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1. Find which of the following sequence form a G.P.:
(i) 8, 24, 72, 216, ………
(ii) 1/8, 1/24, 1/72, 1/216, ………
(iii) 9, 12, 16, 24, ………
Solution:

(i) Given sequence: 8, 24, 72, 216, ………
Since,
24/8 = 3, 72/24 = 3, 216/72 = 3
⇒ 24/8 = 72/24 = 216/72 = ……….. = 3
Therefore 8, 24, 72, 216, ……… is a G.P. with common ratio 3.

(ii) Given sequence: 1/8, 1/24, 1/72, 1/216, ………
Since,
(1/24)/ (1/8) = 1/3, (1/72)/ (1/24) = 1/3, (1/216)/ (1/72) = 3
⇒ (1/24)/ (1/8) = (1/72)/ (1/24) = (1/216)/ (1/72) = ……….. = 1/3
Therefore 8, 24, 72, 216, ……… is a G.P. with common ratio 1/3.

(iii) Given sequence: 9, 12, 16, 24, ………
Since,
12/9 = 16/12 ≠ 24/16 , given sequence is not a G.P.

2. Find the 9th term of the series: 1, 4, 16, 64, …..
Solution:

It's seen that, the first term is (a) = 1
And, common ratio(r) = 4/1 = 4
We know that, the general term is
$t_n = ar^{n-1}$
Thus,
$t_9 = (1)(4)^{9-1} = 4^8 = 65536$

3. Find the seventh term of the G.P: 1, √3, 3, 3 √3, …..
Solution:

It's seen that, the first term is (a) = 1
And, common ratio(r) = √3/1 = √3
We know that, the general term is
$t_n = ar^{n-1}$
Thus,
$t_7 = (1)(\sqrt{3})^{7-1} = (\sqrt{3})^6 = 27$

4. Find the 8th term of the sequence:

$\frac{3}{4}, 1\frac{1}{2}, 3, \ldots\ldots\ldots$

Solution:

The given sequence can be rewritten as,
3/4, 3/2, 3, …..
It's seen that, the first term is (a) = 3/4
And, common ratio(r) = (3/2)/ (3/4) = 2
We know that, the general term is
$t_n = ar^{n-1}$
Thus,
$t_8 = (3/4)(2)^{8-1} = (3/4)(2)^7 = 3 \times 2^5 = 3 \times 32 = 96$

5. Find the 10th term of the G.P. :

$12, 4, 1\frac{1}{3}, \ldots\ldots\ldots$

Solution:

The given sequence can be rewritten as,
12, 4, 4/3, …..
It's seen that, the first term is (a) = 12
And, common ratio(r) = (4)/ (12) = 1/3
We know that, the general term is
$t_n = ar^{n-1}$
Thus,
$t_{10} = (12)(1/3)^{10-1} = (12)(1/3)^9 = 12 \times 1/(19683) = 4/6561$

6. Find the nth term of the series:
1, 2, 4, 8, ……..
Solution:

It's seen that, the first term is (a) = 1
And, common ratio(r) = 2/ 1 = 2
We know that, the general term is
$t_n = ar^{n-1}$
Thus,
$t_n = (1)(2)^{n-1} = 2^{n-1}$

Exercise 11(B)

1. Which term of the G.P. :

$$-10, \frac{5}{\sqrt{3}}, -\frac{5}{6}, \ldots.., is \;\; -\frac{5}{72}?$$

Solution:

In the given G.P.
First term, a = -10
Common ratio, r = (5/√3)/ (-10) = 1/(-2√3)
We know that, the general term is
$t_n = ar^{n-1}$
So,
$t_n = (-10)(1/(-2\sqrt{3}))^{n-1} = -5/72$

$$-\frac{5}{72} = -10 \times \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\frac{1}{144} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\frac{1}{2\times2\times2\times2\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}\times\sqrt{3}} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

$$\left(\frac{1}{2\sqrt{3}}\right)^{4} = \left(\frac{1}{2\sqrt{3}}\right)^{n-1}$$

Now, equating the exponents we have
n – 1 = 4
n = 5
Thus, the 5th of the given G.P. is -5/72

2. The fifth term of a G.P. is 81 and its second term is 24. Find the geometric progression.
Solution:

Given,
$t_5 = 81$ and $t_2 = 24$
We know that, the general term is
$t_n = ar^{n-1}$
So,
$t_5 = ar^{5-1} = ar^4 = 81$ …. (1)
And,
$t_2 = ar^{2-1} = ar^1 = 24$ …. (2)
Dividing (1) by (2), we have
$ar^4/ ar = 81/ 24$
$r^3 = 27/ 8$
r = 3/2
Using r in (2), we get

$a(3/2) = 24$
$a = 16$
Hence, the G.P. is
16, 24, 36, 54, ……

3. Fourth and seventh terms of a G.P. are 1/18 and -1/486 respectively. Find the G.P.
Solution:

Given,
$t_4 = 1/18$ and $t_7 = -1/486$
We know that, the general term is
$t_n = ar^{n-1}$
So,
$t_4 = ar^{4-1} = ar^3 = 1/18$ …. (1)
And,
$t_7 = ar^{7-1} = ar^6 = -1/486$ …. (2)
Dividing (2) by (1), we have
$ar^6/ ar^3 = (-1/486)/ (1/18)$
$r^3 = -1/27$
$r = -1/3$
Using r in (1), we get
$a(-1/3)^3 = 1/18$
$a = -27/ 18 = -3/2$
Hence, the G.P. is
$-3/2, -3/2(-1/3), -3/2(-1/3)^2, -3/2(-1/3)^3, \ldots\ldots$
$-3/2, 1/2, -1/6, 1/18, \ldots..$

4. If the first and the third terms of a G.P are 2 and 8 respectively, find its second term.
Solution:

Given,
$t_1 = 2$ and $t_3 = 8$
We know that, the general term is
$t_n = ar^{n-1}$
So,
$t_1 = ar^{1-1} = a = 2$ …. (1)
And,
$t_3 = ar^{3-1} = ar^2 = 8$ …. (2)
Dividing (2) by (1), we have
$ar^2/ a = 8/ 2$
$r^2 = 4$
$r = \pm 2$
Hence, the 2nd term of the G.P. is
When $a = 2$ and $r = 2$ is $2(2) = 4$
Or when $a = 2$ and $r = -2$ is $2(-2) = -4$

5. The product of 3rd and 8th terms of a G.P. is 243. If its 4th term is 3, find its 7th term
Solution:

Given,
Product of 3rd and 8th terms of a G.P. is 243
The general term of a G.P. with first term a and common ratio r is given by,
$t_n = ar^{n-1}$
So,
$t_3 \times t_8 = ar^{3-1} \times ar^{8-1} = ar^2 \times ar^7 = a^2r^9 = 243$
Also given,
$t_4 = ar^{4-1} = ar^3 = 3$
Now,
$a^2r^9 = (ar^3)\ ar^6 = 243$
$(3)\ ar^6 = 243$
$ar^6 = 81$
$ar^{7-1} = 81 = t_7$
Thus, the 7th term of the G.P is 81.

Exercise 11(C)

1. Find the seventh term from the end of the series: √2, 2, 2√2, …… , 32
Solution:

Given series: √2, 2, 2√2, …… , 32
Here,
$a = \sqrt{2}$
$r = 2/\sqrt{2} = \sqrt{2}$
And, the last term (l) = 32
$l = t_n = ar^{n-1} = 32$
$(\sqrt{2})(\sqrt{2})^{n-1} = 32$
$(\sqrt{2})^n = 32$
$(\sqrt{2})^n = (2)^5 = (\sqrt{2})^{10}$
Equating the exponents, we have
n = 10
So, the 7th term from the end is $(10 – 7 + 1)^{th}$ term from the front.
i.e. 4th term of the G.P
Hence,
$t_{11} = (\sqrt{2})(\sqrt{2})^{4-1} = (\sqrt{2})(\sqrt{2})^3 = (\sqrt{2}) \times 2\sqrt{2} = 4$

2. Find the third term from the end of the G.P.
2/27, 2/9, 2/3, ……., 162
Solution:

Given series: 2/27, 2/9, 2/3, ……., 162
Here,
$a = \sqrt{2}$
$r = 2/\sqrt{2} = \sqrt{2}$
And, the last term (l) = 32
$l = t_n = ar^{n-1} = 32$
$(\sqrt{2})(\sqrt{2})^{n-1} = 32$
$(\sqrt{2})^n = 32$
$(\sqrt{2})^n = (2)^5 = (\sqrt{2})^{10}$
Equating the exponents, we have
n = 10
So, the 7th term from the end is $(10 – 7 + 1)^{th}$ term from the front.
i.e. 4th term of the G.P
Hence,
$t_{11} = (\sqrt{2})(\sqrt{2})^{4-1} = (\sqrt{2})(\sqrt{2})^3 = (\sqrt{2}) \times 2\sqrt{2} = 4$

3. Find the G.P. 1/27, 1/9, 1/3, ……, 81; find the product of fourth term from the beginning and the fourth term from the end.
Solution:

Given G.P. 1/27, 1/9, 1/3, ……, 81

Here, $a = 1/27$, common ratio $(r) = (1/9)/ (1/27) = 3$ and $l = 81$
We know that,
$l = t_n = ar^{n-1} = 81$
$(1/27)(3)^{n-1} = 81$
$3^{n-1} = 81 \times 27 = 2187$
$3^{n-1} = 3^7$
$n - 1 = 7$
$n = 8$
Hence, there are 8 terms in the given G.P.
Now,
4^{th} term from the beginning is t_4 and the 4^{th} term from the end is $(8 - 4 + 1) = 5^{th}$ term (t_5)
Thus,
the product of t_4 and $t_5 = ar^{4-1} \times ar^{5-1} = ar^3 \times ar^4 = a^2r^7 = (1/27)^2(3)^7 = 3$

4. If for a G.P., p^{th}, q^{th} and r^{th} terms are a, b and c respectively; prove that:
$(q - r) \log a + (r - p) \log b + (p - q) \log c = 0$
Solution:

Let's take the first term of the G.P. to be A and its common ratio be R.
Then,
p^{th} term $= a \Rightarrow AR^{p-1} = a$
q^{th} term $= b \Rightarrow AR^{q-1} = b$
r^{th} term $= c \Rightarrow AR^{p-1} = c$
Now,

$$a^{q-r} \times b^{r-p} \times c^{p-q} = (AR^{p-1})^{q-r} \times (AR^{q-1})^{r-p} \times (AR^{r-1})^{p-q}$$
$$= A^{q-r}.R^{(p-1)(q-r)} \times A^{r-p}.R^{(q-1)(r-p)} \times A^{p-q}.R^{(r-1)(p-q)}$$
$$= A^{q-r+r-p+p-q} \times R^{(p-1)(q-r)+(q-1)(r-p)+(r-1)(p-q)}$$
$$= A^0 \times R^0$$
$$= 1$$

On taking log on both the sides, we get
$\log(a^{q-r} \times b^{r-p} \times c^{p-q}) = \log 1$
$\Rightarrow (q - r)\log a + (r - p)\log b + (p - q)\log c = 0$
- Hence Proved

Exercise 11(D)

1. Find the sum of G.P.:

(i) 1 + 3 + 9 + 27 + ………. to 12 terms

(ii) 0.3 + 0.03 + 0.003 + 0.0003 +….. to 8 terms.

(iii) 1 – 1/2 + 1/4 – 1/8 + …….. to 9 terms

(iv) 1 - 1/3 + 1/3^2 – 1/3^3 + ……… to n terms

(v) $\frac{x+y}{x-y}+1+\frac{x-y}{x+y}+ \ldots\ldots \text{ upto } n \text{ terms}$

(vi) $\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}+\ldots\ldots \text{to } n \text{ terms.}$

Solution:

(i) Given G.P: 1 + 3 + 9 + 27 + ……….

Here,

a = 1 and r = 3/1 = 3 (r > 1)

Number of terms, n = 12

Hence,

$S_n = a(r^n - 1)/ r – 1$

⇒ $S_{12} = (1)((3)^{10} - 1)/ 3 – 1$

$= (3^{12} – 1)/ 2$

$= (531441 – 1)/ 2$

$= 531440/2$

$= 265720$

(ii) Given G.P: 0.3 + 0.03 + 0.003 + 0.0003 +…..

Here,

a = 0.3 and r = 1/10 = 0.1 (r < 1)

Number of terms, n = 8

Hence,

$S_n = a(1 - r^n)/ 1 - r$

⇒ $S_8 = (0.3)(1 - 0.1^8)/ (1 – 0.1)$

$= 0.3(1 – 0.1^8)/ 0.9$

$= (1 – 0.1^8)/ 3$

$= 1/3(1 – (1/10)^8)$

(iii) Given G.P: 1 – 1/2 + 1/4 – 1/8 + ……..

Here,

a = 1 and r = (-1/2)/ 1 = -1/2 (| r | < 1)

Number of terms, n = 9

Hence,

$S_n = a(1 - r^n)/ 1 - r$

⇒ $S_9 = (1)(1 - (-1/2)^9)/ (1 – (-1/2))$

$= (1 + (1/2)^9)/ (3/2)$

$= 2/3 \times (1 + 1/512)$

= 2/3 x (513/512)
= 171/ 256

(iv) Given G.P: $1 - 1/3 + 1/3^2 - 1/3^3 + \ldots\ldots\ldots$
Here,
a = 1 and r = (-1/3)/ 1 = -1/3 (| r | < 1)
Number of terms is n
Hence,
$S_n = a(1 - r^n)/ 1 - r$

$$S_n = \frac{1\left(1-\left(-\frac{1}{3}\right)^n\right)}{1-\left(-\frac{1}{3}\right)}$$

$$= \frac{1\left(1-\left(-\frac{1}{3}\right)^n\right)}{1+\frac{1}{3}}$$

$$= \frac{\left[1-\left(-\frac{1}{3}\right)^n\right]}{\frac{4}{3}}$$

$$= \frac{3}{4}\left[1-\left(-\frac{1}{3}\right)^n\right]$$

(v) Given G.P:

$$\frac{x+y}{x-y} + 1 + \frac{x-y}{x+y} + \ldots\ldots \textit{ upto n terms}$$

Here,
a = (x + y)/ (x – y) and r = 1/[(x + y)/ (x – y)] = (x - y)/ (x + y) (| r | < 1)
Number of terms = n
Hence,
$S_n = a(1 - r^n)/ 1 - r$

$$S_n = \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{1-\left(\frac{x-y}{x+y}\right)}$$

$$= \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{\frac{x+y-x+y}{x+y}}$$

$$= \frac{\frac{x+y}{x-y}\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{\frac{2y}{x+y}}$$

$$= \frac{(x+y)^2\left(1-\left(\frac{x-y}{x+y}\right)^n\right)}{2y(x-y)}$$

(vi) Given G.P:

$$\sqrt{3}+\frac{1}{\sqrt{3}}+\frac{1}{3\sqrt{3}}+\ldots\ldots to\ n\ terms.$$

Here,
$a = \sqrt{3}$ and $r = 1/\sqrt{3}/\sqrt{3} = 1/3$ $(|r| < 1)$
Number of terms = n
Hence,
$S_n = a(1 - r^n)/ 1 - r$

$$S_n = \frac{\sqrt{3}\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}} = \frac{\sqrt{3}\left(1-\frac{1}{3^n}\right)}{\frac{2}{3}}$$

$$= \frac{3\sqrt{3}}{2}\left(1-\frac{1}{3^n}\right)$$

2. How many terms of the geometric progression 1 + 4 + 16 + 64 + …….. must be added to get sum equal to 5461?
Solution:

Given G.P: 1 + 4 + 16 + 64 + ……..
Here,
a = 1 and r = 4/1 = 4 (r > 1)
And,

$S_n = 5461$
We know that,
$S_n = a(r^n - 1)/ r - 1$
$\Rightarrow S_n = (1)((4)^n - 1)/ 4 - 1$
$= (4^n - 1)/3$
$5461 = (4^n - 1)/3$
$16383 = 4^n - 1$
$4^n = 14384 = 4^7$
$n = 7$
Therefore, 7 terms of the G.P must be added to get a sum of 5461.

3. The first term of a G.P. is 27 and its 8th term is 1/81. Find the sum of its first 10 terms.
Solution:

Given,
First term (a) of a G.P = 27
And, 8th term = $t_8 = ar^{8-1} = 1/81$
$(27)r^7 = 1/81$
$r^7 = 1/(81 \times 27)$
$r^7 = (1/3)^7$
$r = 1/3$ (r > 1)
Now,
Sum of first 10 terms = S_{10}

$$S_{10} = \frac{27\left(1-\left(\frac{1}{3}\right)^{10}\right)}{1-\frac{1}{3}} = \frac{27\left(1-\frac{1}{3^{10}}\right)}{\frac{2}{3}}$$

$$= \frac{81}{2}\left(1-\frac{1}{3^{10}}\right)$$

4. A boy spends Rs.10 on first day, Rs.20 on second day, Rs.40 on third day and so on. Find how much, in all, will he spend in 12 days?
Solution:

Given,
Amount spent on 1st day = Rs 10
Amount spent on 2nd day = Rs 20
And amount spent on 3rd day = Rs 40
It's seen that,
10, 20, 40, …… forms an G.P with first term, a = 10 and common ratio, r = 20/10 = 2 (r > 1)
The number of days, n = 12
Hence, the sum of money spend in 12 days is the sum of 12 terms of the G.P.
$S_{12} = (10)(2^{12} - 1)/ 2 - 1 = 10 (2^{12} - 1) = 10 (4096 - 1) = 10 \times 4095 = 40950$
Therefore, the amount he will spend = Rs 40950

5. The 4th term and the 7th term of a G.P. are 1/27 and 1/729 respectively. Find the sum of n terms of the G.P.
Solution:

Given,
$t_4 = 1/27$ and $t_7 = 1/729$
We know that,
$t_n = ar^{n-1}$
So,
$t_4 = ar^{4-1} = ar^3 = 1/27$ …. (1)
$t_7 = ar^{7-1} = ar^6 = 1/729$ …. (2)
Dividing (2) and (1) we get,
$ar^6/ ar^3 = (1/729)/ (1/27)$
$r^3 = (1/3)^3$
$r = 1/3$ $(r < 1)$
In (1)
a x 1/27 = 1/27
a = 1
Hence,
$S_n = (1 – (1/3)^n)/ 1 – (1/3)$
$= (1 – (1/3)^n)/ (2/3)$
$= 3/2 (1 – (1/3)^n)$

6. A geometric progression has common ratio = 3 and last term = 486. If the sum of its terms is 728; find its first term.
Solution:

Given,
For a G.P.,
$r = 3$, $l = 486$ and $S_n = 728$
$$\frac{lr - a}{r - 1} = 728$$
$$\frac{486 \times 3 - a}{3 - 1} = 728$$
$$\frac{1458 - a}{2} = 728$$
1458 – a = 728 x 2 = 1456
Thus, a = 2

7. Find the sum of G.P.: 3, 6, 12, …., 1536.
Solution:

Given G.P: 3, 6, 12, …., 1536
Here,

a = 3, l = 1536 and r = 6/3 = 2
So,
The sum of terms = (lr – a)/ (r - 1)
= (1536 x 2 – 3)/ (2 - 1)
= 3072 – 3
= 3069

8. How many terms of the series 2 + 6 + 18 + ….. must be taken to make the sum equal to 728?
Solution:

Given G.P: 2 + 6 + 18 + …..
Here,
a = 2 and r = 6/2 = 3
Also given,
S_n = 728
728 = (2)(3^n - 1)/ 3 – 1 = 3^n – 1
729 = 3^n
$3^6 = 3^n$
n = 6
Therefore, 6 terms must be taken to make the sum equal to 728.

9. In a G.P., the ratio between the sum of first three terms and that of the first six terms is 125: 152.
Find its common ratio.
Solution:

Given,

$$\frac{a(r^3-1))}{r-1} : \frac{a(r^6-1))}{r-1} = 125 : 152$$

$$\frac{\frac{a(r^3-1))}{r-1}}{\frac{a(r^6-1))}{r-1}} = \frac{125}{152}$$

$$\frac{(r^3-1)}{(r^6-1)} = \frac{125}{152}$$

$$\frac{r^3-1}{(r^3)^2-(1)^2} = \frac{125}{152}$$

$$\frac{r^3-1}{(r^3-1)(r^3+1)} = \frac{125}{152}$$

$$\frac{1}{r^3+1} = \frac{125}{152}$$

$$r^3 + 1 = \frac{152}{125}$$

$$r^3 = \frac{152}{125} - 1 = \frac{152-125}{125} = \frac{27}{125}$$

$$r^3 = \left(\frac{3}{5}\right)^3$$

r = 3/5

Therefore, the common ratio is 3/5.