Exercise 7(C)

1. If a : b = c : d, prove that:
(i) 5a + 7b : 5a - 7b = 5c + 7d : 5c - 7d.
(ii) (9a + 13b) (9c - 13d) = (9c + 13d) (9a - 13b).
(iii) xa + yb : xc + yd = b : d.
Solution:

(i) Given, a/b = c/d

$$\frac{5a}{7b} = \frac{5c}{7d} \text{ (Multiplying each by 5/7)}$$

$$\frac{5a+7b}{5a-7b} = \frac{5c+7d}{5c-7d} \text{ (By componendo and Dividendo)}$$

(ii) Given, a/b = c/d

$$\frac{9a}{13b} = \frac{9c}{13d} \text{ (Multiplying each by 9/13)}$$

$$\frac{9a+13b}{9a-13b} = \frac{9c+13d}{9c-13d} \text{ (By componendo and Dividendo)}$$

On cross-multiplication we have,
(9a + 13b)(9c – 13d) = (9c + 13d)(9a – 13b)

(iii) Given, a/b = c/d

$$\frac{xa}{yb} = \frac{xc}{yd} \text{ (Multiplying each by x/y)}$$

$$\frac{xa+yb}{yb} = \frac{xc+yd}{yd} \text{ (By compenendo)}$$

$$\frac{xa+yb}{xc+yd} = \frac{yb}{yd}$$

$$\frac{xa+yb}{xc+yd} = \frac{b}{d}$$

- Hence Proved

2. If a : b = c : d, prove that:
(6a + 7b) (3c - 4d) = (6c + 7d) (3a - 4b).
Solution:

Given, a/b = c/d

$\frac{6a}{7b} = \frac{6c}{7d}$ (Multiplying each by 6/7)

$\frac{6a+7b}{7b} = \frac{6c+7d}{7d}$ (By compenendo)

$\frac{6a+7b}{6c+7d} = \frac{7b}{7d} = \frac{b}{d}$ .... (1)

Also, a/b = c/d

$\frac{3a}{4b} = \frac{3c}{4d}$ (Multiplying each by 3/4)

$\frac{3a-4b}{4b} = \frac{3c-4d}{4d}$ (By dividendo)

$\frac{3a-4b}{3c-4d} = \frac{4b}{4d} = \frac{b}{d}$ .... (2)

Fromo (1) and (2), we have

$\frac{6a+7b}{6c+7d} = \frac{3a-4b}{3c-4d}$

(6a + 7b)(3c – 4d) = (3a – 4b)(6c + 7d)

- Hence Proved

3. Given, a/b = c/d, prove that:
(3a – 5b)/ (3a + 5b) = (3c – 5d)(3c + 5d)
Solution:

Given,

$\frac{a}{b} = \frac{c}{d}$

$\frac{3a}{5b} = \frac{3c}{5d}$ (Multiplying both by 3/5)

$\frac{3a+5b}{3a-5b} = \frac{3c+5d}{3c-5d}$ (By compnendo and Dividendo)

$\frac{3a-5b}{3a+5b} = \frac{3c-5d}{3c+5d}$ (By alternendo)

4. If $\frac{5x+6y}{5u+6v} = \frac{5x-6y}{5u-6v}$**;**
Then prove that x: y = u: v
Solution:

$$\frac{5x+6y}{5u+6v} = \frac{5x-6y}{5u-6v} \text{ (By alternendo)}$$

$$\frac{5x+6y}{5x-6y} = \frac{5u+6v}{5u-6v}$$

$$\frac{5x+6y+5x-6y}{5x+6y-5x+6y} = \frac{5u+6v+5u-6v}{5u+6v-5u+6v}$$

(By componendo and dividendo)

10x/ 12y = 10u/ 12v

Thus,

x/y = u/v ⇒ x: y = u: v

5. If (7a + 8b) (7c – 8d) = (7a – 8b) (7c + 8d);
Prove that a: b = c: d
Solution:

The given can the rewritten as,

$$\frac{7a+8b}{7a-8b} = \frac{7c+8d}{7c-8d}$$

Applying componendo and dividendo, we have

$$\frac{7a+8b+7a-8b}{7a+8b-7a+8b} = \frac{7c+8d+7c-8d}{7c+8d-7c+8d}$$

$$\frac{14a}{16b} = \frac{14c}{16d}$$

$$\frac{a}{b} = \frac{c}{d}$$

6. (i) If x = 6ab/ (a + b), find the value of:

$$\frac{x+3a}{x-3a} + \frac{x+3b}{x-3b}$$

Solution:

Given, x = 6ab/ (a + b)

⇒ x/3a = 2b/ a + b

Now, applying componendo and dividendo we have

$$\frac{x+3a}{x-3a} = \frac{2b+a+b}{2b-a-b}$$

$$\frac{x+3a}{x-3a} = \frac{3b+a}{b-a} \quad \ldots (1)$$

Again, x = 6ab/ (a + b)

⇒ x/3b = 2a/ a + b

Now, applying componendo and dividendo we have

$$\frac{x+3b}{x-3b}=\frac{2a+a+b}{2a-a-b}$$
$$\frac{x+3b}{x-3b}=\frac{3a+b}{a-b} \quad \ldots (2)$$

From (1) and (2), we get

$$\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{3b+a}{b-a}+\frac{3a+b}{a-b}$$
$$\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{-3b-a+3a+b}{a-b}$$
$$\frac{x+3a}{x-3a}+\frac{x+3b}{x-3b}=\frac{2a-2b}{a-b}=2$$

(ii) If a = 4√6/ (√2 + √3), find the value of:

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}+\frac{a+2\sqrt{3}}{a-2\sqrt{3}}$$

Solution:

Given, a = 4√6/ (√2 + √3)
a/2√2 = 2√3/ (√2 + √3)
Now, applying componendo and dividendo we have

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}=\frac{2\sqrt{3}+\sqrt{2}+\sqrt{3}}{2\sqrt{3}-\sqrt{2}-\sqrt{3}}$$
$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \quad \ldots (1)$$

Again, a = 4√6/ (√2 + √3)
a/2√3 = 2√2/ (√2 + √3)
Now, applying componendo and dividendo we have

$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}}=\frac{2\sqrt{2}+\sqrt{2}+\sqrt{3}}{2\sqrt{2}-\sqrt{2}-\sqrt{3}}$$
$$\frac{a+2\sqrt{3}}{a-2\sqrt{3}}=\frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}} \quad \ldots (2)$$

From (1) and (2), we have

$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}+\frac{a+2\sqrt{3}}{a-2\sqrt{3}}=\frac{3\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}+\frac{3\sqrt{2}+\sqrt{3}}{\sqrt{2}-\sqrt{3}}$$
$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}+\frac{a+2\sqrt{3}}{a-2\sqrt{3}}=\frac{3\sqrt{2}+\sqrt{3}-3\sqrt{3}-\sqrt{2}}{\sqrt{2}-\sqrt{3}}$$
$$\frac{a+2\sqrt{2}}{a-2\sqrt{2}}+\frac{a+2\sqrt{3}}{a-2\sqrt{3}}=\frac{2\sqrt{2}-2\sqrt{3}}{\sqrt{2}-\sqrt{3}}=2$$

7. If (a + b + c + d) (a - b - c + d) = (a + b - c - d) (a - b + c - d), prove that a: b = c: d.
Solution:

Rewriting the given, we have

$$\frac{a+b+c+d}{a+b-c-d}=\frac{a-b+c-d}{a-b-c+d}$$

Now, applying componendo and dividendo

$$\frac{(a+b+c+d)+(a+b-c-d)}{(a+b+c+d)-(a+b-c-d)}=\frac{(a-b+c-d)+(a-b-c+d)}{(a-b+c-d)-(a-b-c+d)}$$

$$\frac{2(a+b)}{2(c+d)}=\frac{2(a-b)}{2(c-d)}$$

$$\frac{a+b}{c+d}=\frac{a-b}{c-d}$$

$$\frac{a+b}{a-b}=\frac{c+d}{c-d}$$

Applying componendo and dividendo again, we get

$$\frac{a+b+a-b}{a+b-a+b}=\frac{c+d+c-d}{c+d-c+d}$$

$$\frac{2a}{2b}=\frac{2c}{2d}$$

$$\frac{a}{b}=\frac{c}{d}$$

- Hence Proved