

Exercise 11(C)

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1. Find the seventh term from the end of the series: $\sqrt{2}, 2, 2\sqrt{2}, \dots, 32$

Solution:

Given series: $\sqrt{2}, 2, 2\sqrt{2}, \dots, 32$

Here,

$$a = \sqrt{2}$$

$$r = 2/\sqrt{2} = \sqrt{2}$$

And, the last term (l) = 32

$$l = t_n = ar^{n-1} = 32$$

$$(\sqrt{2})(\sqrt{2})^{n-1} = 32$$

$$(\sqrt{2})^n = 32$$

$$(\sqrt{2})^n = (2)^5 = (\sqrt{2})^{10}$$

Equating the exponents, we have

$$n = 10$$

So, the 7th term from the end is $(10 - 7 + 1)$ th term from the front.

i.e. 4th term of the G.P

Hence,

$$t_{11} = (\sqrt{2})(\sqrt{2})^{4-1} = (\sqrt{2})(\sqrt{2})^3 = (\sqrt{2}) \times 2\sqrt{2} = 4$$

2. Find the third term from the end of the G.P.

$2/27, 2/9, 2/3, \dots, 162$

Solution:

Given series: $2/27, 2/9, 2/3, \dots, 162$

Here,

$$a = \sqrt{2}$$

$$r = 2/\sqrt{2} = \sqrt{2}$$

And, the last term (l) = 32

$$l = t_n = ar^{n-1} = 32$$

$$(\sqrt{2})(\sqrt{2})^{n-1} = 32$$

$$(\sqrt{2})^n = 32$$

$$(\sqrt{2})^n = (2)^5 = (\sqrt{2})^{10}$$

Equating the exponents, we have

$$n = 10$$

So, the 7th term from the end is $(10 - 7 + 1)$ th term from the front.

i.e. 4th term of the G.P

Hence,

$$t_{11} = (\sqrt{2})(\sqrt{2})^{4-1} = (\sqrt{2})(\sqrt{2})^3 = (\sqrt{2}) \times 2\sqrt{2} = 4$$

3. Find the G.P. $1/27, 1/9, 1/3, \dots, 81$; find the product of fourth term from the beginning and the fourth term from the end.

Solution:

Given G.P. $1/27, 1/9, 1/3, \dots, 81$

Here, $a = 1/27$, common ratio (r) = $(1/9)/(1/27) = 3$ and $l = 81$

We know that,

$$l = t_n = ar^{n-1} = 81$$

$$(1/27)(3)^{n-1} = 81$$

$$3^{n-1} = 81 \times 27 = 2187$$

$$3^{n-1} = 3^7$$

$$n - 1 = 7$$

$$n = 8$$

Hence, there are 8 terms in the given G.P.

Now,

4th term from the beginning is t_4 and the 4th term from the end is $(8 - 4 + 1) = 5^{\text{th}}$ term (t_5)

Thus,

$$\text{the product of } t_4 \text{ and } t_5 = ar^{4-1} \times ar^{5-1} = ar^3 \times ar^4 = a^2r^7 = (1/27)^2(3)^7 = 3$$

**4. If for a G.P., p^{th} , q^{th} and r^{th} terms are a , b and c respectively; prove that:
 $(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$**

Solution:

Let's take the first term of the G.P. to be A and its common ratio be R .

Then,

$$p^{\text{th}} \text{ term} = a \Rightarrow AR^{p-1} = a$$

$$q^{\text{th}} \text{ term} = b \Rightarrow AR^{q-1} = b$$

$$r^{\text{th}} \text{ term} = c \Rightarrow AR^{r-1} = c$$

Now,

$$\begin{aligned} a^{q-r} \times b^{r-p} \times c^{p-q} &= (AR^{p-1})^{q-r} \times (AR^{q-1})^{r-p} \times (AR^{r-1})^{p-q} \\ &= A^{q-r} \cdot R^{(p-1)(q-r)} \times A^{r-p} \cdot R^{(q-1)(r-p)} \times A^{p-q} \cdot R^{(r-1)(p-q)} \\ &= A^{q-r+r-p+p-q} \times R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)} \\ &= A^0 \times R^0 \\ &= 1 \end{aligned}$$

On taking log on both the sides, we get

$$\log(a^{q-r} \times b^{r-p} \times c^{p-q}) = \log 1$$

$$\Rightarrow (q - r)\log a + (r - p)\log b + (p - q)\log c = 0$$

- Hence Proved