

Exercise 9(C)

1. Evaluate: if possible:

$$(i) \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$(ii) \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix}$$

$$(iii) \begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \end{bmatrix}$$

If not possible, give reason.

Solution:

$$(i) \begin{bmatrix} 3 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} = [6 + 0] = [6]$$

$$(ii) \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ -1 & 4 \end{bmatrix} = [-2+2 \quad 3-8] = [0 \quad -5]$$

$$(iii) \begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} -6 + 12 \\ -3 - 3 \end{bmatrix} = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$$

$$(iv) \begin{bmatrix} 6 & 4 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} -1 & 3 \end{bmatrix}$$

The multiplication of these matrices is not possible as the rule for the number of columns in the first is not equal to the number of rows in the second matrix.

2. If $A = \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$ and I is a unit matrix of order 2×2 , find:

- (i) AB (ii) BA (iii) AI
(iv) IB (v) A^2 (vi) B^2A

Solution:

$$\begin{aligned} (i) AB &= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 0+6 & 0+4 \\ 5-6 & -5-4 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 4 \\ -1 & -9 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{(ii) } BA &= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 0 - 5 & 2 + 2 \\ 0 + 10 & 6 - 4 \end{bmatrix} \\ &= \begin{bmatrix} -5 & 4 \\ 10 & 2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{(iii) } AI &= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 0 + 0 & 0 + 2 \\ 5 + 0 & 0 - 2 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{(iv) } IB &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 + 0 & -1 + 0 \\ 0 + 3 & 0 + 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{(v) } A^2 &= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 0 + 10 & 0 - 4 \\ 0 - 10 & 10 + 4 \end{bmatrix} \\ &= \begin{bmatrix} 10 & -4 \\ -10 & 14 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \text{(vi) } B^2 &= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 1 - 3 & -1 - 2 \\ 3 + 6 & -3 + 4 \end{bmatrix} \\ &= \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix} \end{aligned}$$

$$\begin{aligned} B^2A &= \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \\ &= \begin{bmatrix} 0 - 15 & -4 + 6 \\ 0 + 5 & 18 - 2 \end{bmatrix} \\ &= \begin{bmatrix} -15 & 2 \\ 5 & 16 \end{bmatrix} \end{aligned}$$

3. If $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$, find x and y when $A^2 = B$.

Solution:

$$\begin{aligned} A^2 &= \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 9 + 0 & 3x + x \\ 0 + 0 & 0 + 1 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} \end{aligned}$$

$$A^2 = B$$

$$\begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$$

On comparing corresponding elements, we have

$$4x = 16$$

$$x = 4$$

And,

$$1 = -y$$

$$y = -1$$

4. Find x and y, if:

$$(i) \begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$$

$$(ii) \begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

Solution:

$$(i) \begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 20 + 3x \\ 5x - 2 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$$

On comparing the corresponding terms, we have

$$5x - 2 = 8$$

$$5x = 10$$

$$x = 2$$

And,

$$20 + 3x = y$$

$$20 + 3(2) = y$$

$$20 + 6 = y$$

$$y = 26$$

$$(ii) \begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

$$\begin{bmatrix} x+0 & x \\ -3+0 & -3+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$$

On comparing the corresponding terms, we have

$$x = 2$$

And,

$$-3 + y = -2$$

$$y = 3 - 2 = 1$$

5. If $A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, find :

(i) (AB) C

(ii) A (BC)

Solution:

$$(i) (AB) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+12 & 2+9 \\ 2+16 & 4+12 \end{bmatrix}$$

$$= \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix}$$

$$(AB) C = \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 52+11 & 39+22 \\ 72+16 & 54+32 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$$

$$(ii) BC = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+2 & 3+4 \\ 16+3 & 12+6 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix}$$

$$A (BC) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix} = \begin{bmatrix} 6+57 & 7+54 \\ 12+76 & 14+72 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$$

Therefore, its seen that (AB) C = A (BC)

6. Given $A = \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}$, is the following possible:

(i) AB (ii) BA (iii) A^2

Solution:

$$(i) AB = \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix} = \begin{bmatrix} 0 - 4 - 30 & 0 + 8 - 36 \\ 0 - 0 + 5 & 3 + 0 + 6 \end{bmatrix} = \begin{bmatrix} -34 & -28 \\ 5 & 9 \end{bmatrix}$$

$$(ii) BA = \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix} \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0 + 3 & 0 + 0 & 0 - 1 \\ 0 + 6 & -4 + 0 & -6 - 2 \\ 0 - 18 & -20 + 0 & -30 + 6 \end{bmatrix} = \begin{bmatrix} 3 & 0 & -1 \\ 6 & -4 & -8 \\ -18 & -20 & -24 \end{bmatrix}$$

(iii) $A^2 = A \times A$, is not possible since the number of columns of matrix A is not equal to its number of rows.

7. Let $A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}$, $B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$. Find $A^2 + AC - 5B$.

Solution:

$$A^2 = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4 + 0 & 2 - 2 \\ 0 & 0 + 4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -6 - 1 & 4 + 4 \\ 0 + 2 & 0 - 8 \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix}$$

$$5B = 5 \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$$

$$A^2 + AC - 5B =$$

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix} = \begin{bmatrix} 4 - 7 - 20 & 8 - 5 \\ 2 + 15 & 4 - 8 + 10 \end{bmatrix} = \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}$$

8. If $M = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and I is a unit matrix of the same order as that of M ; show that:

$$M^2 = 2M + 3I$$

Solution:

$$M^2 = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

$$2M + 3I = 2 \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} + 3 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+3 & 4+0 \\ 4+0 & 2+3 \end{bmatrix} = \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$$

Thus, $M^2 = 2M + 3I$

9. If $A = \begin{bmatrix} a & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -b \\ 1 & 0 \end{bmatrix}$, $M = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$ and $BA = M^2$, find the values of a and b.

Solution:

$$BA = \begin{bmatrix} 0 & -b \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & -2b \\ a & 0 \end{bmatrix}$$

$$M^2 = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1-1 & -1-1 \\ 1+1 & -1+1 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

So, $BA = M^2$

$$\begin{bmatrix} 0 & -2b \\ a & 0 \end{bmatrix} = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

On comparing the corresponding elements, we have

$$-2b = -2$$

$$b = 1$$

And,

$$a = 2$$

10. Given $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$, find:

(i) $A - B$ (ii) A^2 (iii) AB (iv) $A^2 - AB + 2B$

Solution:

$$(i) A - B = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 4-1 & 1-0 \\ 2+2 & 3-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 4 & 2 \end{bmatrix}$$

$$(ii) A^2 = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 16+2 & 4+3 \\ 8+6 & 2+6 \end{bmatrix} = \begin{bmatrix} 18 & 7 \\ 14 & 8 \end{bmatrix}$$

$$(iii) AB = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 4-2 & 0+1 \\ 2-6 & 0+3 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix}$$

(iv) $A^2 - AB + 2B =$

$$\begin{aligned}
 &= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix} + 2 \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 18-2 & 7-1 \\ 14+4 & 11-3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -4 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 16 & 6 \\ 18 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -4 & 2 \end{bmatrix} \\
 &= \begin{bmatrix} 18 & 6 \\ 14 & 10 \end{bmatrix}
 \end{aligned}$$

11. If $A = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$, find :

- (i) $(A + B)^2$ (ii) $A^2 + B^2$
 (iii) Is $(A + B)^2 = A^2 + B^2$?

Solution:

$$(i) (A + B) = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix}$$

So, $(A + B)^2 = (A + B)(A + B) =$

$$= \begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix} \begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 4+0 & 12-24 \\ 0+0 & 0+16 \end{bmatrix} = \begin{bmatrix} 4 & -12 \\ 0 & 16 \end{bmatrix}$$

$$(ii) A^2 = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1+4 & 4-12 \\ 1-3 & 4+9 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ -2 & 13 \end{bmatrix}$$

$$B^2 = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 1-2 & 2-2 \\ -1+1 & -2+1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A^2 + B^2 = \begin{bmatrix} 5 & -8 \\ -2 & 13 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ -2 & 12 \end{bmatrix}$$

Thus, its seen that $(A + B)^2 \neq A^2 + B^2$

12. Find the matrix A, if $B = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ and $B^2 = B + \frac{1}{2}A$.

Solution:

$$B^2 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+0 & 2+1 \\ 0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$$

$$B^2 = B + \frac{1}{2}A$$

$$\frac{1}{2}A = B^2 - B$$

$$A = 2(B^2 - B)$$

$$= 2\left(\begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\right) = 2\begin{bmatrix} 2 & 2 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 0 & 0 \end{bmatrix}$$

13. If $A = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix}$ and $A^2 = I$, find a and b.

Solution:

$$A^2 = \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix} \begin{bmatrix} -1 & 1 \\ a & b \end{bmatrix} = \begin{bmatrix} 1+a & -1+b \\ -a+ab & a+b^2 \end{bmatrix}$$

And, given $A^2 = I$

So on comparing the corresponding terms, we have

$$1 + a = 1$$

Thus, $a = 0$

$$\text{And, } -1 + b = 0$$

Thus, $b = 1$

14. If $A = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix}$, then show that:

(i) $A(B + C) = AB + AC$

(ii) $(B - A)C = BC - AC$.

Solution:

$$(i) A(B + C) = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \left(\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} \right) = \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 3 & 7 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 6+4 & 14+3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 10 & 17 \\ 0 & 0 \end{bmatrix}$$

$$\begin{aligned} AB + AC &= \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 4+4 & 6+1 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2+0 & 8+2 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 8 & 7 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 10 & 17 \\ 0 & 0 \end{bmatrix} \end{aligned}$$

Thus, $A(B + C) = AB + AC$

$$(ii) (B - A)C = \left(\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \right) \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0+4 \\ 4+0 & 16+2 \end{bmatrix} \\ = \begin{bmatrix} 0 & 4 \\ 4 & 18 \end{bmatrix}$$

$$\begin{aligned}BC - AC &= \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & 8+6 \\ 4+0 & 16+2 \end{bmatrix} - \begin{bmatrix} 2+0 & 8+2 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 2 & 14 \\ 4 & 18 \end{bmatrix} - \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 0 & 4 \\ 4 & 18 \end{bmatrix}\end{aligned}$$

Thus, $(B - A)C = BC - AC$

15. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, simplify: $A^2 + BC$.

Solution:

$$\begin{aligned}A^2 + BC &= \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \\ &= \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix} + \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}\end{aligned}$$