= 6 \times 36 \\ &= 216 \text{ dm}^2\end{aligned}$$

5. Find the volume of a cube whose surface area is

(i) 96 cm^2

(ii) 150 m^2

Solution:

(i) Given details are,

$$\text{Surface area of cube} = 96 \text{ cm}^2$$

$$6 \times \text{side}^2 = 96 \text{ cm}^2$$

$$\text{Side}^2 = 96/6$$

$$= 16$$

$$\text{Side} = \sqrt{16} = 4 \text{ cm}$$

$$\therefore \text{Volume of a cube} = 4^3 = 64 \text{ cm}^3$$

(ii) Given details are,

$$\text{Surface area of cube} = 150 \text{ m}^2$$

$$6 \times \text{side}^2 = 150 \text{ cm}^2$$

$$\text{Side}^2 = 150/6$$

$$= 25$$

$$\text{Side} = \sqrt{25} = 5 \text{ cm}$$

$$\therefore \text{Volume of a cube} = 5^3 = 125 \text{ m}^3$$

6. The dimensions of a cuboid are in the ratio 5: 3: 1 and its total surface area is 414 m^2 . Find the dimensions.

Solution:

Given details are,

Ratio of dimensions of a cuboid = 5:3:1

Total surface area of cuboid = 414 m^2

The dimensions are = $5x \times 3x \times x$

Surface area of cuboid = 414 m^2

We know that,

Surface area of cuboid = $2 (lb + bh + hl) \text{ cm}^2$

$2 (lb + bh + hl) \text{ cm}^2 = 414$

$2 (15x^2 + 3x^2 + 5x^2) = 414$

$2 (23x^2) = 414$

$46x^2 = 414$

$x^2 = 414/46$

$= 9$

$x = \sqrt{9}$

$= 3$

\therefore Dimensions are,

$5x = 5 (3) = 15\text{m}$

$3x = 3 (3) = 9\text{m}$

$x = 3\text{m}$

7. Find the area of the cardboard required to make a closed box of length 25 cm, 0.5 m and height 15 cm.

Solution:

Given details are,

Dimensions of closed box = $25\text{cm} \times 0.5\text{m} \times 15\text{cm} = 25\text{cm} \times 50\text{cm} \times 15\text{cm}$

We know that,

$$\begin{aligned} \text{Area of cardboard required} &= 2 (lb + bh + hl) \text{ cm}^2 \\ &= 2 (25 \times 50 + 50 \times 15 + 15 \times 25) \\ &= 2 (1250 + 750 + 375) \\ &= 2 (2375) \\ &= 4750 \text{ cm}^2 \end{aligned}$$

8. Find the surface area of a wooden box whose shape is of a cube, and if the edge of the box is 12 cm.

Solution:

Given details are,

Edge of a cubic wooden box = 12 cm

We know that,

$$\begin{aligned} \text{Surface area of cubic wooden box} &= 6 \times \text{side}^2 \\ &= 6 \times 12^2 \end{aligned}$$

$$\begin{aligned} &= 6 \times 144 \\ &= 864 \text{ cm}^2 \end{aligned}$$

9. The dimensions of an oil tin are 26 cm × 26 cm × 45 cm. Find the area of the tin sheet required for making 20 such tins. If 1 square metre of the tin sheet costs Rs. 10, find the cost of tin sheet used for these 20 tins.

Solution:

Given details are,

Dimensions of oil tin = 26cm × 26cm × 45cm

Then,

$$\begin{aligned} \text{Area of tin sheet required for making one oil tin} &= \text{total surface area of oil tin} \\ &= 2 (lb + bh + hl) \text{ cm}^2 \\ &= 2 (26 \times 26 + 26 \times 45 + 45 \times 26) \\ &= 2 (676 + 1170 + 1170) \\ &= 2 (3016) \\ &= 6032 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of tin sheet required for 20 oil tins} &= 20 \times 6032 \\ &= 120640 \text{ cm}^2 \\ &= 12.064 \text{ m}^2 \end{aligned}$$

Given, Cost of 1 m² tin sheet = Rs 10

$$\begin{aligned} \text{So, Cost of 12.064 m}^2 \text{ tin sheet} &= 10 \times 12.064 \\ &= \text{Rs } 120.60 \end{aligned}$$

10. A classroom is 11 m long, 8 m wide and 5 m high. Find the sum of the areas of its floor and the four walls (including doors, windows etc.)

Solution:

Given details are,

Dimensions of class room = 11m × 8m × 5m

Where, Length = 11m, Breadth = 8m, Height = 5m

We know,

$$\begin{aligned} \text{Area of floor} &= \text{length} \times \text{breadth} \\ &= 11 \times 8 \\ &= 88 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of four walls (including doors \& windows)} &= 2 (lh + bh) \text{ cm}^2 \\ &= 2 (11 \times 5 + 8 \times 5) \\ &= 2 (55 + 40) \\ &= 2 (95) \\ &= 190 \text{ m}^2 \end{aligned}$$

∴ Sum of areas of floor and four walls = area of floor + area of four walls

$$\begin{aligned} &= 88 + 190 \\ &= 278 \text{ m}^2 \end{aligned}$$

11. A swimming pool is 20 m long 15 m wide and 3 m deep. Find the cost of repairing the floor and wall at the rate of Rs. 25 per square metre.

Solution:

Given details are,

Dimensions of swimming pool are = $20\text{m} \times 15\text{m} \times 3\text{m}$

Where, Length = 20m , Breadth = 15m , Height = 3m

We know,

$$\begin{aligned} \text{Area of floor} &= \text{length} \times \text{breadth} \\ &= 20 \times 15 \\ &= 300 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of walls of swimming pool} &= 2 (lh + bh) \text{ cm}^2 \\ &= 2 (20 \times 3 + 15 \times 3) \\ &= 2 (60 + 45) \\ &= 2 (105) \\ &= 210\text{m}^2 \end{aligned}$$

$$\begin{aligned} \text{Sum of areas of floor and four walls} &= \text{area of floor} + \text{area of walls} \\ &= 300 + 210 \\ &= 510 \text{ m}^2 \end{aligned}$$

Given, Cost for repairing 1m^2 area = Rs 25

$$\begin{aligned} \therefore \text{Cost for repairing } 510 \text{ m}^2 &= 510 \times 25 \\ &= \text{Rs } 12750 \end{aligned}$$

12. The perimeter of a floor of a room is 30 m and its height is 3 m. Find the area of four walls of the room.

Solution:

Given details are,

Height of floor = 3m

Perimeter of floor = 30m

So, perimeter = 30

$$2(l+b) = 30$$

$$l+b = 30/2$$

$$l+b = 15\text{m}$$

$$\begin{aligned} \therefore \text{Area of four walls of room} &= 2 (lh + bh) \text{ m}^2 \\ &= 2h (l+b) \\ &= 2 (3) (15) \\ &= 90\text{m}^2 \end{aligned}$$

13. Show that the product of the areas of the floor and two adjacent walls of a cuboid is the square of its volume.

Solution:

Let us consider length of cuboid as = l cm

Let us consider breadth of cuboid as = b cm

Let us consider height of cuboid as = h cm

We know,

$$\text{Area of floor} = l \times b = lb \text{ cm}^2$$

Then,

$$\text{Product of areas of two adjacent walls} = (l \times h) \times (b \times h) = lbh^2 \text{ cm}^4$$

$$\begin{aligned} \text{Product of areas of floor and two adjacent walls} &= lb \times lbh^2 \text{ cm}^6 \\ &= l^2 \times b^2 \times h^2 \text{ cm}^6 \\ &= (lbh)^2 \text{ cm}^6 \end{aligned}$$

We know, volume of cuboid = lbh cm

Hence, areas of the floor and two adjacent walls of a cuboid is the square of its volume.

14. The walls and ceiling of a room are to be plastered. The length, breadth and height of the room are 4.5 m, 3 m and 350 cm, respectively. Find the cost of plastering at the rate of Rs. 8 per square metre.

Solution:

Given details are,

$$\text{Length of room} = 4.5\text{m}$$

$$\text{Breadth of wall} = 3\text{m}$$

$$\text{Height of wall} = 350\text{cm} = 350/100 = 3.5\text{m}$$

$$\begin{aligned} \text{Area of ceiling} &= l \times b \\ &= 4.5 \times 3 \\ &= 13.5 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of walls} &= 2(lh + bh) \text{ m}^2 \\ &= 2(4.5 \times 3.5 + 3 \times 3.5) \\ &= 2(15.75 + 10.5) \\ &= 52.5 \text{ m}^2 \end{aligned}$$

$$\begin{aligned} \text{Sum of Area of ceiling} + \text{area of walls} &= 13.5\text{m}^2 + 52.5\text{m}^2 \\ &= 66\text{m}^2 \end{aligned}$$

Given, Cost for plastering 1m^2 area = Rs 8

$$\therefore \text{Cost for plastering } 66 \text{ m}^2 \text{ area} = 66 \times 8 = \text{Rs } 528$$

15. A cuboid has total surface area of 50 m^2 and lateral surface area is 30 m^2 . Find the area of its base.

Solution:

Given details are,

$$\text{Total surface area of cuboid} = 50 \text{ m}^2$$

$$\text{Lateral surface area of cuboid} = 30 \text{ m}^2$$

$$\text{Total Surface area} = 2 (\text{surface area of base}) + (\text{surface area of 4 walls})$$

$$50 = 2 (\text{surface area of base}) + (\text{lateral surface area})$$

$$50 = 2 (\text{surface area of base}) + 30$$

$$50 - 30 = 2 (\text{surface area of base})$$

$$20 = 2 (\text{surface area of base})$$

$$\begin{aligned} \text{Surface area of base} &= 20/2 \\ &= 10 \text{ m}^2 \end{aligned}$$

$$\therefore \text{Area of base is } 10 \text{ m}^2$$

16. A classroom is 7 m long, 6 m broad and 3.5 m high. Doors and windows occupy an area of 17 m^2 . What is the cost of white washing the walls at the rate of Rs 1.50 per m^2 ?

Solution:

Given details are,

$$\text{Dimensions of class room} = 7 \text{ m} \times 6 \text{ m} \times 3.5 \text{ m}$$

$$\text{Where, Length} = 7 \text{ m, Breadth} = 6 \text{ m, Height} = 3.5 \text{ m}$$

$$\begin{aligned} \text{Area of four walls (including doors \& windows)} &= 2 (lh + bh) \text{ m}^2 \\ &= 2 (7 \times 3.5 + 6 \times 3.5) \\ &= 91 \text{ m}^2 \end{aligned}$$

$$\text{Area of four walls (without doors \& windows)} =$$

$$\begin{aligned} \text{Area including doors \& windows} - \text{area occupied by doors \& windows} \\ &= 91 - 17 = 74 \text{ m}^2 \end{aligned}$$

Then,

$$\text{Cost for white washing } 1 \text{ m}^2 \text{ area of walls} = \text{Rs } 1.50$$

$$\therefore \text{Total cost for white washing the walls} = 74 \times 1.50 = \text{Rs } 111$$

17. The central hall of a school is 80 m long and 8 m high. It has 10 doors each of size $3 \text{ m} \times 1.5 \text{ m}$ and 10 windows each of size $1.5 \text{ m} \times 1 \text{ m}$. If the cost of white washing the walls of the hall at the rate of Rs 1.20 per m^2 is Rs 2385.60, find the breadth of the hall.

Solution:

Given details are,

Dimensions of central hall of a school = Length = 80 m , height = 8m

Let breadth of hall be 'b' m

So,

$$\text{Area of each door} = 3\text{m} \times 1.5\text{m} = 4.5\text{m}^2$$

$$\text{Area of 10 doors} = 10 \times 4.5 = 45\text{m}^2$$

$$\text{Area of each window} = 1.5\text{m} \times 1\text{m} = 1.5 \text{ m}^2$$

$$\text{Area of 10 windows} = 10 \times 1.5 = 15\text{m}^2$$

$$\text{Area occupied by doors and windows} = 45 + 15 = 60 \text{ m}^2$$

$$\begin{aligned} \text{Area of the walls of the hall including doors and windows} &= 2(lh + bh) \text{ m}^2 \\ &= 2(80 \times 8 + b \times 8) \\ &= 2(640 + 8b) \text{ m}^2 \end{aligned}$$

Then,

Area of only walls = area of walls including doors & windows – area occupied by doors & windows

$$\begin{aligned} &= 2(640 + 8b) - 60 \\ &= 1280 + 16b - 60 \\ &= (1220 + 16b) \text{ m}^2 \end{aligned}$$

Given, Total cost for white washing = Rs 2385.60

Rate of white washing = Rs 1.20 per m^2

So,

Total cost = Rate \times (areas of walls only)

$$2385.60 = 1.20 \times (1220 + 16b)$$

$$2385.60 / 1.20 = (1220 + 16b)$$

$$1988 = 1220 + 16b$$

$$16b = 1988 - 1220$$

$$= 768$$

$$b = 768 / 16$$

$$= 48$$

\therefore Breadth of hall is 48 m

EXERCISE 21.4**PAGE NO: 21.30**

1. Find the length of the longest rod that can be placed in a room 12 m long, 9 m broad and 8 m high.

Solution:

Given details are,

Length of room = 12 m

Breadth of room = 9m

Height of room = 8m

So,

$$\begin{aligned} \text{Length of longest rod that can be placed in room} &= \text{diagonal of room (cuboid)} \\ &= \sqrt{l^2 + b^2 + h^2} \\ &= \sqrt{12^2 + 9^2 + 8^2} \\ &= \sqrt{144 + 81 + 64} \\ &= \sqrt{289} \\ &= 17\text{m} \end{aligned}$$

2. If V is the volume of a cuboid of dimensions a, b, c and S is its surface area, then prove that $1/V = 2/S (1/a + 1/b + 1/c)$

Solution:

Let us consider,

V = volume of cuboid

S = surface area of cuboid

Dimensions of cuboid = a, b, c

So,

$$S = 2(ab + bc + ca)$$

$$V = abc$$

$$S/V = 2(ab + bc + ca) / abc$$

$$= 2[(ab/abc) + (bc/abc) + (ca/abc)]$$

$$= 2(1/a + 1/b + 1/c)$$

$$1/V = 2/S (1/a + 1/b + 1/c)$$

Hence proved.

3. The areas of three adjacent faces of a cuboid are x, y, and z. If the volume is V, prove that $V^2 = xyz$.

Solution:

Let us consider,

Areas of three faces of cuboid as x,y,z

So, Let length of cuboid be = l

Breadth of cuboid be = b

Height of cuboid be = h

Let, $x = l \times b$

$y = b \times h$

$z = h \times l$

Else we can write as

$xyz = l^2 b^2 h^2 \dots\dots$ (i)

If 'V' is volume of cuboid = $V = lbh$

$V^2 = l^2 b^2 h^2 = xyz \dots\dots$ from (i)

$\therefore V^2 = xyz$

Hence proved.

4. A rectangular water reservoir contains 105 m^3 of water. Find the depth of the water in the reservoir if its base measures 12 m by 3.5 m.

Solution:

Given details are,

Capacity of water reservoir = 105 m^3

Length of base of reservoir = 12 m

Width of base = 3.5 m

Let the depth of reservoir be 'h' m

$l \times b \times h = 105$

$h = 105 / (l \times b)$

$= 105 / (12 \times 3.5)$

$= 105/42$

$= 2.5\text{m}$

\therefore Depth of reservoir is 2.5 m

5. Cubes A, B, C having edges 18 cm, 24 cm and 30 cm respectively are melted and moulded into a new cube D. Find the edge of the bigger cube D.

Solution:

Given details are,

Edge length of cube A = 18 cm

Edge length of cube B = 24 cm

Edge length of cube C = 30 cm

Then,

Volume of cube A = $v_1 = 18^3 = 5832\text{cm}^3$

Volume of cube B = $v_2 = 24^3 = 13824\text{cm}^3$

Volume of cube C = $v_3 = 30^3 = 27000\text{cm}^3$

Total volume of cube A,B,C = $5832 + 13824 + 27000 = 46656 \text{ cm}^3$

Let 'a' be the length of edge of newly formed cube.

$$a^3 = 46656$$

$$a = \sqrt[3]{46656}$$
$$= 36$$

∴ Edge of bigger cube is 36cm

6. The breadth of a room is twice its height, one half of its length and the volume of the room is 512 cu. Dm. Find its dimensions.

Solution:

Given,

Breadth of room is twice of its height, $b = 2h$ or $h = b/2$... (i)

Breadth is one half of length, $b = l/2$ or $l = 2b$... (ii)

Volume of the room = $lbh = 512 \text{ dm}^3$... (iii)

By substituting (i) and (ii) in (iii)

$$2b \times b \times b/2 = 512$$

$$b^3 = 512$$

$$b = \sqrt[3]{512}$$
$$= 8$$

∴ Breadth of cube = $b = 8 \text{ dm}$

Length of cube = $2b = 2 \times 8 = 16 \text{ dm}$

Height of cube = $b/2 = 8/2 = 4 \text{ dm}$

7. A closed iron tank 12 m long, 9 m wide and 4 m deep is to be made. Determine the cost of iron sheet used at the rate of Rs. 5 per metre sheet, sheet being 2 m wide.

Solution:

Given,

Length of tank, $l = 12 \text{ m}$

Width of tank, $b = 9 \text{ m}$

Depth of tank, $h = 4 \text{ m}$

Area of sheet required = total surface area of tank

$$\begin{aligned} &= 2 (lb \times bh \times hl) \\ &= 2 (12 \times 9 + 9 \times 4 + 4 \times 12) \\ &= 2 (108 + 36 + 48) \\ &= 2 (192) \\ &= 384 \text{ m}^2 \end{aligned}$$

Let length be l_1

Breadth be b_1

Given, $b_1 = 2\text{m}$

$$l_1 \times b_1 = 384$$

$$l_1 = 384/b_1$$

$$= 384/2$$

$$= 192\text{m}$$

\therefore Cost of iron sheet at the rate of Rs 5 per metre = $5 \times 192 = \text{Rs } 960$

8. A tank open at the top is made of iron sheet 4 m wide. If the dimensions of the tank are $12\text{m} \times 8\text{m} \times 6\text{m}$, find the cost of iron sheet at Rs. 17.50 per metre.

Solution:

Given details are,

Dimensions of tank = $12\text{m} \times 8\text{m} \times 6\text{m}$

Where, length = 12m

Breadth = 8m

Height = 6m

Area of sheet required = total surface area of tank with one top open

$$= l \times b + 2(l \times h + b \times h)$$

$$= 12 \times 8 + 2(12 \times 6 + 8 \times 6)$$

$$= 96 + 240$$

$$= 336 \text{ m}^2$$

Let length be l_1

Breadth be b_1

Given, $b_1 = 4\text{m}$

$$l_1 \times b_1 = 336$$

$$l_1 = 336/b_1$$

$$= 336/4$$

$$= 84\text{m}$$

\therefore Cost of iron sheet at the rate of Rs 17.50 per metre = $17.50 \times 84 = \text{Rs } 1470$

9. Three equal cubes are placed adjacently in a row. Find the ratio of total surface area of the new cuboid to that of the sum of the surface areas of the three cubes.

Solution:

Given details are,

Let edge length of three equal cubes = a

Then,

$$\text{Sum of surface area of 3 cubes} = 3 \times 6a^2 = 18a^2$$

When these cubes are placed in a row adjacently they form a cuboid.

$$\text{Length of new cuboid formed} = a + a + a = 3a$$

Breadth of cuboid = a

Height of cuboid = a

$$\begin{aligned}\text{Total surface area of cuboid} &= 2 (lb \times bh \times hl) \\ &= 2 (3a \times a + a \times a + a \times 3a) \\ &= 2 (3a^2 + a^2 + 3a^2) \\ &= 2 (7a^2) \\ &= 14 a^2\end{aligned}$$

Total surface area of new cuboid / sum of surface area of 3 cuboids = $14/18 = 7/9 = 7:9$
∴ The ratio is 7:9

10. The dimensions of a room are 12.5 m by 9 m by 7 m. There are 2 doors and 4 windows in the room; each door measures 2.5 m by 1.2 m and each window 1.5 m by 1 m. Find the cost of painting the walls at Rs. 3.50 per square metre.

Solution:

Given details are,

Dimensions of room = 12.5m × 9m × 7m

Dimensions of each door = 2.5m × 1.2m

Dimensions of each window = 1.5m × 1m

$$\begin{aligned}\text{Area of four walls including doors and windows} &= 2 (l \times h + b \times h) \\ &= 2 (12.5 \times 7 + 9 \times 7) \\ &= 2 (87.5 + 63) \\ &= 2 (150.5) \\ &= 301 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of 2 doors and 4 windows} &= 2 (2.5 \times 1.2) + 4 (1.5 \times 1) \\ &= 2(3) + 4 (1.5) \\ &= 6 + 6 \\ &= 12 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Area of only walls} &= 301 - 12 \\ &= 289 \text{ m}^2\end{aligned}$$

∴ Cost of painting the walls at the rate of Rs 3.50 per square metre = Rs $(3.50 \times 289) =$ Rs 1011.50

11. A field is 150m long and 100m wide. A plot (outside the field) 50m long and 30m wide is dug to a depth of 8m and the earth taken out from the plot is spread evenly in the field. By how much is the level of field raised?

Solution:

Given details are,

Length of field = 150m

Width of field = 100m

Area of field = $150\text{m} \times 100\text{m} = 15000\text{m}^2$

Length of plot = 50m

Breadth of the plot = 30m

Depth = 8m

So, volume = $l \times b \times h = 50 \times 30 \times 8 = 12000\text{m}^3$

Let raise in earth level of field on which it spread be 'h' metre

Volume = $l \times b \times h$

$h = \text{volume} / (l \times b)$

$$= 12000 / (50 \times 30)$$

$$= 12000 / 1500$$

$$= 8\text{cm}$$

∴ The level of field is raised by 8cm.

12. Two cubes, each of volume 512 cm^3 are joined end to end. Find the surface area of the resulting cuboid.

Solution:

Given details are,

Volume of each cube = 512 cm^3

Let length of edge of each cube be 'a' cm

So,

Edge, $a^3 = 512$

$$a = \sqrt[3]{512}$$

$$= 8\text{cm}$$

When these two cubes are joined end to end, a cuboid is formed.

Length of cuboid = $8+8 = 16\text{ cm}$

Breadth = 8 cm

Height = 8 cm

$$\begin{aligned} \text{Surface area of resulting cuboid} &= 2(lb + bh + hl) \\ &= 2(16 \times 8 + 8 \times 8 + 8 \times 16) \\ &= 2(128 + 64 + 128) \\ &= 2(320) \\ &= 640\text{ cm}^2 \end{aligned}$$

∴ Surface area of resulting cuboid is 640cm^2 .

13. Three cubes whose edges measure 3 cm, 4 cm, and 5 cm respectively are melted to form a new cube. Find the surface area of the new cube formed.

Solution:

Given details are,

Edge of three cubes are = 3cm, 4cm, 5cm

$$\begin{aligned}\text{Sum of volume of these cubes} &= 3^3 + 4^3 + 5^3 \\ &= 27 + 64 + 125 \\ &= 216 \text{ cm}^3\end{aligned}$$

After these cubes are melted, a new cube is formed.

Let edge length of this new cube be 'a' cm

$$a^3 = 216$$

$$a = \sqrt[3]{216}$$

$$= 6\text{cm}$$

Edge of new cube is = 6cm

$$\begin{aligned}\therefore \text{Surface area of new cube} &= 6 \times a^2 \\ &= 6 \times 6^2 \\ &= 6 \times 36 \\ &= 216\text{cm}^2\end{aligned}$$

14. The cost of preparing the walls of a room 12m long at the rate of Rs 1.35 per square metre is Rs 340.20 and the cost of matting the floor at 85 paise per square metre is Rs 91.80. Find the height of the room.

Solution:

Given details are,

Length of room = 12 m

Let width of room be 'b' m

Let height of room be 'h' metre

Now,

$$\text{Area of floor} = 12 \times b \text{ m}^2 = 12b \text{ m}^2$$

Cost of matting the floor at the rate of 85 paise per square metre = Rs 91.80

$$12b \times 0.85 = 91.80$$

$$12b = 91.80/0.85$$

$$12b = 108$$

$$b = 108/12$$

$$= 9\text{m}$$

Now, Breadth of room = 9m

$$\begin{aligned}\text{Area of 4 walls} &= 2(l \times h + b \times h) \\ &= 2(12 \times h + 9 \times h) \\ &= 2(12h + 9h) \\ &= 2(21h) \\ &= 42h \text{ m}^2\end{aligned}$$

Cost for preparing walls at the rate of Rs 1.35 per square metre = Rs 340.20

$$42h \times 1.35 = 340.20$$

$$42h = 340.20/1.35$$

$$42h = 252$$

$$h = 252/42$$

$$= 6\text{m}$$

∴ Height of room is 6m.

15. The length of a hall is 18 m and the width 12 m. The sum of the areas of the floor and the flat roof is equal to the sum of the areas of the four walls. Find the height of the wall.

Solution:

Given details are,

Length of hall = 18m

Width of hall = 12m

Let height of hall be 'h' metre

$$\begin{aligned}\text{Sum of area of floor and flat roof} &= (l \times b + l \times b) \\ &= (12 \times 18 + 12 \times 18) \\ &= (216 + 216) \\ &= 432 \text{ m}^2\end{aligned}$$

$$\begin{aligned}\text{Sum of area of 4 walls} &= 2(l \times h + b \times h) \\ &= 2(18 \times h + 12 \times h) \\ &= 2(18h + 12h) \\ &= 2(30h) \\ &= 60h \text{ m}^2\end{aligned}$$

Now,

Sum of area of 4 walls = sum of area of floor and flat roof

$$60h = 432$$

$$h = 432/60$$

$$= 7.2\text{m}$$

∴ Height of hall is 7.2m

16. A metal cube of edge 12 cm is melted and formed into three smaller cubes. If the edges of the two smaller cubes are 6 cm and 8 cm, find the edge of the third smaller cube.

Solution:

Given details are,

Edge of metal cube (volume) = 12cm

Edge of smaller two cubes = 6cm, 8cm

Let edge of third cube be 'a' cm

So,

Volume of metal cube = sum of volume of three small cubes

$$12^3 = 6^3 + 8^3 + a^3$$

$$1728 = 216 + 512 + a^3$$

$$a^3 = 1728 - 216 - 512$$

$$= 1000$$

$$a = \sqrt[3]{1000}$$

$$= 10\text{cm}$$

∴ Edge of third smaller cube is 10cm.

17. The dimensions of a cinema hall are 100 m, 50 m and 18 m. How many persons can sit in the hall, if each person required 150 m³ of air?

Solution:

Given details are,

Dimensions of cinema hall = 100m × 50m × 18m

Where,

length = 100m, breadth = 50m, height = 18 m

Each person requires = 150 m³ of air

So,

Volume of cinema hall = l × b × h

$$= 100 \times 50 \times 18$$

$$= 90000 \text{ cm}^3$$

Number of person who can sit in cinema hall = volume of hall / volume of air required by one person

$$= 90000 / 150$$

$$= 600$$

∴ 600 people can sit in the cinema hall.

18. The external dimensions of a closed wooden box are 48 cm, 36 cm and 30 cm. The box is made of 1.5 cm thick wood. How many bricks of size 6 cm × 3 cm × 0.75 cm can be put in this box?

Solution:

Given details are,

External dimensions of wooden box = $48\text{cm} \times 36\text{cm} \times 30\text{cm}$

Dimensions of bricks = $6\text{cm} \times 3\text{cm} \times 0.75\text{cm}$

Thickness of wood = 1.5cm

Internal dimensions of box = $48 - (2 \times 1.5)\text{cm} \times 36 - (2 \times 1.5)\text{cm} \times 30 - (2 \times 1.5)\text{cm}$
 $= (48 - 3)\text{cm} \times (36 - 3)\text{cm} \times (30 - 3)\text{cm}$
 $= (45 \times 33 \times 27)\text{cm}$

Hence,

Number of bricks can be put in box = internal volume of box / volume of one brick
 $= (45 \times 33 \times 27) / (6 \times 3 \times 0.75)$
 $= 40095 / 13.5$
 $= 2970\text{ bricks}$

\therefore 2970 bricks can be put in the box.

19. The dimensions of a rectangular box are in the ratio of 2: 3: 4 and the difference between the cost of covering it with sheet of paper at the rates of Rs 8 and Rs 9.50 per m^2 is Rs 1248. Find the dimensions of the box.

Solution:

Given details are,

Ratio of dimensions of rectangular box = 2:3:4

Let length of box be ' $2x$ ' m

Let breadth of box be ' $3x$ ' m

Let height of box be ' $4x$ ' m

Area of sheet of paper required for covering it = total surface area of cuboid
 $= 2(lb + bh + hl)$
 $= 2(2x \times 3x + 3x \times 4x + 4x \times 2x)$
 $= 2(6x^2 + 12x^2 + 8x^2)$
 $= 2(26x^2)$
 $= 52x^2\text{ m}^2$

Cost for covering with sheet of paper at the rate of Rs 9.50 / $\text{m}^2 = 52x^2 \times 9.50$
 $= \text{Rs } 494x^2$

Cost for covering with sheet of paper at the rate of Rs 8 / $\text{m}^2 = 52x^2 \times 8$
 $= \text{Rs } 416x^2$

Given, the difference between the cost of covering it with sheet of paper at the rates of Rs 8 and Rs 9.50 per m^2 is Rs 1248

$$494x^2 - 416x^2 = 12448$$

$$78x^2 = 1248$$

$$x^2 = 1248/78$$

$$= 16$$

$$x = \sqrt{16}$$

$$= 4$$

$$\therefore \text{Length of box} = 2x = 2 \times 4 = 8\text{m}$$

$$\text{Breadth of box} = 3x = 3 \times 4 = 12\text{m}$$

$$\text{Height of box} = 4x = 4 \times 4 = 16\text{m}$$

