Exercise 8(A)

1. Find, in each case, the remainder when:
(i) $x^4 - 3x^2 + 2x + 1$ is divided by $x - 1$.
(ii) $x^3 + 3x^2 - 12x + 4$ is divided by $x - 2$.
(ii) $x^4 + 1$ is divided by $x + 1$.
Solution:

From remainder theorem, we know that when a polynomial f (x) is divided by (x – a), then the remainder is f(a).
(i) Given, $f(x) = x^4 - 3x^2 + 2x + 1$ is divided by x - 1
So, remainder $= f(1) = (1)^4 - 3(1)^2 + 2(1) + 1 = 1 - 3 + 2 + 1 = 1$

(ii) Given, $f(x) = x^3 + 3x^2 - 12x + 4$ is divided by x – 2
So, remainder $= f(2) = (2)^3 + 3(2)^2 - 12(2) + 4 = 8 + 12 - 24 + 4 = 0$

(iii) Given, $f(x) = x^4 + 1$ is divided by x + 1
So, remainder $= f(-1) = (-1)^4 + 1 = 2$

2. Show that:
(i) $x - 2$ is a factor of $5x^2 + 15x - 50$
(ii) $3x + 2$ is a factor of $3x^2 - x - 2$
Solution:

(x - a) is a factor of a polynomial f(x) if the remainder, when f(x) is divided by (x - a), is 0, i.e., if f(a) = 0.
(i) $f(x) = 5x^2 + 15x - 50$
$f(2) = 5(2)^2 + 15(2) - 50 = 20 + 30 - 50 = 0$
As the remainder is zero for x = 2
Thus, we can conclude that (x – 2) is a factor of $5x^2 + 15x - 50$

(ii) $f(x) = 3x^2 - x - 2$
$f(-2/3) = 3(-2/3)^2 - (-2/3) - 2 = 4/3 + 2/3 - 2 = 2 - 2 = 0$
As the remainder is zero for x = -2/3
Thus, we can conclude that (3x + 2) is a factor of $3x^2 - x - 2$

3. Use the Remainder Theorem to find which of the following is a factor of $2x^3 + 3x^2 - 5x - 6$.
(i) x + 1 (ii) 2x - 1
(iii) x + 2
Solution:

From remainder theorem we know that when a polynomial f (x) is divided by x - a, then the remainder is f(a).
Here, $f(x) = 2x^3 + 3x^2 - 5x - 6$

(i) $f(-1) = 2(-1)^3 + 3(-1)^2 - 5(-1) - 6 = -2 + 3 + 5 - 6 = 0$

⇒ Remainder is zero for x = -1
Therefore, (x + 1) is a factor of the polynomial f(x).

(ii) $f(1/2) = 2(1/2)^3 + 3(1/2)^2 – 5(1/2) – 6$
$= ¼ + ¾ - 5/2 – 6$
$= -5/2 – 5 = -15/2$
⇒ Remainder is not equals to zero for x = 1/2
Therefore, (2x - 1) is not a factor of the polynomial f(x).

(iii) $f(-2) = 2(-2)^3 + 3(-2)^2 - 5(-2) - 6 = -16 + 12 + 10 - 6 = 0$
⇒ Remainder is zero for x = -2
Therefore, (x + 2) is a factor of the polynomial f(x).

4. (i) If 2x + 1 is a factor of $2x^2 + ax - 3$, find the value of a.
(ii) Find the value of k, if 3x - 4 is a factor of expression $3x^2 + 2x - k$.
Solution:

(i) Given, 2x + 1 is a factor of $f(x) = 2x^2 + ax - 3$.
So, f(-1/2) = 0
$2(-1/2)^2 + a(-1/2) - 3 = 0$
½ - a/2 – 3 = 0
1 – a – 6 = 0
a = -5

(ii) Given, 3x - 4 is a factor of $g(x) = 3x^2 + 2x - k$.
So, f(4/3) = 0
$3(4/3)^2 + 2(4/3) – k = 0$
16/3 + 8/3 – k = 0
24/3 = k
k = 8

5. Find the values of constants a and b when x - 2 and x + 3 both are the factors of expression $x^3 + ax^2 + bx - 12$.
Solution:

Here, $f(x) = x^3 + ax^2 + bx – 12$
Given, x – 2 and x + 3 both are the factors of f(x)
So,
f(2) and f(-3) both should be equal to zero.
$f(2) = (2)^3 + a(2)^2 + b(2) – 12$
0 = 8 + 4a + 2b – 12
0 = 4a + 2b – 4
2a + b = 2 …. (1)

Now,
$f(-3) = (-3)^3 + a(-3)^2 + b(-3) – 12$

0 = -27 + 9a – 3b – 12
9a – 3b – 39 = 0
3a – b = 13 …. (2)

Adding (1) and (2), we get,
5a = 15
Thus, a = 3
Putting the value of a in (1), we have
6 + b = 2
Thus, b = -4

6. Find the value of k, if 2x + 1 is a factor of $(3k + 2)x^3 + (k - 1)$.
Solution:

Let take $f(x) = (3k + 2)x^3 + (k - 1)$
Now, 2x + 1 = 0
x = -1/2
As, 2x + 1 is a factor of f(x) then the remainder should be 0.
$f(-1/2) = (3k + 2)(-1/2)^3 + (k - 1) = 0$

$$\Rightarrow \frac{-(3k+2)}{8} + (k-1) = 0$$

$$\Rightarrow \frac{-3k-2+8k-8}{8} = 0$$

5k = 10 = 0
k = 2

7. Find the value of a, if x - 2 is a factor of $2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$.
Solution:

Given, $f(x) = 2x^5 - 6x^4 - 2ax^3 + 6ax^2 + 4ax + 8$ and x – 2 is a factor of f(x).
So, x - 2 = 0; x = 2
Hence, f(2) = 0
$2(2)^5 - 6(2)^4 - 2a(2)^3 + 6a(2)^2 + 4a(2) + 8 = 0$
64 - 96 - 16a + 24a + 8a + 8 = 0
-24 + 16a = 0
16a = 24
Thus, a = 1.5

8. Find the values of m and n so that x - 1 and x + 2 both are factors of $x^3 + (3m + 1)x^2 + nx - 18$.
Solution:

Let $f(x) = x^3 + (3m + 1)x^2 + nx – 18$
Given, (x – 1) and (x + 2) are the factors of f(x).
So,
x - 1 = 0; x = 1 and x + 2 = 0; x = -2
f(1) and f(-2) both should be equal to zero.

$(1)^3 + (3m + 1)(1)^2 + n(1) – 18 = 0$
$1 + 3m + 1 + n – 18 = 0$
$3m + n – 16 = 0$ ….. (1)
And,
$(-2)^3 + (3m + 1)(-2)^2 + n(-2) – 18 = 0$
$8 + 12m + 4 - 2n – 18 = 0$
$12m – 2n – 22 = 0$
$6m – n – 11 = 0$ ….. (2)

Adding (1) and (2), we get,
$9m - 27 = 0$
Thus, $m = 3$
Putting the value of m in (1), we have
$3(3) + n - 16 =0$
$9 + n - 16 = 0$
Therefore, $n = 7$

Exercise 8(B)

1. Using the Factor Theorem, show that:
(i) (x - 2) is a factor of $x^3 - 2x^2 - 9x + 18$. Hence, factorise the expression $x^3 - 2x^2 - 9x + 18$ completely.
(ii) (x + 5) is a factor of $2x^3 + 5x^2 - 28x - 15$. Hence, factorise the expression $2x^3 + 5x^2 - 28x - 15$ completely.
(iii) (3x + 2) is a factor of $3x^3 + 2x^2 - 3x - 2$. Hence, factorise the expression $3x^3 + 2x^2 - 3x - 2$ completely.
Solution:

(i) Here, $f(x) = x^3 - 2x^2 - 9x + 18$
So, $x - 2 = 0 \Rightarrow x = 2$

Thus, remainder = f(2)
$= (2)^3 - 2(2)^2 - 9(2) + 18$
$= 8 - 8 - 18 + 18$
$= 0$
Therefore, (x - 2) is a factor of f(x).

Now, performing division of polynomial f(x) by (x – 2) we have

$$
\begin{array}{r|l}
 & x^2 - 9 \\
\hline
x - 2 & x^3 - 2x^2 - 9x + 18 \\
 & x^3 - 2x^2 \\
\hline
 & \quad -9x + 18 \\
 & \quad -9x + 18 \\
\hline
 & \quad 0
\end{array}
$$

Thus, $x^3 - 2x^2 - 9x + 18 = (x - 2)(x^2 - 9) = (x - 2)(x + 3)(x - 3)$

(ii) Here, $f(x) = 2x^3 + 5x^2 - 28x - 15$
So, $x + 5 = 0 \Rightarrow x = -5$

Thus, remainder = f(-5)
$= 2(-5)^3 + 5(-5)^2 - 28(-5) - 15$
$= -250 + 125 + 140 - 15$
$= -265 + 265$
$= 0$
Therefore, (x + 5) is a factor of f(x).

Now, performing division of polynomial f(x) by (x + 5) we get

$$\begin{array}{r|l} & 2x^2 - 5x - 3 \\ \hline x + 5 & 2x^3 + 5x^2 - 28x - 15 \\ & 2x^3 + 10x^2 \\ \hline & -5x^2 - 28x - 15 \\ & -5x^2 - 25x \\ \hline & -3x - 15 \\ & -3x - 15 \\ \hline & 0 \end{array}$$

So, $2x^3 + 5x^2 - 28x - 15 = (x + 5)(2x^2 - 5x - 3)$
Further, on factorisation
$= (x + 5)[2x^2 - 6x + x - 3]$
$= (x + 5)[2x(x - 3) + 1(x - 3)] = (x + 5)(2x + 1)(x - 3)$
Thus, f(x) is factorised as $(x + 5)(2x + 1)(x - 3)$

(iii) Here, $f(x) = 3x^3 + 2x^2 - 3x - 2$
So, $3x + 2 = 0 \Rightarrow x = -2/3$

Thus, remainder = f(-2/3)
$= 3(-2/3)^3 + 2(-2/3)^2 - 3(-2/3) - 2$
$= -8/9 + 8/9 + 2 - 2$
$= 0$
Therefore, (3x + 2) is a factor of f(x).

Now, performing division of polynomial f(x) by (3x + 2) we get

$$\begin{array}{r|l} & x^2 - 1 \\ \hline 3x + 2 & 3x^3 + 2x^2 - 3x - 2 \\ & 3x^3 + 2x^2 \\ \hline & -3x - 2 \\ & -3x - 2 \\ \hline & 0 \end{array}$$

Thus, $3x^3 + 2x^2 - 3x - 2 = (3x + 2)(x^2 - 1) = (3x + 2)(x - 1)(x + 1)$

2. Using the Remainder Theorem, factorise each of the following completely.
(i) $3x^3 + 2x^2 - 19x + 6$
(ii) $2x^3 + x^2 - 13x + 6$
(iii) $3x^3 + 2x^2 - 23x - 30$
(iv) $4x^3 + 7x^2 - 36x - 63$
(v) $x^3 + x^2 - 4x - 4$

Solution:

(i) Let $f(x) = 3x^3 + 2x^2 - 19x + 6$

For $x = 2$, the value of $f(x)$ will be

$= 3(2)^3 + 2(2)^2 - 19(2) + 6$

$= 24 + 8 - 38 + 6 = 0$

As $f(2) = 0$, so $(x - 2)$ is a factor of $f(x)$.

Now, performing long division we have

$$\begin{array}{r|l} & 3x^2 + 8x - 3 \\ \hline x - 2 & 3x^3 + 2x^2 - 19x + 6 \\ & 3x^3 - 6x^2 \\ \hline & \quad 8x^2 - 19x + 6 \\ & \quad 8x^2 - 16x \\ \hline & \qquad -3x + 6 \\ & \qquad -3x + 6 \\ \hline & \qquad\qquad 0 \end{array}$$

Thus, $f(x) = (x - 2)(3x^2 + 8x - 3)$

$= (x - 2)(3x^2 + 9x - x - 3)$

$= (x - 2)[3x(x + 3) - 1(x + 3)]$

$= (x - 2)(x + 3)(3x - 1)$

(ii) Let $f(x) = 2x^3 + x^2 - 13x + 6$

For $x = 2$, the value of $f(x)$ will be

$f(2) = 2(2)^3 + (2)^2 - 13(2) + 6 = 16 + 4 - 26 + 6 = 0$

As $f(2) = 0$, so $(x - 2)$ is a factor of $f(x)$.

Now, performing long division we have

$$\begin{array}{r|l} & 2x^2 + 5x - 3 \\ \hline x - 2 & 2x^3 + x^2 - 13x + 6 \\ & 2x^3 - 4x^2 \\ \hline & \quad 5x^2 - 13x + 6 \\ & \quad 5x^2 - 10x \\ \hline & \qquad -3x + 6 \\ & \qquad -3x + 6 \\ \hline & \qquad\qquad 0 \end{array}$$

Thus, $f(x) = (x - 2)(2x^2 + 5x - 3)$

$= (x - 2)[2x^2 + 6x - x - 3]$

$= (x - 2)[2x(x + 3) - 1(x + 3)]$

$= (x - 2)\,[2x(x + 3) - 1(x + 3)]$
$= (x - 2)\,(2x – 1)\,(x + 3)$

(iii) Let $f(x) = 3x^3 + 2x^2 - 23x - 30$
For $x = -2$, the value of f(x) will be
$f(-2) = 3(-2)^3 + 2(-2)^2 - 23(-2) - 30$
$= -24 + 8 + 46 - 30 = -54 + 54 = 0$
As $f(-2) = 0$, so $(x + 2)$ is a factor of f(x).
Now, performing long division we have

$$
\begin{array}{r|l}
 & 3x^2 - 4x - 15 \\
\hline
x + 2 & 3x^3 + 2x^2 - 23x - 30 \\
 & 3x^3 + 6x^2 \\
\hline
 & -4x^2 - 23x - 30 \\
 & -4x^2 - 8x \\
\hline
 & -15x - 30 \\
 & -15x - 30 \\
\hline
 & 0
\end{array}
$$

Thus, $f(x) = (x + 2)\,(3x^2 – 4x - 15)$
$= (x + 2)\,(3x^2 – 9x + 5x - 15)$
$= (x + 2)\,[3x(x – 3) + 5(x – 3)]$
$= (x + 2)\,(3x + 5)\,(x – 3)$

(iv) Let $f(x) = 4x^3 + 7x^2 - 36x - 63$
For $x = 3$, the value of f(x) will be
$f(3) = 4(3)^3 + 7(3)^2 - 36(3) - 63$
$= 108 + 63 - 108 - 63 = 0$
As $f(3) = 0$, $(x + 3)$ is a factor of f(x).
Now, performing long division we have

$$
\begin{array}{r|l}
 & 4x^2 + 19x + 21 \\
\hline
x - 3 & 4x^3 + 7x^2 - 36x - 63 \\
 & 4x^3 - 12x^2 \\
\hline
 & 19x^2 - 36x - 63 \\
 & 19x^2 - 57x \\
\hline
 & 21x - 63 \\
 & 21x - 63 \\
\hline
 & 0
\end{array}
$$

Thus, $f(x) = (x + 3)(4x^2 – 5x - 21)$
$= (x + 3)(4x^2 – 12x + 7x - 21)$
$= (x + 3)[4x(x – 3) + 7(x - 3)]$
$= (x + 3)(4x + 7)(x - 3)$

(v) Let $f(x) = x^3 + x^2 - 4x - 4$
For $x = -1$, the value of $f(x)$ will be
$f(-1) = (-1)^3 + (-1)^2 - 4(-1) - 4$
$= -1 + 1 + 4 - 4 = 0$
As, $f(-1) = 0$ so $(x + 1)$ is a factor of $f(x)$.
Now, performing long division we have

$$\begin{array}{r|l} & x^2 - 4 \\ \hline x+1 & x^3 + x^2 - 4x - 4 \\ & x^3 + x^2 \\ \hline & \qquad -4x - 4 \\ & \qquad -4x - 4 \\ \hline & \qquad\qquad 0 \end{array}$$

Thus, $f(x) = (x + 1)(x^2 - 4)$
$= (x + 1)(x - 2)(x + 2)$

3. Using the Remainder Theorem, factorise the expression $3x^3 + 10x^2 + x - 6$. Hence, solve the equation $3x^3 + 10x^2 + x - 6 = 0$.
Solution:

Let's take $f(x) = 3x^3 + 10x^2 + x - 6$
For $x = -1$, the value of $f(x)$ will be
$f(-1) = 3(-1)^3 + 10(-1)^2 + (-1) - 6 = -3 + 10 - 1 - 6 = 0$
As, $f(-1) = 0$ so $(x + 1)$ is a factor of $f(x)$.
Now, performing long division we have

$$\begin{array}{r|l} & 3x^2 + 7x - 6 \\ \hline x+1 & 3x^3 + 10x^2 + x - 6 \\ & 3x^3 + 3x^2 \\ \hline & \qquad 7x^2 + x - 6 \\ & \qquad 7x^2 + 7x \\ \hline & \qquad\qquad -6x - 6 \\ & \qquad\qquad -6x - 6 \\ \hline & \qquad\qquad\qquad 0 \end{array}$$

Thus, $f(x) = (x + 1) (3x^2 + 7x - 6)$
$= (x + 1) (3x^2 + 9x – 2x - 6)$
$= (x + 1) [3x(x + 3) -2(x + 3)]$
$= (x + 1) (x + 3) (3x - 2)$

Now, $3x^3 + 10x^2 + x - 6 = 0$
$(x + 1) (x + 3) (3x - 2) = 0$
Therefore,
$x = -1, -3$ or $2/3$

4. Factorise the expression f (x) = $2x^3 - 7x^2 - 3x + 18$. Hence, find all possible values of x for which f(x) = 0.
Solution:

Let $f(x) = 2x^3 - 7x^2 - 3x + 18$
For x = 2, the value of f(x) will be
$f(2) = 2(2)^3 - 7(2)^2 - 3(2) + 18$
$= 16 - 28 - 6 + 18 = 0$
As f(2) = 0, (x - 2) is a factor of f(x).
Now, performing long division we have

$$
\begin{array}{r|l}
 & 2x^2 - 3x - 9 \\
\hline
x - 2 & 2x^3 - 7x^2 - 3x + 18 \\
 & 2x^3 - 4x^2 \\
\hline
 & -3x^2 - 3x + 18 \\
 & -3x^2 + 6x \\
\hline
 & -9x + 18 \\
 & -9x + 18 \\
\hline
 & 0
\end{array}
$$

Thus, $f(x) = (x - 2) (2x^2 – 3x - 9)$
$= (x - 2) (2x^2 – 6x + 3x - 9)$
$= (x - 2) [2x(x – 3) + 3(x - 3)]$
$= (x - 2) (x - 3) (2x + 3)$
Now, for f(x) = 0
$(x - 2) (x - 3) (2x + 3) = 0$
Hence x = 2, 3 or -3/2

5. Given that x - 2 and x + 1 are factors of f(x) = $x^3 + 3x^2 + ax + b$; calculate the values of a and b. Hence, find all the factors of f(x).
Solution:

Let $f(x) = x^3 + 3x^2 + ax + b$

As, (x - 2) is a factor of f(x), so f(2) = 0
$(2)^3 + 3(2)^2 + a(2) + b = 0$
$8 + 12 + 2a + b = 0$
$2a + b + 20 = 0$ ... (1)

And as, (x + 1) is a factor of f(x), so f(-1) = 0
$(-1)^3 + 3(-1)^2 + a(-1) + b = 0$
$-1 + 3 - a + b = 0$
$-a + b + 2 = 0$ ... (2)

Subtracting (2) from (1), we have
$3a + 18 = 0$
$a = -6$
On substituting the value of a in (ii), we have
$b = a - 2 = -6 – 2 = -8$

Thus, $f(x) = x^3 + 3x^2 - 6x - 8$
Now, for x = -1
$f(-1) = (-1)^3 + 3(-1)^2 - 6(-1) - 8 = -1 + 3 + 6 - 8 = 0$
Therefore, (x + 1) is a factor of f(x).
Now, performing long division we have

$$
\begin{array}{r|l}
 & x^2 + 2x - 8 \\
\hline
x + 1 & x^3 + 3x^2 - 6x - 8 \\
 & x^3 + x^2 \\
\hline
 & \quad 2x^2 - 6x - 8 \\
 & \quad 2x^2 + 2x \\
\hline
 & \qquad -8x - 8 \\
 & \qquad -8x - 8 \\
\hline
 & \qquad\qquad 0
\end{array}
$$

Hence, $f(x) = (x + 1)(x^2 + 2x - 8)$
$= (x + 1)(x^2 + 4x – 2x - 8)$
$= (x + 1)[x(x + 4) – 2(x + 4)]$
$= (x + 1)(x + 4)(x - 2)$

Exercise 8(C) Page No: 112

1. Show that (x - 1) is a factor of $x^3 - 7x^2 + 14x - 8$. Hence, completely factorise the given expression.

Solution:

Let $f(x) = x^3 - 7x^2 + 14x - 8$

Then, for x = 1

$f(1) = (1)^3 - 7(1)^2 + 14(1) - 8 = 1 - 7 + 14 - 8 = 0$

Thus, (x - 1) is a factor of f(x).

Now, performing long division we have

$$\begin{array}{r|l} & x^2 - 6x + 8 \\ \hline x - 1 & x^3 - 7x^2 + 14x - 8 \\ & x^3 - x^2 \\ \hline & \quad -6x^2 + 14x - 8 \\ & \quad -6x^2 + 6x \\ \hline & \qquad\qquad 8x - 8 \\ & \qquad\qquad 8x - 8 \\ \hline & \qquad\qquad\quad 0 \end{array}$$

Hence, $f(x) = (x - 1)(x^2 – 6x + 8)$

$= (x - 1)(x^2 – 4x – 2x + 8)$

$= (x - 1)[x(x - 4) -2(x - 4)]$

$= (x - 1)(x - 4)(x - 2)$

2. Using Remainder Theorem, factorise:
$x^3 + 10x^2 - 37x + 26$ completely.

Solution:

Let $f(x) = x^3 + 10x^2 - 37x + 26$

From remainder theorem, we know that

For x = 1, the value of f(x) is the remainder

$f(1) = (1)^3 + 10(1)^2 – 37(1) + 26 = 1 + 10 – 37 + 26 = 0$

As f(1) = 0, x – 1 is a factor of f(x).

Now, performing long division we have

$$
\begin{array}{r|l}
 & x^2 + 11x - 26 \\
x - 1 & x^3 + 10x^2 - 37x + 26 \\
 & x^3 - x^2 \\
\hline
 & 11x^2 - 37x + 26 \\
 & 11x^2 - 11x \\
\hline
 & -26x + 26 \\
 & -26x + 26 \\
\hline
 & 0
\end{array}
$$

Thus, $f(x) = (x - 1)(x^2 + 11x - 26)$
$= (x - 1)(x^2 + 13x - 2x - 26)$
$= (x - 1)[x(x + 13) – 2(x + 13)]$
$= (x - 1)(x + 13)(x - 2)$

3. When $x^3 + 3x^2 - mx + 4$ is divided by $x - 2$, the remainder is $m + 3$. Find the value of m.
Solution:

Let $f(x) = x^3 + 3x^2 - mx + 4$
From the question, we have
$f(2) = m + 3$
$(2)^3 + 3(2)^2 - m(2) + 4 = m + 3$
$8 + 12 - 2m + 4 = m + 3$
$24 - 3 = m + 2m$
$3m = 21$
Thus, $m = 7$

4. What should be subtracted from $3x^3 - 8x^2 + 4x - 3$, so that the resulting expression has $x + 2$ as a factor?
Solution:

Let's assume the required number to be k.
And let $f(x) = 3x^3 - 8x^2 + 4x - 3 - k$
From the question, we have
$f(-2) = 0$
$3(-2)^3 - 8(-2)^2 + 4(-2) - 3 - k = 0$
$-24 - 32 - 8 - 3 - k = 0$
$-67 - k = 0$
$k = -67$
Therefore, the required number is -67.

5. If $(x + 1)$ and $(x - 2)$ are factors of $x^3 + (a + 1)x^2 - (b - 2)x - 6$, find the values of a and b. And then, factorise the given expression completely.

Solution:

Let's take $f(x) = x^3 + (a + 1)x^2 - (b - 2)x - 6$
As, $(x + 1)$ is a factor of $f(x)$.
Then, remainder $= f(-1) = 0$
$(-1)^3 + (a + 1)(-1)^2 - (b - 2)(-1) - 6 = 0$
$-1 + (a + 1) + (b - 2) - 6 = 0$
$a + b - 8 = 0$ ... (1)

And as, $(x - 2)$ is a factor of $f(x)$.
Then, remainder $= f(2) = 0$
$(2)^3 + (a + 1)(2)^2 - (b - 2)(2) - 6 = 0$
$8 + 4a + 4 - 2b + 4 - 6 = 0$
$4a - 2b + 10 = 0$
$2a - b + 5 = 0$ ... (2)

Adding (1) and (2), we get
$3a - 3 = 0$
Thus, $a = 1$
Substituting the value of a in (i), we get,
$1 + b - 8 = 0$
Thus, $b = 7$

Hence, $f(x) = x^3 + 2x^2 - 5x - 6$
Now as $(x + 1)$ and $(x - 2)$ are factors of $f(x)$.
So, $(x + 1)(x - 2) = x^2 - x - 2$ is also a factor of $f(x)$.

$$\begin{array}{r|l} & x + 3 \\ \hline x^2 - x - 2 & x^3 + 2x^2 - 5x - 6 \\ & x^3 - x^2 - 2x \\ \hline & 3x^2 - 3x - 6 \\ & 3x^2 - 3x - 6 \\ \hline & 0 \end{array}$$

Therefore, $f(x) = x^3 + 2x^2 - 5x - 6 = (x + 1)(x - 2)(x + 3)$

6. If $x - 2$ is a factor of $x^2 + ax + b$ and $a + b = 1$, find the values of a and b.
Solution:

Let $f(x) = x^2 + ax + b$
Given, $(x - 2)$ is a factor of $f(x)$.
Then, remainder $= f(2) = 0$
$(2)^2 + a(2) + b = 0$
$4 + 2a + b = 0$
$2a + b = -4$ ... (1)

And also given that,
a + b = 1 ... (2)

Subtracting (2) from (1), we have
a = -5
On substituting the value of a in (2), we have
b = 1 - (-5) = 6

7. Factorise $x^3 + 6x^2 + 11x + 6$ completely using factor theorem.
Solution:

Let $f(x) = x^3 + 6x^2 + 11x + 6$
For x = -1, the value of f(x) is
$f(-1) = (-1)^3 + 6(-1)^2 + 11(-1) + 6$
$= -1 + 6 - 11 + 6 = 12 - 12 = 0$
Thus, (x + 1) is a factor of f(x).

$$\begin{array}{r|l} & x^2 + 5x + 6 \\ \hline x + 1 & x^3 + 6x^2 + 11x + 6 \\ & x^3 + x^2 \\ \hline & \quad 5x^2 + 11x + 6 \\ & \quad 5x^2 + 5x \\ \hline & \qquad 6x + 6 \\ & \qquad 6x + 6 \\ \hline & \qquad\quad 0 \end{array}$$

Therefore, $f(x) = (x + 1)(x^2 + 5x + 6)$
$= (x + 1)(x^2 + 3x + 2x + 6)$
$= (x + 1)[x(x + 3) + 2(x + 3)]$
$= (x + 1)(x + 3)(x + 2)$

8. Find the value of 'm', if $mx^3 + 2x^2 - 3$ and $x^2 - mx + 4$ leave the same remainder when each is divided by x - 2.
Solution:

Let $f(x) = mx^3 + 2x^2 – 3$ and $g(x) = x^2 - mx + 4$
From the question, it's given that f(x) and g(x) leave the same remainder when divided by (x - 2). So, we have:
f(2) = g(2)
$m(2)^3 + 2(2)^2 - 3 = (2)^2 - m(2) + 4$
8m + 8 - 3 = 4 - 2m + 4
10m = 3
Thus, m = 3/10