JEE Main 2020 Paper

Date of Exam: **8th January (Shift I)**

Time: **9:30 am – 12:30 pm**

Subject: **Physics**

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1. A particle of mass m is fixed to one end of a light spring having force constant k and ustreatch length l. The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is

   a. $\dfrac{l m \omega^2}{k - m \omega^2}$

   b. $\dfrac{l m \omega^2}{k + m \omega^2}$

   c. $\dfrac{l m \omega^2}{k - m \omega}$
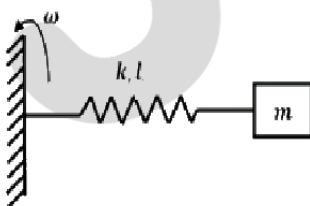
   d. $\dfrac{l m \omega^2}{k + m \omega}$

   Solution: (a)

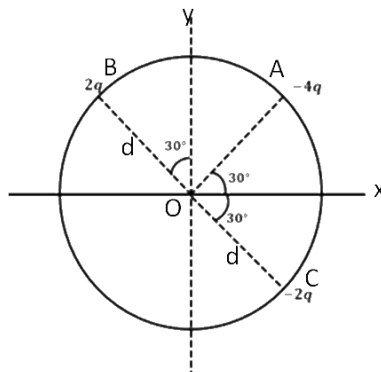   The centripetal force is provided by the spring force.

   $$m\omega^2 (l + x) = kx$$

   $$\left(\frac{l}{x} + 1\right) = \frac{k}{m\omega^2}$$

   $$x = \frac{l m \omega^2}{k - m \omega^2}$$

2. Three charged particles A, B and, C with charge -4q, +2q and -2q are present on the circumference of a circle of radius d. The charges particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x- direction is:

a. $\dfrac{\sqrt{3}q}{4\pi\varepsilon_o d^2}$

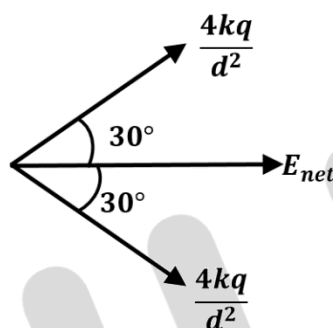b. $\dfrac{2\sqrt{3}q}{\pi\varepsilon_o d^2}$

c. $\dfrac{\sqrt{3}q}{\pi\varepsilon_o d^2}$

d. $\dfrac{3\sqrt{3}q}{4\pi\varepsilon_o d^2}$
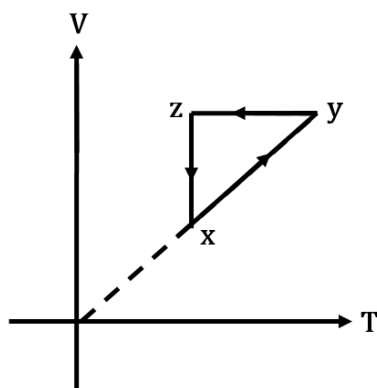
Solution: (c)

Applying superposition principle,

$$\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$$
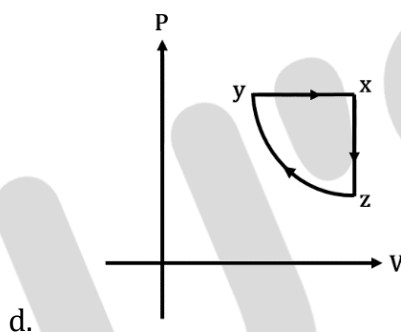

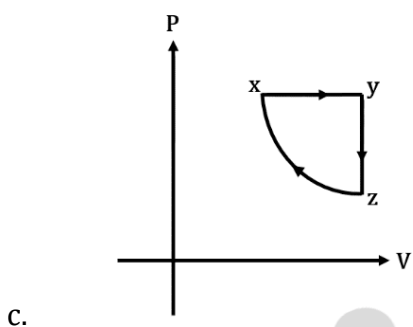By symmetry, net electric field along the x-axis.

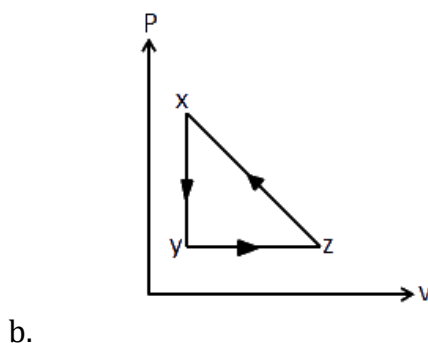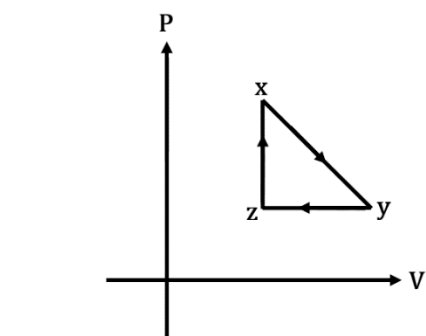$$\left|\vec{E}_{net}\right| = \dfrac{4kq}{d^2} \times 2\cos 30° = \dfrac{\sqrt{3}q}{\pi\varepsilon_o d^2}$$

3. A thermodynamic cycle xyzx is shown on a $V - T$ diagram.

The P-V diagram that best describes this cycle is : (Diagrams are schematic and not upto scale)

a.

b.

c.

d.

Solution: (c)
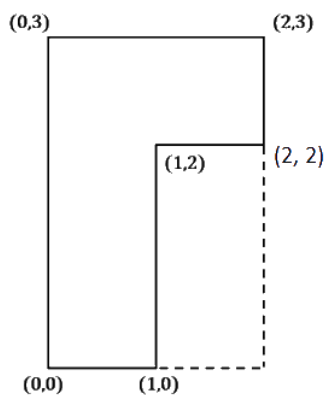
For the given $V - T$ graph

For the process $x \rightarrow y$; $V \propto T$; $P =$ constant

For the process $y \rightarrow z$; $V =$ constant

Only $'c'$ satisfies these two conditions.

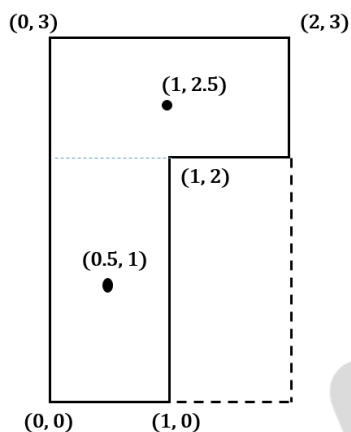4. Find the co-ordinates of center of mass of the lamina shown in the figure below.

a. (0.75 m, 1.75 m)   b. (0.75 m, 0.75 m)
c. (1.25 m, 1.5 m)   d. (1 m, 1.75 m)

Solution: (a)

The Lamina can be divided into two parts having equal mass m each.

(0, 3)   (2, 3)

(1, 2.5)

(1, 2)

(0.5, 1)

(0, 0)   (1, 0)

$$\vec{r}_{cm} = \frac{m \times \left(\frac{\hat{i}}{2} + \hat{j}\right) + m \times \left(\hat{i} + \frac{5\hat{j}}{2}\right)}{2m}$$

$$\vec{r}_{cm} = \frac{3}{4}\hat{i} + \frac{7}{4}\hat{j}$$

5. The plot that depicts the behavior of the mean free time τ (time between two successive collisions) fot the molecules of an ideal gas, as a function of temperatire (T), qualitatively, is: (Graph are schematic and not drawn to scale)

a.

τ vs $\frac{1}{\sqrt{T}}$

b.

τ vs $\sqrt{T}$

c.

τ vs T

d.

τ vs $\sqrt{T}$

Solution: (a)

$$\tau \propto \frac{1}{\sqrt{T}}$$
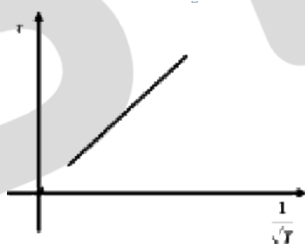
6. Effective capacitance of parallel combination of two capacitors C_1 and C_2 is 10 µF. When these capacitor are individually connectes to a voltage source of $1\,V$, the ene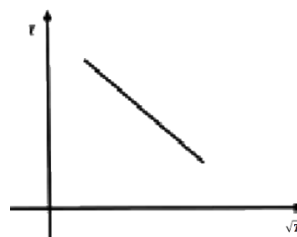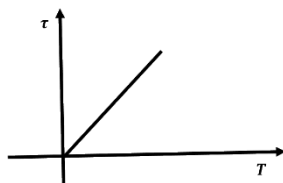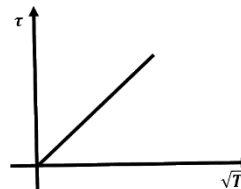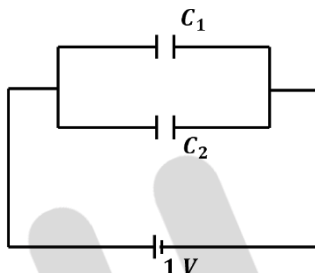rgy stored in the capacitor C_2 is 4 times of that in C_1. If these capacitors are connected in series, their effective capacitance will be:

   a. 1.6 µF
   b. 3.2 µF
   c. 4.2 µF
   d. 8.4 µF

   Solution: (a)

   Given that,

   $C_1 + C_2 = 10$ µF   .....(i)

   $4\left(\frac{1}{2}C_1V^2\right) = \frac{1}{2}C_2V^2$

   $\Rightarrow 4C_1 = C_2$ ...(ii)

   From equations (i) and (ii)

   $C_1 = 2$ µF

   $C_2 = 8$ µF

   If they are in series

   $C_{eq} = \dfrac{C_1C_2}{C_1 + C_2} = 1.6$ µF

7. Consider a uniform rod of mass $4m$ and length L pivoted about its centre. A mass m is moving with a velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod rod and stick to it.. The angular speed of the rod-mass system just after collision is

   a. $\dfrac{3\sqrt{2}v}{7L}$
   b. $\dfrac{4v}{7L}$
   c. $\dfrac{3v}{7\sqrt{2}L}$
   d. $\dfrac{3v}{7L}$

Solution: (a)
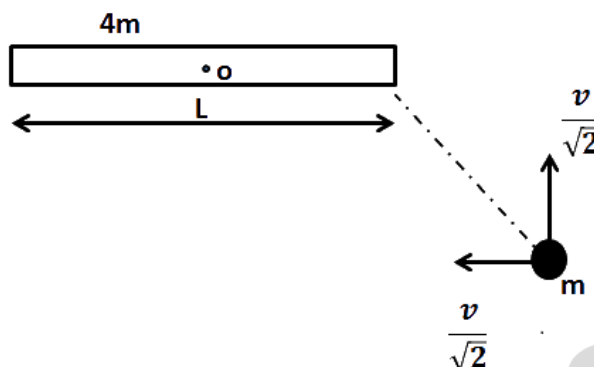
There is no external torque on the system about the hinge point.

So,

$$\vec{L}_1 = \vec{L}_f$$

$$\frac{mv}{\sqrt{2}} \times \frac{1}{2} = \left[\frac{4mL^2}{12} + \frac{mL^2}{4} \right] \times \omega$$

$$\omega = \frac{6v}{7\sqrt{2}L} = \frac{3\sqrt{2}v}{7L}$$

8. When photons of energy 4 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy T_A eV and de-Broglie wavelength λ_A. The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is
$T_B = (T_A - 1.5)$ eV. If the de-Broglie wavelength of these photoelectrons $\lambda_B = 2\lambda_A$, then the work function of metal B is

a. 3 eV

b. 1.5 eV

c. 2 eV

d. 4 eV

Solution: (d)

$$\lambda = \frac{h}{\sqrt{2(KE)m_B}} = \lambda \propto \frac{1}{\sqrt{KE}}$$

$$\frac{\lambda_A}{\lambda_B} = \frac{\sqrt{KE_B}}{\sqrt{KE_A}}$$

$$\frac{1}{2} = \sqrt{\frac{T_A - 1.5}{T_A}}$$

$$T_A = 2 \text{ eV}$$

$$KE_B = 2 - 1.5 = 0.5 \text{ eV}$$

$$\emptyset_B = 4.5 - 0.5 = 4 \text{ eV}$$

9. The length of a potentiometer wire of length $1200\ cm$ and it carries a current of $60\ mA$. For a cell of emf $5\ V$ and internal resistance of $20\ \Omega$, the null point on it is found to be at $1000\ cm$. The resistance of whole wire is

   a. $80\ \Omega$ b. $100\ \Omega$
   c. $120\ \Omega$ d. $60\ \Omega$

   Solution: (b)

Assume the terminal voltage of the primary battery as V_p. As long as this potentiometer is operating on balanced length, V_p will remain constant.

As we know, potential gradient $= \dfrac{5}{1000} = \dfrac{V_p}{1200}$

$$V_p = 6 \text{ V}$$

And $R_p = \dfrac{V_p}{i} = \dfrac{6}{60 \times 10^{-3}} = 100\ \Omega$

10. The magnifying power of a telescope with tube length 60 cm is 5. What is the focal length of its eyepiece?

a. 10 cm
b. 20 cm
c. 30 cm
d. 40 cm

Solution: (a)
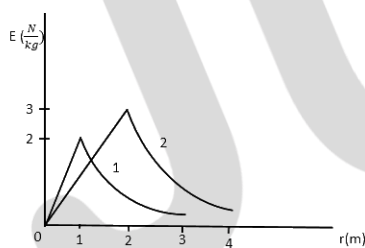
$$m = \frac{f_0}{f_e} = 5$$

$$\Rightarrow f_0 = 5f_e$$

$$f_0 + f_e = 5f_e + f_e = 6f_e = \text{length of the tube}$$

$$\Rightarrow 6f_e = 60 \text{ cm}$$

$$\Rightarrow f_e = 10 \text{ cm}$$

11. Consider two solid spheres of radii $R_1 = 1$ m, $R_2 = 2$ m and masses M_1 & M_2, respectively. The gravitational field due to two spheres 1 and 2 are shown. The value of $\frac{M_1}{M_2}$ is

a. $\frac{1}{6}$
b. $\frac{1}{3}$

c. $\frac{1}{2}$
d. $\frac{2}{3}$

Solution: (a)

Gravitation field will be maximum at the surface of a sphere. Therefore,

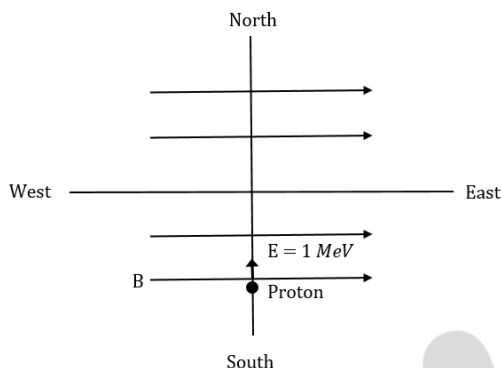$$\frac{GM_2}{2^2} = 3 \ \& \ \frac{GM_1}{1^2} = 2$$

$$\Rightarrow \frac{M_2}{M_1} \times \frac{1}{4} = \frac{3}{2}$$

$$\Rightarrow \frac{M_1}{M_2} = \frac{1}{6}$$

12. Proton with kinetic energy of 1 MeV moves from south to north. It gets an acceleration of 10^{12} m/s^2 by an applied magnetic field (west to east). The value of magnetic field: (Rest mass of proton is 1.6×10^{-27} kg)

a. 0.71 mT

b. 7.1 mT

c. 71 mT

d. 0.071 mT

Solution: (a)

$K.E = 1 \times 10^6$ eV $= 1.6 \times 10^{-13}$ J

$= \dfrac{1}{2} m_e v^2$

Where m_e is the mass of the electron $= 1.6 \times 10^{-27}$ kg

$\Rightarrow 1.6 \times 10^{-13} = \dfrac{1}{2} \times 1.6 \times 10^{-27} \times v^2$

$\therefore v = \sqrt{2} \times 10^7$ m/s

$Bqv = m_e a$

$\therefore B = \dfrac{1.6 \times 10^{-27} \times 10^{12}}{1.6 \times 10^{-19} \times \sqrt{2} \times 10^7}$

$= 0.71 \times 10^{-3}$ T $= 0.71$ mT

13. If finding the electric field around a surface is given by $\left|\vec{E}\right| = \dfrac{q_{enclosed}}{\varepsilon_0 |A|}$ is applicable. In the formula ϵ_0 is permittivity of free space, A is area of Gaussian and q_{enc} is charge enclosed by the Gaussian surface. This equation can be used in which of the following equation?

a. Only when the Gaussian surface is an equipotential surface.

b. Only when $\left|\vec{E}\right|$ = constant on the surface..

c. Equipotential surface and $\left|\vec{E}\right|$ is constant on the surface .

d. for any choice of Gaussian surfaces.

Solution: (c)

The magnitude of the electric field is constant and the electric field must be along the area vector i.e. the surface is equipotential.

14. The dimension of stopping potential V_0 in photoelectric effect in units of Planck's constant (h), speed of light (c), and gravitational constant (G) and Ampere (A) is

a. $h^{2/3}c^{5/3}G^{1/3}A^{-1}$

b. $h^2c^{1/3}G^{3/2}A^{-1}$

c. $h^{1/3}G^{2/3}c^{1/3}A^{-1}$

d. $h^{-2/3}c^{-1/3}G^{4/3}A^{-1}$

Solution: (challenge question)

$V = K(h)^a(I)^b(G)^c(c)^d$ Unit of stopping potential is (V_0) Volt.

We know $[h] = ML^2T^{-1}$

$[I] = A$

$[G] = M^{-1}L^3T^{-2}$

$[C] = LT^{-1}$

$[V] = ML^2T^{-3}A^{-1}$

$ML^2T^{-3}A^{-1} = (ML^2T^{-1})^a(A)^b \ (M^{-1}L^3T^{-2})^c(LT^{-1})^d$

$ML^2T^{-3}A^{-1} = M^{a-c}L^{2a+3c+d}T^{-a-2c-d}A^b$

$a - c = 1$

$2a + 3c + d = 2$

$-a - 2c - d = -3$

$b = -1$

On solving,

$c = -1$

$a = 0$

$d = 5$

$b = -1$

$V = K(h)^0(I)^{-1}(G)^{-1}(C)^5$

15. A leak proof cylinder of length 1 m, made of metal which has very low coefficient of expansion is floating in water at 0° C such that its height above the water surface is 20 cm. When the temperature of water is increases to 4° C, the height of the cylinder above the water surface becomes 21 cm. The density of water at $T = 4^o$ C relative to the density at $T = 0^o$ C is close to
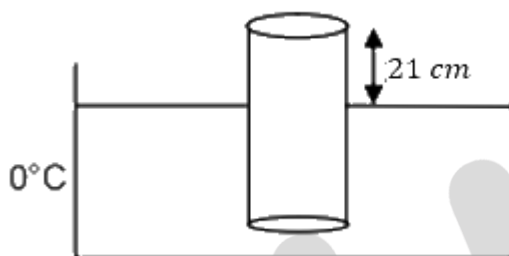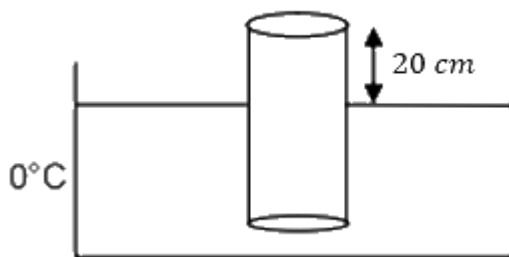
a. 1.01

b. 1.03

c. 1.26

d. 1.04

Solution: (a)

Since the cylinder is in equilibrium, it's weight is balanced by the Buoyant force.

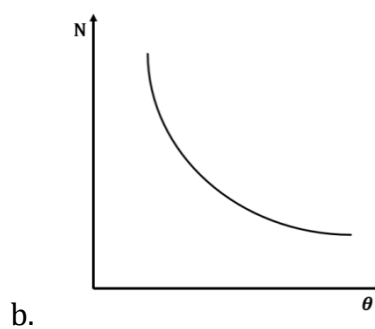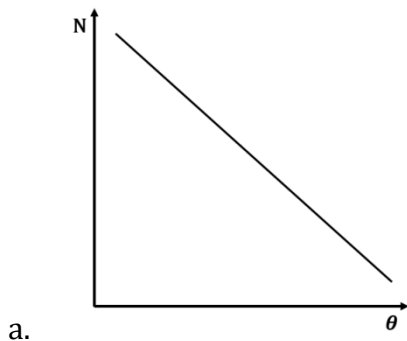$mg = A(80)(\rho_{0^\circ C})g$

$mg = A(79)(\rho_{4^\circ C})g$

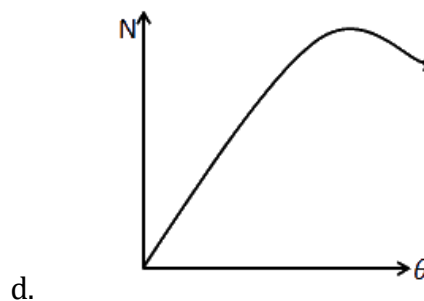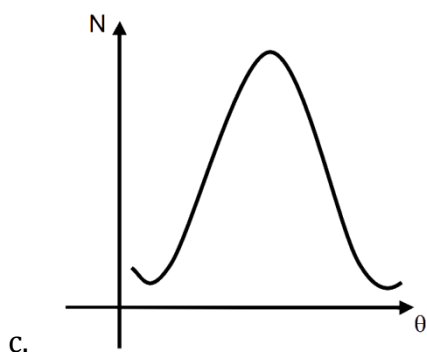$\dfrac{\rho_{4^\circ C}}{\rho_{0^\circ C}} = \dfrac{80}{79} = 1.01$

16. The graph which depicts the result of Rutherford gold foil experiement with α- particle is:
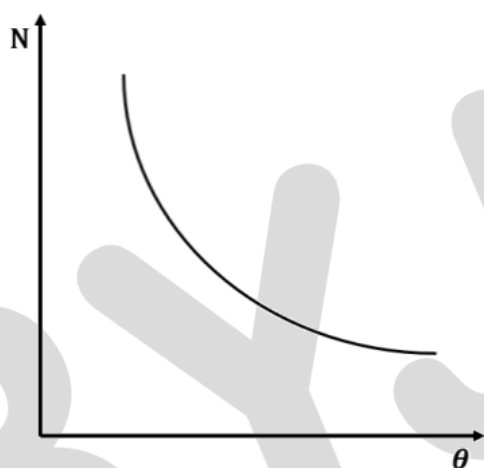
θ: Scattering angle

N : Number of scattered $\alpha -$ particles is detected

(Plots are schematic and not to scale)
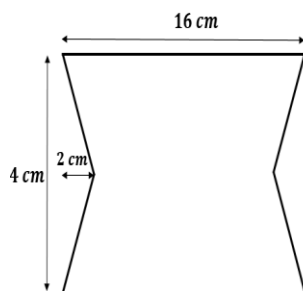

a.

b.

c.

d.

Solution: (b)

$$N \propto \frac{1}{\sin^4(\theta/2)}$$

17. At time $t = 0$ magnetic field of 1000 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5 s, then induced EMF in the loop is:

a. 56 μV
c. 30 μV

b. 28 μV
d. 48 μV

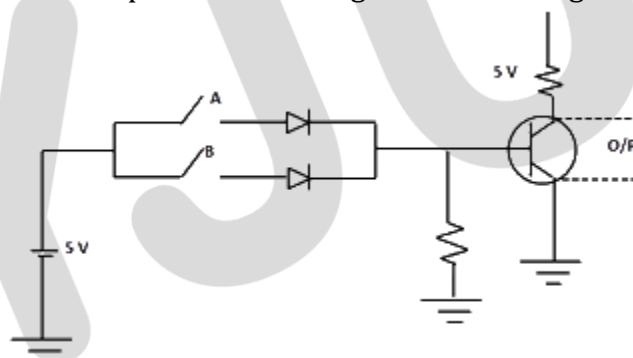Solution: (a)

$$\epsilon = \left| -\frac{d\phi}{dt} \right| = \left| -\frac{AdB}{dt} \right|$$

$$= (16 \times 4 - 4 \times 2)\frac{(1000 - 500)}{5} \times 10^{-4} \times 10^{-4}$$

$$= 56 \times \frac{500}{5} \times 10^{-8} = 56 \times 10^{-6} \text{ V}$$

18. Choose the correct Boolean expression for the given circuit diagram:

a. A.B

b. $\bar{A} + \bar{B}$

c. $A + B$

d. $\bar{A}.\bar{B}$

Solution: (d)
First part of figure shown OR gate and second part of figure shown NOT gate.
So, $Y = \overline{A + B} = \bar{A}.\bar{B}$

19. Consider a solid sphere of density $\rho(r) = \rho_o \left(1 - \frac{r^2}{R^2}\right), 0 < r \le R$. The minimum density of a liquid in which it float isjust

a. $\frac{2}{5}\rho_o$

b. $\frac{2}{3}\rho_o$

c. $\frac{\rho_0}{5}$

d. $\frac{\rho_0}{3}$

Solution: (a)

Let the mass of the sphere be m and the density of the liquid be ρ_L

$$\rho = \rho_o \left(1 - \frac{r^2}{R^2}\right), 0 < r \leq R$$

Since the sphere is floating in the liquid, buoyancy force (F_B) due to liquid will balance the weight of the sphere.

$$F_B = mg$$

$$\rho_L \frac{4}{3}\pi R^3 g = \int \rho(4\pi r^2 dr)g$$

$$\rho_L \frac{4}{3}\pi R^3 = \int \rho_o \left(1 - \frac{r^2}{R^2}\right) 4\pi r^2 dr$$

$$\rho_L \frac{4}{3}\pi R^3 = \int_0^R \rho_o 4\pi \left(r^2 - \frac{r^4}{R^2}\right) dr = \rho_o 4\pi \left(\frac{r^3}{3} - \frac{r^5}{5R^2}\right)_0^R$$

$$\rho_L = \frac{2}{5}\rho_o$$

20. The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability $\frac{4}{3}$ for this wavelength, will be

   a. 15°   b. 30°

   c. 45°   d. 60°

Solution: (b)

If the speed of light in the given medium is V then,

$$V = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\text{We know that,} \quad n = \frac{c}{v}$$

$$n = \sqrt{\mu_r \epsilon_r} = 2$$

$$\sin \theta_c = \frac{1}{2}$$

$$\theta_c = 30°$$

21. A body of mass $m = 0.10\ kg$ has an initial velocity of $3\hat{\imath}\ m/s$. It collides elastically with another body, B of the mass which has an initial velocity of $5\hat{\jmath}\ m/s$. After collision, A moves with a velocity $v = 4(\hat{\imath} + \hat{\jmath})\ m/s$. The energy of B after collision is written as $\frac{x}{10}\ J$, the value of x is

Solution: (1)(Challenge question)

Mass of each object, $m_1 = m_2 = 0.1 \, kg$
Initial velocity of 1st object, $u_1 = 5 \, m/s$
Initial velocity of 2nd object, $u_2 = 3 \, m/s$
Final velocity of 1st object, $V_1 = 4\hat{i} + 4\hat{j} \, m/s = \sqrt{4^2 + 4^2} = 16\sqrt{2} \, m/s$

For elastic collision, kinetic energy remains conserved

$$Initial \; kinetic \; energy \; (K_i) = Final \; kinetic \; energy (K_f)$$

$$\frac{1}{2}mu_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2$$

$$\frac{1}{2}m(5)^2 + \frac{1}{2}m(3)^2 = \frac{1}{2}m(16\sqrt{2})^2 + \frac{1}{2}mV_2^2$$

$$V_2 = \sqrt{2} \, m/s$$

Kinetic energy of second object $= \frac{1}{2}mV_2^2 = \frac{1}{2} \times 0.1 \times \sqrt{2}^2 = \frac{1}{10}$

$$\Rightarrow x = 1$$

22. A point object in air is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of lens material is 1.5, then the focal length of the lens (in cm) is

Solution: (60 cm)

Applying Lens makers' formula,

$$\frac{1}{f} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$R_1 = \infty$$

$$R_2 = -30 \text{ cm}$$

$$\frac{1}{f} = (1.5 - 1)\left(\frac{1}{\infty} - \frac{1}{-30}\right)$$

$$\frac{1}{f} = \frac{0.5}{30}$$

$$f = 60 \text{ cm}$$

23. A particle is moving along the x-axis with its coordinate with time t given by $x(t) = -3t^2 + 8t + 10 \, m$. Another particle is moving along the y-axis with its coordinate as a function of time given by $y = 5 - 8t^3 \, m$. At t = 1 s, the speed of the second particle as measured in the frame of the first particle is given as $\sqrt{v}$. Then v (m/s) is _____.

Solution: (580 m/s)(Challenge question)

$$x = -3t^2 + 8t + 10$$

$$\vec{v_A} = (-6t + 8)\hat{i}$$

$$= 2\hat{i}$$

$$y = 5 - 8t^3$$

$$\vec{v_B} = -24t^2\hat{j}$$

$$\left|\vec{v_{B/A}}\right| = \left|\vec{v_B} - \vec{v_A}\right| = \left|-24\hat{j} - 2\hat{i}\right|$$

$$v = \sqrt{24^2 + 2^2}$$

$$v = 580 \text{ m/s}$$

24. A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300 m/s, the frequency difference between the fundamental and second harmonic of this pipe is_____Hz.

Solution: (106.05 Hz)

$$V = \sqrt{\frac{B}{\rho}}$$

$$\frac{V_{pipe}}{V_{air}} = \frac{\sqrt{\frac{B}{2\rho}}}{\sqrt{\frac{B}{\rho}}} = \frac{1}{\sqrt{2}}$$
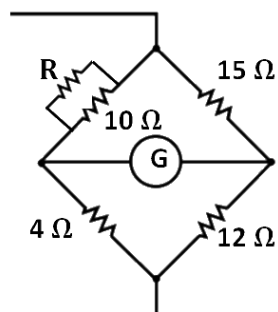
$$V_{pipe} = \frac{V_{air}}{\sqrt{2}}$$

$$f_n = \frac{(n + 1)}{2l} V_{pipe}$$

$$f_1 - f_0 = \frac{V_{pipe}}{2l} = \frac{300}{2\sqrt{2}} = 106.06 \text{ Hz (if } \sqrt{2} = 1.414) \approx 106 \, Hz$$

25. Four resistors of resistance 15 Ω, 12 Ω, 4 Ω and 10 Ω respectively in cyclic order to form a wheatstone's network. The resistance that is to be connected in parallel with the resistance of 10 Ω to balance the network is _____ Ω.

Solution: (10 Ω)

$$\frac{10\,R}{10+R} \times 12 = 15 \times 4 \Rightarrow R = 10\ \Omega$$