Exercise 9(A) Page No: 120

1. State, whether the following statements are true or false. If false, give a reason.
(i) If A and B are two matrices of orders 3 x 2 and 2 x 3 respectively; then their sum A + B is possible.
(ii) The matrices $A_{2 \times 3}$ and $B_{2 \times 3}$ are conformable for subtraction.
(iii) Transpose of a 2 x 1 matrix is a 2 x 1 matrix.
(iv) Transpose of a square matrix is a square matrix.
(v) A column matrix has many columns and one row.
Solution:

(i) False.
The sum of matrices A + B is possible only when the order of both the matrices A and B are same.
(ii) True
(iii) False
Transpose of a 2 x 1 matrix is a 1 x 2 matrix.
(iv) True
(v) False
A column matrix has only one column and many rows.

2. Given: $\begin{bmatrix} x & y+2 \\ 3 & z-1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 3 & 2 \end{bmatrix}$**, find x, y and z.**
Solution:

If two matrices are said to be equal, then their corresponding elements are also equal.
Therefore,
x = 3,
y + 2 = 1 so, y = -1
z - 1 = 2 so, z = 3

3. Solve for a, b and c if
(i) $\begin{bmatrix} -4 & a+5 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} b+4 & 2 \\ 3 & c-1 \end{bmatrix}$
(ii) $\begin{bmatrix} a & a-b \\ b+c & 0 \end{bmatrix} = \begin{bmatrix} 3 & -1 \\ 2 & 0 \end{bmatrix}$
Solution:

If two matrices are said to be equal, then their corresponding elements are also equal.
Then,
(i) $a + 5 = 2 \Rightarrow a = -3$
$-4 = b + 4 \Rightarrow b = -8$
$2 = c - 1 \Rightarrow c = 3$
(ii) $a = 3$
$a - b = -1$

$\Rightarrow b = a + 1 = 4$
$b + c = 2$
$\Rightarrow c = 2 - b = 2 - 4 = -2$

4. If A = [8 -3] and B = [4 -5]; find:
(i) A + B (ii) B - A
Solution:
(i) A + B = [8 -3] + [4 -5] = [8+4 -3-5] = [12 -8]
(ii) B – A = [4 -5] - [8 -3] = [4-8 -5-(-3)] = [-4 -2]

5. If A= $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$**, B =** $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ **and C =** $\begin{bmatrix} 6 \\ -2 \end{bmatrix}$**; find:**
(i) B + C (ii) A - C
(iii) A + B - C (iv) A - B +C
Solution:

(i)B + C = $\begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} 1+6 \\ 4-2 \end{bmatrix} = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$

(ii)A - C = $\begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \end{bmatrix} = \begin{bmatrix} 2-6 \\ 5+2 \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \end{bmatrix}$

(iii)
$$A + B - C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$
$$= \begin{bmatrix} 2+1-6 \\ 5+4+2 \end{bmatrix} = \begin{bmatrix} -3 \\ 11 \end{bmatrix}$$

(iv)
$$A - B + C = \begin{bmatrix} 2 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 4 \end{bmatrix} + \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$
$$= \begin{bmatrix} 2-1+6 \\ 5-4-2 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$$

Exercise 9(B)

1. Evaluate:
(i) 3[5 -2]
Solution:

3[5 -2] = [3x5 3x-2] = [15 -6]

(ii) $7\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$
Solution:

$$7\begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 14 \\ 0 & 7 \end{bmatrix}$$

(iii) $2\begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix}$
Solution:

$$2\begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 4 & -6 \end{bmatrix} + \begin{bmatrix} 3 & 3 \\ 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 9 & -6 \end{bmatrix}$$

(iv) $6\begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2\begin{bmatrix} -8 \\ 1 \end{bmatrix}$
Solution:

$$6\begin{bmatrix} 3 \\ -2 \end{bmatrix} - 2\begin{bmatrix} -8 \\ 1 \end{bmatrix} = \begin{bmatrix} 18 \\ -12 \end{bmatrix} - \begin{bmatrix} -16 \\ 2 \end{bmatrix} = \begin{bmatrix} 34 \\ -14 \end{bmatrix}$$

2. Find x and y if:
(i) 3[4 x] + 2[y -3] = [10 0]
Solution:

Taking the L.H.S, we have
3[4 x] + 2[y -3] = [12 3x] + [2y -6] = [(12 + 2y)   (3x - 6)]
Now, equating with R.H.S we get
[(12 + 2y)   (3x - 6)] = [10 0]
12 + 2y = 10   and   3x – 6 = 0
2y = -2   and   3x = 6
y = -1   and   x = 2

(ii) $x\begin{bmatrix}-1\\2\end{bmatrix}-4\begin{bmatrix}-2\\y\end{bmatrix}=\begin{bmatrix}7\\-8\end{bmatrix}$

Solution:

We have,

$$x\begin{bmatrix}-1\\2\end{bmatrix}-4\begin{bmatrix}-2\\y\end{bmatrix}=\begin{bmatrix}7\\-8\end{bmatrix}$$

$$\begin{bmatrix}-x\\2x\end{bmatrix}-\begin{bmatrix}-8\\4y\end{bmatrix}=\begin{bmatrix}-x+8\\2x-4y\end{bmatrix}=\begin{bmatrix}7\\-8\end{bmatrix}$$

So, equating the matrices we get

-x + 8 = 7 and 2x – 4y = -8

x = 1 and 2(1) – 4y = -8

2 – 4y = -8

4y = 10

y = 5/2

3. $Given\ A=\begin{bmatrix}2&1\\3&0\end{bmatrix}, B=\begin{bmatrix}1&1\\5&2\end{bmatrix}\ and\ C=\begin{bmatrix}-3&-1\\0&0\end{bmatrix}; find:$

(i) 2A – 3B + C **(ii) A + 2C - B**

Solution:

(i) 2A – 3B + C

$$= 2\begin{bmatrix}2&1\\3&0\end{bmatrix}-3\begin{bmatrix}1&1\\5&2\end{bmatrix}+\begin{bmatrix}-3&-1\\0&0\end{bmatrix}$$

$$=\begin{bmatrix}4&2\\6&0\end{bmatrix}-\begin{bmatrix}3&3\\15&6\end{bmatrix}+\begin{bmatrix}-3&-1\\0&0\end{bmatrix}$$

$$=\begin{bmatrix}4-3-3&2-3-1\\6-15+0&0-6+0\end{bmatrix}$$

$$=\begin{bmatrix}-2&-2\\-9&-6\end{bmatrix}$$

(ii) A + 2C – B

$$= \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} + 2\begin{bmatrix} -3 & -1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 2 & 1 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} -6 & -2 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 5 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 2-6-1 & 1-2-1 \\ 3+0-5 & 0+0-2 \end{bmatrix}$$
$$= \begin{bmatrix} -5 & -2 \\ -2 & -2 \end{bmatrix}$$

4. $If \begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + 3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}; find\ A.$

Solution:

Given,

$$\begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix} + 3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix}$$
$$3A = \begin{bmatrix} -2 & -2 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} 4 & -2 \\ 4 & 0 \end{bmatrix}$$
$$3A = \begin{bmatrix} -2-4 & -2+2 \\ 1-4 & -3-0 \end{bmatrix} = \begin{bmatrix} -6 & 0 \\ -3 & -3 \end{bmatrix}$$
$$A = \begin{bmatrix} -2 & 0 \\ -1 & -1 \end{bmatrix}$$

5. $Given\ A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix} and\ B = \begin{bmatrix} -4 & -1 \\ -3 & -2 \end{bmatrix}$

(i) find the matrix 2A + B.

(ii) find a matrix C such that:

$$C + B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Solution:

(i) 2A + B

$$= 2\begin{bmatrix}1 & 4\\2 & 3\end{bmatrix} + \begin{bmatrix}-4 & -1\\-3 & -2\end{bmatrix}$$
$$= \begin{bmatrix}2 & 8\\4 & 6\end{bmatrix} + \begin{bmatrix}-4 & -1\\-3 & -2\end{bmatrix}$$
$$= \begin{bmatrix}-2 & 7\\1 & 4\end{bmatrix}$$

(ii)

$$C + B = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix}$$
$$C = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} - B$$
$$C = \begin{bmatrix}0 & 0\\0 & 0\end{bmatrix} - \begin{bmatrix}-4 & -1\\-3 & -2\end{bmatrix}$$
$$C = \begin{bmatrix}4 & 1\\3 & 2\end{bmatrix}$$

Exercise 9(C)

1. Evaluate: if possible:

$$(i)\ \begin{bmatrix}3 & 2\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix}$$

$$(ii)\ \begin{bmatrix}1 & -2\end{bmatrix}\begin{bmatrix}-2 & 3\\-1 & 4\end{bmatrix}$$

$$(iii)\ \begin{bmatrix}6 & 4\\3 & -1\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix}$$

$$(iv)\ \begin{bmatrix}6 & 4\\3 & -1\end{bmatrix}\begin{bmatrix}-1 & 3\end{bmatrix}$$

If not possible, give reason.

Solution:

$(i)\ \begin{bmatrix}3 & 2\end{bmatrix}\begin{bmatrix}2\\0\end{bmatrix}$ = [6 + 0] = [6]

$(ii)\ \begin{bmatrix}1 & -2\end{bmatrix}\begin{bmatrix}-2 & 3\\-1 & 4\end{bmatrix}$ = [-2+2   3-8] = [0   -5]

$$(iii)\ \begin{bmatrix}6 & 4\\3 & -1\end{bmatrix}\begin{bmatrix}-1\\3\end{bmatrix} = \begin{bmatrix}-6+12\\-3-3\end{bmatrix} = \begin{bmatrix}6\\-6\end{bmatrix}$$

$$(iv)\ \begin{bmatrix}6 & 4\\3 & -1\end{bmatrix}\begin{bmatrix}-1 & 3\end{bmatrix}$$

The multiplication of these matrices is not possible as the rule for the number of columns in the first is not equal to the number of rows in the second matrix.

2. If $A = \begin{bmatrix}0 & 2\\5 & -2\end{bmatrix}, B = \begin{bmatrix}1 & -1\\3 & 2\end{bmatrix}$ **and I is a unit matrix of order 2x2, find:**

(i) AB **(ii) BA** **(iii) AI**
(iv) IB **(v) A^2** **(iv) B^2A**

Solution:

$$\text{(i) AB} = \begin{bmatrix}0 & 2\\5 & -2\end{bmatrix}\begin{bmatrix}1 & -1\\3 & 2\end{bmatrix}$$
$$= \begin{bmatrix}0+6 & 0+4\\5-6 & -5-4\end{bmatrix}$$
$$= \begin{bmatrix}6 & 4\\-1 & -9\end{bmatrix}$$

(ii) BA $= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$

$= \begin{bmatrix} 0-5 & 2+2 \\ 0+10 & 6-4 \end{bmatrix}$

$= \begin{bmatrix} -5 & 4 \\ 10 & 2 \end{bmatrix}$

(iii) AI $= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 0+0 & 0+2 \\ 5+0 & 0-2 \end{bmatrix}$

$= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$

(iv) IB $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1+0 & -1+0 \\ 0+3 & 0+2 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$

(v) A^2 $= \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$

$= \begin{bmatrix} 0+10 & 0-4 \\ 0-10 & 10+4 \end{bmatrix}$

$= \begin{bmatrix} 10 & -4 \\ -10 & 14 \end{bmatrix}$

(vi) B^2 $= \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1-3 & -1-2 \\ 3+6 & -3+4 \end{bmatrix}$

$= \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix}$

$$B^2A = \begin{bmatrix} -2 & -3 \\ 9 & 1 \end{bmatrix} \begin{bmatrix} 0 & 2 \\ 5 & -2 \end{bmatrix}$$
$$= \begin{bmatrix} 0-15 & -4+6 \\ 0+5 & 18-2 \end{bmatrix}$$
$$= \begin{bmatrix} -15 & 2 \\ 5 & 16 \end{bmatrix}$$

3. If $A = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix},$ **find x and y when x and y when $A^2 = B$.**

Solution:

$$A^2 = \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & x \\ 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 9+0 & 3x+x \\ 0+0 & 0+1 \end{bmatrix}$$
$$= \begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix}$$

$A^2 = B$

$$\begin{bmatrix} 9 & 4x \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 16 \\ 0 & -y \end{bmatrix}$$

On comparing corresponding elements, we have

$4x = 16$

$x = 4$

And,

$1 = -y$

$y = -1$

4. Find x and y, if:

$(i)\ \begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$

$(ii)\ \begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

Solution:

(i)
$$\begin{bmatrix} 4 & 3x \\ x & -2 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$$
$$\begin{bmatrix} 20+3x \\ 5x-2 \end{bmatrix} = \begin{bmatrix} y \\ 8 \end{bmatrix}$$

On comparing the corresponding terms, we have
5x – 2 = 8
5x = 10
x = 2
And,
20 + 3x = y
20 + 3(2) = y
20 + 6 = y
y = 26

(ii) $\begin{bmatrix} x & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

$\begin{bmatrix} x+0 & x \\ -3+0 & -3+y \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -3 & -2 \end{bmatrix}$

On comparing the corresponding terms, we have
x = 2
And,
-3 + y = -2
y = 3 – 2 = 1

5. $If\ A = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}\ and\ C = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}, find:$

(i) (AB) C **(ii) A (BC)**

Solution:

(i) (AB) $= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1+12 & 2+9 \\ 2+16 & 4+12 \end{bmatrix}$

$= \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix}$

(AB) C $= \begin{bmatrix} 13 & 11 \\ 18 & 16 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 52+11 & 39+22 \\ 72+16 & 54+32 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$

(ii) BC $= \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4+2 & 3+4 \\ 16+3 & 12+6 \end{bmatrix} = \begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix}$

A (BC) $= \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 6 & 7 \\ 19 & 18 \end{bmatrix} = \begin{bmatrix} 6+57 & 7+54 \\ 12+76 & 14+72 \end{bmatrix} = \begin{bmatrix} 63 & 61 \\ 88 & 86 \end{bmatrix}$

Therefore, its seen that (AB) C = A (BC)

6. $Given\ A = \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix}\ and\ B = \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix}$, **is the following possible:**

(i) AB **(ii) BA** **(iii) A^2**

Solution:

(i) AB $= \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix} = \begin{bmatrix} 0-4-30 & 0+8-36 \\ 0-0+5 & 3+0+6 \end{bmatrix} = \begin{bmatrix} -34 & -28 \\ 5 & 9 \end{bmatrix}$

(ii) BA $= \begin{bmatrix} 0 & 1 \\ -1 & 2 \\ -5 & -6 \end{bmatrix} \begin{bmatrix} 0 & 4 & 6 \\ 3 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 0+3 & 0+0 & 0-1 \\ 0+6 & -4+0 & -6-2 \\ 0-18 & -20+0 & -30+6 \end{bmatrix} = \begin{bmatrix} 3 & 0 & -1 \\ 6 & -4 & -8 \\ -18 & -20 & -24 \end{bmatrix}$

(iii) A^2 = A x A, is not possible since the number of columns of matrix A is not equal to its number of rows.

7. $Let\ A = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix}, B = \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}\ and\ C = \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix}$. **Find A^2 + AC – 5B.**

Solution:

$A^2 = \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} = \begin{bmatrix} 4+0 & 2-2 \\ 0 & 0+4 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

AC $= \begin{bmatrix} 2 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} -3 & 2 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} -6-1 & 4+4 \\ 0+2 & 0-8 \end{bmatrix} = \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix}$

5B $= 5 \begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix} = \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix}$

A^2 + AC – 5B =

$$= \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} + \begin{bmatrix} -7 & 8 \\ 2 & -8 \end{bmatrix} - \begin{bmatrix} 20 & 5 \\ -15 & -10 \end{bmatrix} = \begin{bmatrix} 4-7-20 & 8-5 \\ 2+15 & 4-8+10 \end{bmatrix} = \begin{bmatrix} -23 & 3 \\ 17 & 6 \end{bmatrix}$$

8. If $M = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ **and I is a unit matrix of the same order as that of M; show that:**

M^2 = 2M + 3I

Solution:

$M^2 = \begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix} = \begin{bmatrix}1+4 & 2+2\\2+2 & 4+1\end{bmatrix} = \begin{bmatrix}5 & 4\\4 & 5\end{bmatrix}$

$2M + 3I = 2\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix} + 3\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix} = \begin{bmatrix}2+3 & 4+0\\4+0 & 2+3\end{bmatrix} = \begin{bmatrix}5 & 4\\4 & 5\end{bmatrix}$

Thus, $M^2 = 2M + 3I$

9. $If\ A = \begin{bmatrix}a & 0\\0 & 2\end{bmatrix}, B = \begin{bmatrix}0 & -b\\1 & 0\end{bmatrix}, M = \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}$ **and BA = M^2, find the values of a and b.**

Solution:

$BA = \begin{bmatrix}0 & -b\\1 & 0\end{bmatrix}\begin{bmatrix}a & 0\\0 & 2\end{bmatrix} = \begin{bmatrix}0 & -2b\\a & 0\end{bmatrix}$

$M^2 = \begin{bmatrix}1 & -1\\1 & 1\end{bmatrix}\begin{bmatrix}1 & -1\\1 & 1\end{bmatrix} = \begin{bmatrix}1-1 & -1-1\\1+1 & -1+1\end{bmatrix} = \begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}$

So, BA $=M^2$

$\begin{bmatrix}0 & -2b\\a & 0\end{bmatrix} = \begin{bmatrix}0 & -2\\2 & 0\end{bmatrix}$

On comparing the corresponding elements, we have
-2b = -2
b = 1
And,
a = 2

10. $Given\ A = \begin{bmatrix}4 & 1\\2 & 3\end{bmatrix}\ and\ B = \begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix}, find:$

(i) A – B (ii) A^2 (iii) AB (iv) A^2 – AB + 2B

Solution:

(i) $A - B = \begin{bmatrix}4 & 1\\2 & 3\end{bmatrix} - \begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix} = \begin{bmatrix}4-1 & 1-0\\2+2 & 3-1\end{bmatrix} = \begin{bmatrix}3 & 1\\4 & 2\end{bmatrix}$

(ii) $A^2 = \begin{bmatrix}4 & 1\\2 & 3\end{bmatrix}\begin{bmatrix}4 & 1\\2 & 3\end{bmatrix} = \begin{bmatrix}16+2 & 4+3\\8+6 & 2+6\end{bmatrix} = \begin{bmatrix}18 & 7\\14 & 8\end{bmatrix}$

(iii) $AB = \begin{bmatrix}4 & 1\\2 & 3\end{bmatrix}\begin{bmatrix}1 & 0\\-2 & 1\end{bmatrix} = \begin{bmatrix}4-2 & 0+1\\2-6 & 0+3\end{bmatrix} = \begin{bmatrix}2 & 1\\-4 & 3\end{bmatrix}$

(iv) $A^2 - AB + 2B =$

$$= \begin{bmatrix} 18 & 7 \\ 14 & 11 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ -4 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 18-2 & 7-1 \\ 14+4 & 11-3 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -4 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 16 & 6 \\ 18 & 8 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ -4 & 2 \end{bmatrix}$$
$$= \begin{bmatrix} 18 & 6 \\ 14 & 10 \end{bmatrix}$$

11. $If = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} and\ B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}, find:$

(i) $(A + B)^2$ (ii) $A^2 + B^2$
(iii) Is $(A + B)^2 = A^2 + B^2$?
Solution:

(i) (A + B) $= \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} + \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix}$

So, $(A + B)^2 = (A + B)(A + B) =$
$$= \begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 0 & -4 \end{bmatrix} = \begin{bmatrix} 4+0 & 12-24 \\ 0+0 & 0+16 \end{bmatrix} = \begin{bmatrix} 4 & -12 \\ 0 & 16 \end{bmatrix}$$

(ii) $A^2 = \begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} 1 & 4 \\ 1 & -3 \end{bmatrix} = \begin{bmatrix} 1+4 & 4-12 \\ 1-3 & 4+9 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ -2 & 13 \end{bmatrix}$

$B^2 = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 1-2 & 2-2 \\ -1+1 & -2+1 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$A^2 + B^2 = \begin{bmatrix} 5 & -8 \\ -2 & 13 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 4 & -8 \\ -2 & 12 \end{bmatrix}$

Thus, its seen that $(A + B)^2 \neq A^2 + B^{2+}$

12. Find the matrix A, if B = $\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}$ **and $B^2 = B + ½A$.**
Solution:

$B^2 = \begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4+0 & 2+1 \\ 0 & 0+1 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 0 & 1 \end{bmatrix}$

$B^2 = B + ½A$
$½A = B^2 – B$
$A = 2(B^2 – B)$

$$= 2(\begin{bmatrix}4 & 3\\0 & 1\end{bmatrix} - \begin{bmatrix}2 & 1\\0 & 1\end{bmatrix}) = 2\begin{bmatrix}2 & 2\\0 & 0\end{bmatrix} = \begin{bmatrix}4 & 4\\0 & 0\end{bmatrix}$$

13. If $A = \begin{bmatrix}-1 & 1\\a & b\end{bmatrix}$ and A^2 = I, find a and b.

Solution:

$$A^2 = \begin{bmatrix}-1 & 1\\a & b\end{bmatrix}\begin{bmatrix}-1 & 1\\a & b\end{bmatrix} = \begin{bmatrix}1+a & -1+b\\-a+ab & a+b^2\end{bmatrix}$$

And, given $A^2 = I$
So on comparing the corresponding terms, we have
1 + a = 1
Thus, a = 0
And, -1 + b = 0
Thus, b = 1

14. If $A = \begin{bmatrix}2 & 1\\0 & 0\end{bmatrix}, B = \begin{bmatrix}2 & 3\\4 & 1\end{bmatrix}\ and\ C = \begin{bmatrix}1 & 4\\0 & 2\end{bmatrix}$, then show that:

(i) A(B + C) = AB + AC
(ii) (B - A)C = BC – AC.
Solution:

(i) $A(B + C) = \begin{bmatrix}2 & 1\\0 & 0\end{bmatrix}(\begin{bmatrix}2 & 3\\4 & 1\end{bmatrix} + \begin{bmatrix}1 & 4\\0 & 2\end{bmatrix}) = \begin{bmatrix}2 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}3 & 7\\4 & 3\end{bmatrix} = \begin{bmatrix}6+4 & 14+3\\0 & 0\end{bmatrix} = \begin{bmatrix}10 & 17\\0 & 0\end{bmatrix}$

$AB + AC = \begin{bmatrix}2 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}2 & 3\\4 & 1\end{bmatrix} + \begin{bmatrix}2 & 1\\0 & 0\end{bmatrix}\begin{bmatrix}1 & 4\\0 & 2\end{bmatrix} = \begin{bmatrix}4+4 & 6+1\\0 & 0\end{bmatrix} + \begin{bmatrix}2+0 & 8+2\\0 & 0\end{bmatrix}$

$$= \begin{bmatrix}8 & 7\\0 & 0\end{bmatrix} + \begin{bmatrix}2 & 10\\0 & 0\end{bmatrix}$$

$$= \begin{bmatrix}10 & 17\\0 & 0\end{bmatrix}$$

Thus, A(B + C) = AB + AC

(ii) $(B - A)C = (\begin{bmatrix}2 & 3\\4 & 1\end{bmatrix} - \begin{bmatrix}2 & 1\\0 & 0\end{bmatrix})\begin{bmatrix}1 & 4\\0 & 2\end{bmatrix} = \begin{bmatrix}0 & 2\\4 & 1\end{bmatrix}\begin{bmatrix}1 & 4\\0 & 2\end{bmatrix} = \begin{bmatrix}0 & 0+4\\4+0 & 16+2\end{bmatrix}$

$$= \begin{bmatrix}0 & 4\\4 & 18\end{bmatrix}$$

BC – AC $= \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 2+0 & 8+6 \\ 4+0 & 16+2 \end{bmatrix} - \begin{bmatrix} 2+0 & 8+2 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 2 & 14 \\ 4 & 18 \end{bmatrix} - \begin{bmatrix} 2 & 10 \\ 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 4 \\ 4 & 18 \end{bmatrix}$

Thus, (B - A)C = BC – AC

15. If $A = \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix}, B = \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} and\ C = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},$ **simplify: A^2 + BC.**

Solution:

A^2 + BC $= \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 2 & 1 \end{bmatrix} + \begin{bmatrix} -3 & 2 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 1+8 & 4+4 \\ 2+2 & 8+1 \end{bmatrix} + \begin{bmatrix} -3+0 & 0+4 \\ 4+0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 9 & 8 \\ 4 & 9 \end{bmatrix} + \begin{bmatrix} -3 & 4 \\ 4 & 0 \end{bmatrix}$

$= \begin{bmatrix} 6 & 12 \\ 8 & 9 \end{bmatrix}$

Exercise 9(D)

1. Find x and y, if:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ y \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2x \\ 1 \end{bmatrix} + 2 \begin{bmatrix} -4 \\ 5 \end{bmatrix} = 4 \begin{bmatrix} 2 \\ y \end{bmatrix}$$

$$\begin{bmatrix} 6x-2 \\ -2x+4 \end{bmatrix} + \begin{bmatrix} -8 \\ 10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x-2-8 \\ -2x+4+10 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

$$\begin{bmatrix} 6x-10 \\ -2x+14 \end{bmatrix} = \begin{bmatrix} 8 \\ 4y \end{bmatrix}$$

On comparing the corresponding terms, we have

6x – 10 = 8 and -2x + 14 = 4y
6x = 18 and y = (14 - 2x)/4
x = 3 and y = (14 – 2(3))/4
y = (14 - 6)/ 4
y = 8/4 = 2

Thus, x = 3 and y = 2

2. Find x and y, if:

$$\begin{bmatrix} 3x & 8 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3 \begin{bmatrix} 2 & -7 \end{bmatrix} = 5 \begin{bmatrix} 3 & 2y \end{bmatrix}$$

Solution:

$$\begin{bmatrix} 3x & 8 \end{bmatrix} \begin{bmatrix} 1 & 4 \\ 3 & 7 \end{bmatrix} - 3 \begin{bmatrix} 2 & -7 \end{bmatrix} = 5 \begin{bmatrix} 3 & 2y \end{bmatrix}$$

$$\begin{bmatrix} 3x+24 & 12x+56 \end{bmatrix} - \begin{bmatrix} 6 & -21 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

$$\begin{bmatrix} 3x+24-6 & 12x+56+21 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

$$\begin{bmatrix} 3x+18 & 12x+77 \end{bmatrix} = \begin{bmatrix} 15 & 10y \end{bmatrix}$$

On comparing the corresponding terms, we have

3x + 18 = 15 and 12x + 77 = 10y
3x = -3 and y = (12x + 77)/10
x = -1 and y = (12(-1) + 77)/10
y = 65/10 = 6.5

Thus, x = -1 and y = 6.5

3. If; $\begin{bmatrix} x & y \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 25 \end{bmatrix}$ and $\begin{bmatrix} -x & y \end{bmatrix}\begin{bmatrix} 2x \\ y \end{bmatrix} = \begin{bmatrix} -2 \end{bmatrix}$ **; find x and y, if:**

(i) x, y ∈ W (whole numbers)
(ii) x, y ∈ Z (integers)
Solution:

From the question, we have
$x^2 + y^2 = 25$ and $-2x^2 + y^2 = -2$

(i) x, y ∈ W (whole numbers)
It can be observed that the above two equations are satisfied when x = 3 and y = 4.

(ii) x, y ∈ Z (integers)
It can be observed that the above two equations are satisfied when x = ± 3 and y = ± 4.

4. $Given \begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} .X = \begin{bmatrix} 7 \\ 6 \end{bmatrix} .Write :$

(i) The order of the matrix X.
(ii) The matrix X.
Solution:

(i) Let the order of the matrix be a x b.
Then, we know that

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix}_{2X\underline{2}} .X_{\underline{a}Xb} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}_{2X1}$$

Thus, for multiplication of matrices to be possible
a = 2
And, form noticing the order of the resultant matrix
b = 1

(ii)

$$\text{Let} X = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ -3 & 4 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2x+y \\ -3x+4y \end{bmatrix} = \begin{bmatrix} 7 \\ 6 \end{bmatrix}$$

On comparing the corresponding terms, we have
2x + y = 7 and
-3x + 4y = 6
Solving the above two equations, we have

x = 2 and y = 3

Thus, the matrix X is $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

5. Evaluate:

$$\begin{bmatrix} cos45^o & sin30^o \\ \sqrt{2}cos0^o & sin0^o \end{bmatrix} \begin{bmatrix} sin45^o & cos90^o \\ sin90^o & cot45^o \end{bmatrix}$$

Solution:

$$= \begin{bmatrix} cos45^o & sin30^o \\ \sqrt{2}cos0^o & sin0^o \end{bmatrix} \begin{bmatrix} sin45^o & cos90^o \\ sin90^o & cot45^o \end{bmatrix}$$
$$= \begin{bmatrix} cos45^o sin45^o + sin30^o sin90^o & cos45^o cos90^o + sin30^o cot45^o \\ \sqrt{2}cos0^o sin45^o + sin0^o sin90^o & \sqrt{2}cos0^o cos90^o + sin0^o cot45^o \end{bmatrix}$$
$$= \begin{bmatrix} 1/2 + 1/2 & 0 + 1/2 \\ 1 & 0 + 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0.5 \\ 1 & 0 \end{bmatrix}$$

6. $If A = \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}, B = \begin{bmatrix} -5 \\ 6 \end{bmatrix} and$ **3A x M = 2B; find matrix M.**

Solution:

Given,

3A x M = 2B

And let the order of the matric of M be (a x b)

$$3 \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix}_{2\times2} \times M_{a\times b} = 2 \begin{bmatrix} -5 \\ 6 \end{bmatrix}_{2\times1}$$

Now, it's clearly seen that

a = 2 and b = 1

So, the order of the matrix M is (2 x 1)

$$3 \begin{bmatrix} 0 & -1 \\ 4 & -3 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} -5 \\ 6 \end{bmatrix}$$
$$\begin{bmatrix} 0 & -3 \\ 12 & -9 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$
$$\begin{bmatrix} -3y \\ 12x - 9y \end{bmatrix} = \begin{bmatrix} -10 \\ 12 \end{bmatrix}$$

Now, on comparing with corresponding elements we have

-3y = -10 and 12x – 9y = 12

y = 10/3 and 12x – 9(10/3) = 12
12x – 30 = 12
12x = 42
x = 42/12 = 7/2

Therefore,

Matrix M = $\begin{bmatrix} 7/2 \\ 10/3 \end{bmatrix}$

7. $If \begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$,

find the values of a, b and c.
Solution:

$$\begin{bmatrix} a & 3 \\ 4 & 1 \end{bmatrix} + \begin{bmatrix} 2 & b \\ 1 & -2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$
$$\begin{bmatrix} a+2 & 3+b \\ 4+1 & 1-2 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ -2 & c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$
$$\begin{bmatrix} a+2-1 & 3+b-1 \\ 4+1+2 & 1-2-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$
$$\begin{bmatrix} a+1 & b+2 \\ 7 & -1-c \end{bmatrix} = \begin{bmatrix} 5 & 0 \\ 7 & 3 \end{bmatrix}$$

On comparing the corresponding elements, we have
a + 1 = 5 ⇒ a = 4
b + 2 = 0 ⇒ b = -2
-1 – c = 3 ⇒ c = -4

8. $If A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ $and\ B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$; $find:$

(i) A (BA) **(ii) (AB) B.**
Solution:

(i) A (BA) $= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \left(\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \right)$

$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \left(\begin{bmatrix} 2+2 & 4+1 \\ 1+4 & 2+2 \end{bmatrix} \right)$$
$$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 4 & 5 \\ 5 & 4 \end{bmatrix}$$
$$= \begin{bmatrix} 4+10 & 5+8 \\ 8+5 & 10+4 \end{bmatrix}$$
$$= \begin{bmatrix} 14 & 13 \\ 13 & 14 \end{bmatrix}$$

(ii) (AB) B $= \left(\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}\begin{bmatrix}2 & 1\\1 & 2\end{bmatrix}\right)\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$

$$= \begin{bmatrix}2+2 & 1+4\\4+1 & 2+2\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix}$$

$$= \begin{bmatrix}4 & 5\\5 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 1\end{bmatrix} = \begin{bmatrix}4+10 & 8+5\\5+8 & 10+4\end{bmatrix}$$

$$= \begin{bmatrix}14 & 13\\13 & 14\end{bmatrix}$$

9. Find x and y, if: $\begin{bmatrix}x & 3x\\y & 4y\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}5\\12\end{bmatrix}$

Solution:

$$\begin{bmatrix}x & 3x\\y & 4y\end{bmatrix}\begin{bmatrix}2\\1\end{bmatrix} = \begin{bmatrix}5\\12\end{bmatrix}$$

$$\begin{bmatrix}2x+3x\\2y+4y\end{bmatrix} = \begin{bmatrix}5\\12\end{bmatrix}$$

Thus, on comparing the corresponding terms, we have

2x + 3x = 5 and 2y + 4y = 12

5x = 5 and 6y = 12

x = 5 and y = 2

10. If matrix $X = \begin{bmatrix}-3 & 4\\2 & -3\end{bmatrix}\begin{bmatrix}2\\-2\end{bmatrix}$ $and\ 2X - 3Y = \begin{bmatrix}10\\-8\end{bmatrix}$, **find the matrix 'X' and matrix 'Y'.**

Solution:

$$X = \begin{bmatrix}-3 & 4\\2 & -3\end{bmatrix}\begin{bmatrix}2\\-2\end{bmatrix} = \begin{bmatrix}-6-8\\4+6\end{bmatrix} = \begin{bmatrix}-14\\10\end{bmatrix}$$

Now,

$$Let\ Y = \begin{bmatrix}x\\y\end{bmatrix}$$

$$2X - 3Y = 2\begin{bmatrix}-14\\10\end{bmatrix} - 3\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}10\\-8\end{bmatrix}$$

$$\begin{bmatrix}-28\\20\end{bmatrix} - \begin{bmatrix}3x\\3y\end{bmatrix} = \begin{bmatrix}10\\-8\end{bmatrix}$$

$$\begin{bmatrix}-28-3x\\20-3y\end{bmatrix} = \begin{bmatrix}10\\-8\end{bmatrix}$$

On comparing with the corresponding terms, we have

-28 – 3x = 10

3x = -38
x = -38/3
And,
20 – 3y = -8
3y = 38
y = 38/3
Therefore,

$$Y = 1/3\begin{bmatrix}-38\\28\end{bmatrix}$$

11. Given $A = \begin{bmatrix}2 & -1\\2 & 0\end{bmatrix}, B = \begin{bmatrix}-3 & 2\\4 & 0\end{bmatrix}\ and\ \ C = \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$ **find the matrix X such that:**

A + X = 2B + C
Solution:

$$\begin{bmatrix}2 & -1\\2 & 0\end{bmatrix} + X = 2\begin{bmatrix}-3 & 2\\4 & 0\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$$
$$\begin{bmatrix}2 & -1\\2 & 0\end{bmatrix} + X = \begin{bmatrix}-6 & 4\\8 & 0\end{bmatrix} + \begin{bmatrix}1 & 0\\0 & 2\end{bmatrix}$$
$$\begin{bmatrix}2 & -1\\2 & 0\end{bmatrix} + X = \begin{bmatrix}-5 & 4\\8 & 2\end{bmatrix}$$
$$X = \begin{bmatrix}-5 & 4\\8 & 2\end{bmatrix} - \begin{bmatrix}2 & -1\\2 & 0\end{bmatrix}$$
$$X = \begin{bmatrix}-7 & 5\\6 & 2\end{bmatrix}$$

12. Find the value of x, given that A^2 = B,

$$A = \begin{bmatrix}2 & 12\\0 & 1\end{bmatrix}\ and\ B = \begin{bmatrix}4 & x\\0 & 1\end{bmatrix}$$

Solution:

$$A^2 = \begin{bmatrix}2 & 12\\0 & 1\end{bmatrix}\begin{bmatrix}2 & 12\\0 & 1\end{bmatrix} = \begin{bmatrix}4 & x\\0 & 1\end{bmatrix} = B$$
$$A^2 = \begin{bmatrix}4+0 & 24+12\\0 & 1\end{bmatrix} = \begin{bmatrix}4 & x\\0 & 1\end{bmatrix} = B$$
$$A^2 = \begin{bmatrix}4 & 36\\0 & 1\end{bmatrix} = \begin{bmatrix}4 & x\\0 & 1\end{bmatrix} = B$$

Thus, on comparing the terms we get x = 36.