Exercise 8(C) Page No: 112

1. Show that (x - 1) is a factor of $x^3 - 7x^2 + 14x - 8$. Hence, completely factorise the given expression.

Solution:

Let $f(x) = x^3 - 7x^2 + 14x - 8$

Then, for x = 1

$f(1) = (1)^3 - 7(1)^2 + 14(1) - 8 = 1 - 7 + 14 - 8 = 0$

Thus, (x - 1) is a factor of f(x).

Now, performing long division we have

$$\begin{array}{r|l} & x^2 - 6x + 8 \\ \hline x - 1 & x^3 - 7x^2 + 14x - 8 \\ & x^3 - x^2 \\ \hline & -6x^2 + 14x - 8 \\ & -6x^2 + 6x \\ \hline & 8x - 8 \\ & 8x - 8 \\ \hline & 0 \end{array}$$

Hence, $f(x) = (x - 1)(x^2 – 6x + 8)$

$= (x - 1)(x^2 – 4x – 2x + 8)$

$= (x - 1)[x(x - 4) -2(x - 4)]$

$= (x - 1)(x - 4)(x - 2)$

2. Using Remainder Theorem, factorise:
$x^3 + 10x^2 - 37x + 26$ completely.

Solution:

Let $f(x) = x^3 + 10x^2 - 37x + 26$

From remainder theorem, we know that

For x = 1, the value of f(x) is the remainder

$f(1) = (1)^3 + 10(1)^2 – 37(1) + 26 = 1 + 10 – 37 + 26 = 0$

As f(1) = 0, x – 1 is a factor of f(x).

Now, performing long division we have

$$
\begin{array}{r|l}
 & x^2 + 11x - 26 \\
\hline
x - 1 & x^3 + 10x^2 - 37x + 26 \\
 & x^3 - x^2 \\
\hline
 & 11x^2 - 37x + 26 \\
 & 11x^2 - 11x \\
\hline
 & -26x + 26 \\
 & -26x + 26 \\
\hline
 & 0
\end{array}
$$

Thus, $f(x) = (x - 1)(x^2 + 11x - 26)$

$= (x - 1)(x^2 + 13x - 2x - 26)$

$= (x - 1)[x(x + 13) – 2(x + 13)]$

$= (x - 1)(x + 13)(x - 2)$

3. When $x^3 + 3x^2 - mx + 4$ is divided by $x - 2$, the remainder is $m + 3$. Find the value of m.
Solution:

Let $f(x) = x^3 + 3x^2 - mx + 4$
From the question, we have
$f(2) = m + 3$
$(2)^3 + 3(2)^2 - m(2) + 4 = m + 3$
$8 + 12 - 2m + 4 = m + 3$
$24 - 3 = m + 2m$
$3m = 21$
Thus, $m = 7$

4. What should be subtracted from $3x^3 - 8x^2 + 4x - 3$, so that the resulting expression has $x + 2$ as a factor?
Solution:

Let's assume the required number to be k.
And let $f(x) = 3x^3 - 8x^2 + 4x - 3 - k$
From the question, we have
$f(-2) = 0$
$3(-2)^3 - 8(-2)^2 + 4(-2) - 3 - k = 0$
$-24 - 32 - 8 - 3 - k = 0$
$-67 - k = 0$
$k = -67$
Therefore, the required number is -67.

5. If $(x + 1)$ and $(x - 2)$ are factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, find the values of a and b. And then, factorise the given expression completely.

Solution:

Let's take $f(x) = x^3 + (a + 1)x^2 - (b - 2)x - 6$
As, $(x + 1)$ is a factor of $f(x)$.
Then, remainder $= f(-1) = 0$
$(-1)^3 + (a + 1)(-1)^2 - (b - 2)(-1) - 6 = 0$
$-1 + (a + 1) + (b - 2) - 6 = 0$
$a + b - 8 = 0$ ... (1)

And as, $(x - 2)$ is a factor of $f(x)$.
Then, remainder $= f(2) = 0$
$(2)^3 + (a + 1)(2)^2 - (b - 2)(2) - 6 = 0$
$8 + 4a + 4 - 2b + 4 - 6 = 0$
$4a - 2b + 10 = 0$
$2a - b + 5 = 0$ ... (2)

Adding (1) and (2), we get
$3a - 3 = 0$
Thus, $a = 1$
Substituting the value of a in (i), we get,
$1 + b - 8 = 0$
Thus, $b = 7$

Hence, $f(x) = x^3 + 2x^2 - 5x - 6$
Now as $(x + 1)$ and $(x - 2)$ are factors of $f(x)$.
So, $(x + 1)(x - 2) = x^2 - x - 2$ is also a factor of $f(x)$.

$$
\begin{array}{r|l}
 & x + 3 \\
x^2 - x - 2 & x^3 + 2x^2 - 5x - 6 \\
 & x^3 - x^2 - 2x \\
\hline
 & 3x^2 - 3x - 6 \\
 & 3x^2 - 3x - 6 \\
\hline
 & 0
\end{array}
$$

Therefore, $f(x) = x^3 + 2x^2 - 5x - 6 = (x + 1)(x - 2)(x + 3)$

6. If $x - 2$ is a factor of $x^2 + ax + b$ and $a + b = 1$, find the values of a and b.
Solution:

Let $f(x) = x^2 + ax + b$
Given, $(x - 2)$ is a factor of $f(x)$.
Then, remainder $= f(2) = 0$
$(2)^2 + a(2) + b = 0$
$4 + 2a + b = 0$
$2a + b = -4$ ... (1)

And also given that,
a + b = 1 ... (2)

Subtracting (2) from (1), we have
a = -5
On substituting the value of a in (2), we have
b = 1 - (-5) = 6

7. Factorise $x^3 + 6x^2 + 11x + 6$ completely using factor theorem.
Solution:

Let $f(x) = x^3 + 6x^2 + 11x + 6$
For x = -1, the value of f(x) is
$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$
$= -1 + 6 - 11 + 6 = 12 - 12 = 0$
Thus, (x + 1) is a factor of f(x).

$$
\begin{array}{r|l}
 & x^2 + 5x + 6 \\
\hline
x + 1 & x^3 + 6x^2 + 11x + 6 \\
 & x^3 + x^2 \\
\hline
 & \quad 5x^2 + 11x + 6 \\
 & \quad 5x^2 + 5x \\
\hline
 & \qquad 6x + 6 \\
 & \qquad 6x + 6 \\
\hline
 & \qquad\quad 0
\end{array}
$$

Therefore, $f(x) = (x + 1)(x^2 + 5x + 6)$
$= (x + 1)(x^2 + 3x + 2x + 6)$
$= (x + 1)[x(x + 3) + 2(x + 3)]$
$= (x + 1)(x + 3)(x + 2)$

8. Find the value of 'm', if $mx^3 + 2x^2 - 3$ and $x^2 - mx + 4$ leave the same remainder when each is divided by x - 2.
Solution:

Let $f(x) = mx^3 + 2x^2 – 3$ and $g(x) = x^2 - mx + 4$
From the question, it’s given that f(x) and g(x) leave the same remainder when divided by (x - 2). So, we have:
f(2) = g(2)
$m(2)^3 + 2(2)^2 - 3 = (2)^2 - m(2) + 4$
8m + 8 - 3 = 4 - 2m + 4
10m = 3
Thus, m = 3/10