

Exercise 9(D)

1. Find x and y , if:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ y \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ y \end{bmatrix}$$

$$\begin{bmatrix} 6x - 2 \\ -2x + 4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x - 2 - 8 \\ -2x + 4 + 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x - 10 \\ -2x + 14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

On comparing the corresponding terms, we have

$$6x - 10 = 8 \quad \text{and} \quad -2x + 14 = 4y$$

$$6x = 18 \quad \text{and} \quad y = (14 - 2x)/4$$

$$x = 3 \quad \text{and} \quad y = (14 - 2(3))/4$$

$$y = (14 - 6)/4$$

$$y = 8/4 = 2$$

Thus, $x = 3$ and $y = 2$

2. Find x and y , if:

$$\begin{bmatrix} 3x & 8 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3 \begin{bmatrix} 2 & -7 \end{bmatrix} = 5 \begin{bmatrix} 3 & 2y \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 3x & 8 \\ 3 & 7 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3 \begin{bmatrix} 2 & -7 \end{bmatrix} = 5 \begin{bmatrix} 3 & 2y \end{bmatrix}$$

$$\begin{bmatrix} 3x + 24 & 12x + 56 \end{bmatrix} - \begin{bmatrix} 6 & -21 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

$$\begin{bmatrix} 3x + 24 - 6 & 12x + 56 + 21 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

$$\begin{bmatrix} 3x + 18 & 12x + 77 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

On comparing the corresponding terms, we have

$$3x + 18 = 15 \quad \text{and} \quad 12x + 77 = 10y$$

$$3x = -3 \quad \text{and} \quad y = (12x + 77)/10$$

$$x = -1 \quad \text{and} \quad y = (12(-1) + 77)/10$$

$$y = 65/10 = 6.5$$

Thus, $x = -1$ and $y = 6.5$

3. If: $\begin{bmatrix} x & y \\ x & y \end{bmatrix} = \begin{bmatrix} 25 \\ 25 \end{bmatrix}$ and $\begin{bmatrix} -x & y \\ -x & y \end{bmatrix} = \begin{bmatrix} -2 \\ -2 \end{bmatrix}$; find x and y, if:

(i) $x, y \in W$ (whole numbers)

(ii) $x, y \in Z$ (integers)

Solution:

From the question, we have

$$x^2 + y^2 = 25 \quad \text{and} \quad -2x^2 + y^2 = -2$$

(i) $x, y \in W$ (whole numbers)

It can be observed that the above two equations are satisfied when $x = 3$ and $y = 4$.

(ii) $x, y \in Z$ (integers)

It can be observed that the above two equations are satisfied when $x = \pm 3$ and $y = \pm 4$.

4. Given $\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \cdot X = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$. Write :

(i) The order of the matrix X.

(ii) The matrix X.

Solution:

(i) Let the order of the matrix be $a \times b$.

Then, we know that

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}_{2 \times 2} \cdot X_{a \times b} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}_{2 \times 1}$$

Thus, for multiplication of matrices to be possible

$$a = 2$$

And, form noticing the order of the resultant matrix

$$b = 1$$

(ii)

$$\text{Let } X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x + y \\ -3x + 4y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

On comparing the corresponding terms, we have

$$2x + y = 7 \quad \text{and}$$

$$-3x + 4y = 6$$

Solving the above two equations, we have

$x = 2$ and $y = 3$

Thus, the matrix X is $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

5. Evaluate:

$$\begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2}\cos 0^\circ & \sin 0^\circ \end{bmatrix} \begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix}$$

Solution:

$$\begin{aligned} &= \begin{bmatrix} \cos 45^\circ & \sin 30^\circ \\ \sqrt{2}\cos 0^\circ & \sin 0^\circ \end{bmatrix} \begin{bmatrix} \sin 45^\circ & \cos 90^\circ \\ \sin 90^\circ & \cot 45^\circ \end{bmatrix} \\ &= \begin{bmatrix} \cos 45^\circ \sin 45^\circ + \sin 30^\circ \sin 90^\circ & \cos 45^\circ \cos 90^\circ + \sin 30^\circ \cot 45^\circ \\ \sqrt{2}\cos 0^\circ \sin 45^\circ + \sin 0^\circ \sin 90^\circ & \sqrt{2}\cos 0^\circ \cos 90^\circ + \sin 0^\circ \cot 45^\circ \end{bmatrix} \\ &= \begin{bmatrix} 1/2 + 1/2 & 0 + 1/2 \\ 1 & 0 + 0 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0.5 \\ 1 & 0 \end{bmatrix} \end{aligned}$$

6. If $A = \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -5 \\ 6 \end{bmatrix}$ and $3A \times M = 2B$; find matrix M.

Solution:

Given,

$$3A \times M = 2B$$

And let the order of the matrix of M be (a x b)

$$3 \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}_{2 \times 2} \times M_{a \times b} = 2 \begin{bmatrix} -5 \\ 6 \end{bmatrix}_{2 \times 1}$$

Now, it's clearly seen that

$$a = 2 \text{ and } b = 1$$

So, the order of the matrix M is (2 x 1)

$$\begin{aligned} 3 \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} &= 2 \begin{bmatrix} -5 \\ 6 \end{bmatrix} \\ \begin{bmatrix} 0 & -3 \\ 12 & -9 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} &= \begin{bmatrix} -10 \\ 12 \end{bmatrix} \\ \begin{bmatrix} -3y \\ 12x - 9y \end{bmatrix} &= \begin{bmatrix} -10 \\ 12 \end{bmatrix} \end{aligned}$$

Now, on comparing with corresponding elements we have

$$-3y = -10 \quad \text{and} \quad 12x - 9y = 12$$

$$y = 10/3 \quad \text{and} \quad 12x - 9(10/3) = 12$$

$$12x - 30 = 12$$

$$12x = 42$$

$$x = 42/12 = 7/2$$

Therefore,

$$\text{Matrix } M = \begin{bmatrix} 7/2 \\ 10/3 \end{bmatrix}$$

7. If $\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$,

find the values of a, b and c.

Solution:

$$\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a+2 & 3+b \\ 4+1 & 1-2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a+2-1 & 3+b-1 \\ 4+1+2 & 1-2-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

$$\begin{bmatrix} a+1 & b+2 \\ 7 & -1-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

On comparing the corresponding elements, we have

$$a + 1 = 5 \Rightarrow a = 4$$

$$b + 2 = 0 \Rightarrow b = -2$$

$$-1 - c = 3 \Rightarrow c = -4$$

8. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$; find :

(i) $A(BA)$

(ii) $(AB)B$.

Solution:

$$\begin{aligned} \text{(i) } A(BA) &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \left(\begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix} \right) \\ &= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \\ &= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix} \\ &= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix} \end{aligned}$$

$$\begin{aligned}
 \text{(ii) } (AB) B &= \left(\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \right) \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 2+2 & 1+4 \\ 4+1 & 2+2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \\
 &= \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 4+10 & 8+5 \\ 5+8 & 10+4 \end{bmatrix} \\
 &= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}
 \end{aligned}$$

9. Find x and y, if: $\begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 12 \end{bmatrix}$

Solution:

$$\begin{aligned}
 \begin{bmatrix} x & 3x \\ y & 4y \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} &= \begin{bmatrix} 5 \\ 12 \end{bmatrix} \\
 \begin{bmatrix} 2x + 3x \\ 2y + 4y \end{bmatrix} &= \begin{bmatrix} 5 \\ 12 \end{bmatrix}
 \end{aligned}$$

Thus, on comparing the corresponding terms, we have

$$2x + 3x = 5 \quad \text{and} \quad 2y + 4y = 12$$

$$5x = 5 \quad \text{and} \quad 6y = 12$$

$$x = 1 \quad \text{and} \quad y = 2$$

10. If matrix $X = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix}$ and $2X - 3Y = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$, find the matrix 'X' and

Solution:

$$X = \begin{bmatrix} -3 & 4 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -6 - 8 \\ 4 + 6 \end{bmatrix} = \begin{bmatrix} -14 \\ 10 \end{bmatrix}$$

Now,

$$\text{Let } Y = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$2X - 3Y = 2 \begin{bmatrix} -14 \\ 10 \end{bmatrix} - 3 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} -28 \\ 20 \end{bmatrix} - \begin{bmatrix} 3x \\ 3y \end{bmatrix} = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} -28 - 3x \\ 20 - 3y \end{bmatrix} = \begin{bmatrix} 10 \\ -8 \end{bmatrix}$$

On comparing with the corresponding terms, we have

$$-28 - 3x = 10$$

$$3x = -38$$

$$x = -38/3$$

And,

$$20 - 3y = -8$$

$$3y = 38$$

$$y = 38/3$$

Therefore,

$$Y = 1/3 \begin{bmatrix} -38 \\ 28 \end{bmatrix}$$

11. Given $A = \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ find the matrix X such that:

$$A + X = 2B + C$$

Solution:

$$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = 2 \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = \begin{bmatrix} -6 & 4 \\ 8 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix} + X = \begin{bmatrix} -5 & 4 \\ 8 & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} -5 & 4 \\ 8 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -1 \\ 2 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} -7 & 5 \\ 6 & 2 \end{bmatrix}$$

12. Find the value of x, given that $A^2 = B$,

$$A = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix}$$

Solution:

$$A^2 = \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix} = B$$

$$A^2 = \begin{bmatrix} 4 + 0 & 24 + 12 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix} = B$$

$$A^2 = \begin{bmatrix} 4 & 36 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & x \\ 0 & 1 \end{bmatrix} = B$$

Thus, on comparing the terms we get $x = 36$.