JEE Main 2020 Paper

Date of Exam: **9th January 2020 (Shift 2)**
Time: **2:30 P.M. to 5:30 P.M.**
Subject: **Mathematics**

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1. If $A = \{x \in \boldsymbol{R} : |x| < 2\}$ and $B = \{x \in \boldsymbol{R} : |x - 2| \geq 3\}$ then :

a. $A - B = [-1,2]$
b. $B - A = \mathbf{R} - (-2,5)$
c. $A \cup B = \boldsymbol{R} - (2,5)$
d. $A \cap B = (-2,-1)$

Answer: (b)

Solution:

$A = \{x: x \in (-2,2)\}$

$B = \{x: x \in (-\infty,-1] \cup [5,\infty)\}$

$A \cap B = \{x: x \in (-2,-1]\}$

$B - A = \{x: x \in (-\infty,-2] \cup [5,\infty)\}$

$A - B = \{x: x \in (-1,2)\}$

$A \cup B = \{x: x \in (-\infty,2) \cup [5,\infty)\}$

2. If 10 different balls has to be placed in 4 distinct boxes at random, then the probability that two of these boxes contain exactly 2 and 3 balls is :

a. $\frac{965}{2^{10}}$
b. $\frac{945}{2^{10}}$
c. $\frac{945}{2^{11}}$
d. $\frac{965}{2^{11}}$

Answer: (b)

Solution:

Total ways to distribute 10 balls in 4 boxes is $= 4^{10}$

Total ways of placing exactly 2 and 3 balls in any two of these boxes is

$= {}^4C_2! \times {}^{10}C_5 \times \frac{5!}{2!\,3!} \times 2 \times 2^5$

$P(E) = \frac{945}{2^{10}}$

3. If $x = 2\sin\theta - \sin 2\theta$ and $y = 2\cos\theta - \cos 2\theta, \theta \in [0,2\pi]$, then $\frac{d^2y}{dx^2}$ at $\theta = \pi$ is :

a. $-\frac{3}{8}$

b. $\frac{3}{4}$

c. $\frac{3}{2}$

d. $-\frac{3}{4}$

Answer: (Bonus)

Solution:

$$\frac{dx}{d\theta} = 2\cos\theta - 2\cos 2\theta$$

$$\frac{dy}{d\theta} = -2\sin\theta + 2\sin 2\theta$$

$$\frac{dy}{dx} = \frac{2\cos\frac{3\theta}{2}\sin\frac{\theta}{2}}{2\sin\frac{3\theta}{2}\sin\frac{\theta}{2}}$$

$$\frac{dy}{dx} = \cot\frac{3\theta}{2}$$

$$\frac{d^2y}{dx^2} = -\frac{3}{2}\ cosec^2\frac{3\theta}{2}\ \frac{d\theta}{dx}$$

$$\frac{d^2y}{dx^2} = \left(-\frac{3}{2}\ cosec^2\frac{3\theta}{2}\right)\frac{1}{(2\cos\theta - 2\cos 2\theta)}$$

$$\left.\frac{d^2y}{dx^2}\right|_{\theta=\pi} = \frac{3}{8}$$

None of the above option satisfies the answer.

4. Let f and g be differentiable functions on $\boldsymbol{R}$, such that fog is the identity function. If for some $a, b \in \boldsymbol{R}, g'(a) = 5$ and $g(a) = b$, then $f'(b)$ is equal to :

a. $\frac{2}{5}$

b. 5

c. 1

d. $\frac{1}{5}$

Answer: (d)

Solution:

$$f(g(x)) = x$$

$$f'(g(x))g'(x) = 1$$

Put $x = a$

$$f'(g(a))g'(a) = 1 \Rightarrow f'^{(b)} \times 5 = 1 \Rightarrow f'(b) = \frac{1}{5}$$

5. In the expansion of $\left(\frac{x}{\cos\theta} + \frac{1}{x\sin\theta}\right)^{16}$, if l_1 is the least value of the term independent of x when $\frac{\pi}{8} \le \theta \le \frac{\pi}{4}$ and l_2 is the least value of the term independent of x when $\frac{\pi}{16} \le \theta \le \frac{\pi}{8}$, then the ratio $l_2 : l_1$ is equal to :

a. $16:1$

b. $8:1$

c. $1:8$

d. $1:16$

Answer: (a)

Solution:

$$T_{r+1} = {}^{16}C_r\left(\frac{x}{\cos\theta}\right)^{16-r}\left(\frac{1}{x\sin\theta}\right)^r$$

For term independent of x,

$$16 - 2r = 0 \Rightarrow r = 8$$

$$T_9 = {}^{16}C_8\left(\frac{1}{\sin\theta\cos\theta}\right)^8 = {}^{16}C_8 2^8\left(\frac{1}{\sin 2\theta}\right)^8$$

$$l_1 = {}^{16}C_8 2^8 \quad \text{at } \theta = \frac{\pi}{4}$$

$$l_2 = {}^{16}C_8 \frac{2^8}{\left(\frac{1}{\sqrt{2}}\right)^8} = {}^{16}C_8 2^{12} \quad \text{at } \theta = \frac{\pi}{8}$$

$$\frac{l_2}{l_1} = 16:1$$

6. Let $a, b \in \boldsymbol{R}, a \neq 0$, such that the equation, $ax^2 - 2bx + 5 = 0$ has a repeated root α, which is also a root of the equation $x^2 - 2bx - 10 = 0$. If β is the root of this equation, then $\alpha^2 + \beta^2$ is equal to:

a. 24

b. 25

c. 26

d. 28

Answer: (b)

Solution:

$ax^2 - 2bx + 5 = 0$ has both roots as α

$$\Rightarrow 2\alpha = \frac{2b}{a} \Rightarrow \alpha = \frac{b}{a}$$

And $\alpha^2 = \frac{5}{a}$

$\Rightarrow b^2 = 5a(a \neq 0)$ ... (1)

$\Rightarrow \alpha + \beta = 2b \;\&\; \alpha\beta = -10$

$\alpha = \frac{b}{a}$ is also a root of $x^2 - 2bx - 10 = 0$

$\Rightarrow b^2 - 2ab^2 - 10a^2 = 0$

$\because b^2 = 5a \Rightarrow 5a - 10a^2 - 10a^2 = 0$

$\Rightarrow a = \frac{1}{4} \Rightarrow b^2 = \frac{5}{4}$

$\Rightarrow \alpha^2 = 20, \beta^2 = 5 \Rightarrow \alpha^2 + \beta^2 = 25$

7. Let a function $f: [0,5] \rightarrow \boldsymbol{R}$, be continuous, $f(1) = 3$ and $\boldsymbol{F}$ be defined as:

$F(x) = \int_1^x t^2\, g(t)dt$, where $g(t) = \int_1^t f(u)du$ Then for the function $\boldsymbol{F}$, the point $x = 1$ is

a. a point of inflection.

b. a point of local maxima

c. a point of local minima.

d. not a critical point

Answer: (c)

Solution:

$F(x) = x^2 g(x)$

Put $x = 1$

$\Rightarrow F(1) = g(1) = 0$ ... (1)

Now $F''(x) = 2xg(x) + g'(x)x^2$

$F''(1) = 2g(1) + g'(1)$ $\{\because g'(x) = f(x)\}$

$F''(1) = f(1) = 3$ ... (2)

From (1) and (2), $F(x)$ has local minimum at $x = 1$

8. Let $[t]$ denotes the greatest integer $\leq t$ and $\lim\limits_{x\rightarrow 0} x\left[\frac{4}{x}\right] = A$. Then the function, $f(x) = [x^2]\sin \pi x$ is discontinuous, when x is equal to

a. $\sqrt{A+1}$

b. $\sqrt{A}$

c. $\sqrt{A+5}$

d. $\sqrt{A+21}$

Answer: (a)

Solution:

$f(x) = [x^2]\sin \pi x$

It is continuous $\forall x \in \mathbf{Z}$ as $\sin \pi x \to 0$ as $\to \mathbf{Z}$.

$f(x)$ is discontinuous at points where $[x^2]$ is discontinuous i.e. $x^2 \in \mathbf{Z}$ with an exception that $f(x)$ is continuous as x is an integer.

$\therefore$ Points of discontinuity for $f(x)$ would be at

$x = \pm\sqrt{2}, \pm\sqrt{3}, \pm\sqrt{5}, \dots\dots$

Also, it is given that $\lim_{x \to 0} x \left[\frac{4}{x}\right] = A$ (indeterminate form $(0 \times \infty)$)

$\Rightarrow \lim_{x \to 0} x \left(\frac{4}{x} - \left\{\frac{4}{x}\right\}\right) = A$

$\Rightarrow 4 - \lim_{x \to 0} \left\{\frac{4}{x}\right\} = A$

$\Rightarrow A = 4$

$\sqrt{A + 5} = 3$

$\sqrt{A + 1} = \sqrt{5}$

$\sqrt{A + 21} = 5$

$\sqrt{A} = 2$

$\therefore$ Points of discontinuity for $f(x)$ is $x = \sqrt{5}$

9. Let $a - 2b + c = 1$,

If $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$, then:

a. $f(-50) = 501$

b. $f(-50) = -1$

c. $f(50) = 1$

d. $f(50) = -501$

Answer: (c)

Solution:

Given $f(x) = \begin{vmatrix} x+a & x+2 & x+1 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$

$a - 2b + c = 1$

Applying $R_1 \to R_1 - 2R_2 + R_3$

$$f(x) = \begin{vmatrix} a-2b+c & 0 & 0 \\ x+b & x+3 & x+2 \\ x+c & x+4 & x+3 \end{vmatrix}$$

Using $a - 2b + c = 1$

$\therefore f(x) = (x+3)^2 - (x+2)(x+4)$

$\Rightarrow f(x) = 1$

$\Rightarrow f(50) = 1$

$\Rightarrow f(-50) = 1$

10. Given: $f(x) = \begin{cases} x, & 0 \le x < \frac{1}{2} \\ \frac{1}{2}, & x = \frac{1}{2} \\ 1 - x, & \frac{1}{2} < x \le 1 \end{cases}$

and $g(x) = \left(x - \frac{1}{2}\right)^2, x \in \boldsymbol{R}$. Then the area (in sq. units) of the region bounded by the curves $y = f(x)$ and $y = g(x)$ between the lines $2x = 1$ to $2x = \sqrt{3}$ is :

a. $\frac{\sqrt{3}}{4} - \frac{1}{3}$

b. $\frac{1}{3} + \frac{\sqrt{3}}{4}$

c. $\frac{1}{2} + \frac{\sqrt{3}}{4}$

d. $\frac{1}{2} - \frac{\sqrt{3}}{4}$

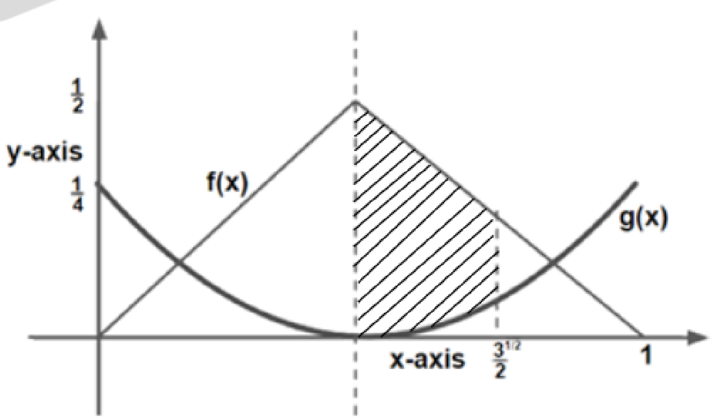
Answer: (a)

Solution:

Given $f(x) = \begin{cases} x & 0 < x < \frac{1}{2} \\ \frac{1}{2} & x = \frac{1}{2} \\ 1 - x & \frac{1}{2} < x < 1 \end{cases}$

$g(x) = \left(x - \frac{1}{2}\right)^2$

The area between $f(x)$ and $g(x)$ from $x = \frac{1}{2}$ to $= \frac{\sqrt{3}}{2}$:



Points of intersection of $f(x)$ and (x) :

$1-x=\left(x-\frac{1}{2}\right)^2$

$\Rightarrow x=\frac{\sqrt{3}}{2}, -\frac{\sqrt{3}}{2}$

Required area $=\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}(f(x)-g(x))dx$

$=\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\left(1-x-\left(x-\frac{1}{2}\right)^2\right)dx$

$=x-\frac{x^2}{2}-\frac{1}{3}\left(x-\frac{1}{2}\right)^3\Big|_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}=\frac{\sqrt{3}}{4}-\frac{1}{3}$

11. The following system of linear equations

$7x+6y-2z=0$,

$3x+4y+2z=0$

$x-2y-6z=0$, has

a. infinitely many solutions, (x,y,z) satisfying $y=2z$
b. infinitely many solutions (x,y,z) satisfying $x=2z$
c. no solution
d. only the trivial solution

Answer: (b)

Solution:

$7x+6y-2z=0$

$3x+4y+2z=0$

$x-2y-6z=0$

As the system of equations are Homogeneous $\Rightarrow$ the system is consistent.

$\Rightarrow \begin{vmatrix} 7 & 6 & -2 \\ 3 & 4 & 2 \\ 1 & -2 & -6 \end{vmatrix}=0$

$\Rightarrow$ Infinite solutions exist (both trivial and non-trivial solutions)

When $y=2z$

Let's take $y=2, z=1$

When $(x,2,1)$is substituted in the system of equations

$\Rightarrow 7x+10=0, 3x+10=0, x-10=0$ (which is not possible)

$\therefore y = 2z \Rightarrow$ Infinitely many solutions does not exist.

For $x = 2z$, lets take $x = 2, z = 1, y = y$

Substitute $(2, y, 1)$in system of equations

$\Rightarrow y = -2$

$\therefore$For each pair of (x, z), we get a value of y.

Therefore, for $x = 2z$ infinitely many solutions exists.

12. If $p \rightarrow (p \wedge \sim q)$ is false. Then the truth values of p and q are respectively

a. F, T

b. T, F

c. F, F

d. T, T

Answer: (d)

Solution:

Given $p \rightarrow (p \wedge \sim q)$

Truth table:

p	q	$\sim q$	$(p \wedge \sim q)$	$p \rightarrow (p \wedge \sim q)$
T	T	F	F	F
T	F	T	T	T
F	T	F	F	T
F	F	T	F	T

$p \rightarrow (p \wedge \sim q)$ is false when p is true and q is true.

13. The length of minor axis (along y-axis) of an ellipse of the standard form is $\frac{4}{\sqrt{3}}$. If this ellipse touches the line $x + 6y = 8$, then its eccentricity is :

a. $\frac{1}{2}\sqrt{\frac{5}{3}}$

b. $\frac{1}{2}\sqrt{\frac{11}{3}}$

c. $\sqrt{\frac{5}{6}}$

d. $\frac{1}{3}\sqrt{\frac{11}{3}}$

Answer: (b)

Solution:

If $2b = \frac{4}{\sqrt{3}}$

$b = \frac{2}{\sqrt{3}}$

Comparing $y = -\frac{x}{6} + \frac{8}{6}$ with $y = mx \pm \sqrt{a^2m^2 + b^2}$

$m = -\frac{1}{6}$ and $a^2m^2 + b^2 = \frac{16}{9}$

$\frac{a^2}{36} + \frac{4}{3} = \frac{16}{9}$

$\Rightarrow \frac{a^2}{36} = \frac{16}{9} - \frac{4}{3}$

$\Rightarrow a^2 = 16$

$e = \sqrt{1 - \frac{b^2}{a^2}}$

$\Rightarrow e = \sqrt{\frac{11}{12}}$

14. If z be a complex number satisfying $|\mathrm{Re}(z)| + |\mathrm{Im}(z)| = 4$, then $|z|$ cannot be:

a. $\sqrt{7}$

b. $\sqrt{\frac{17}{2}}$

c. $\sqrt{10}$

d. $\sqrt{8}$

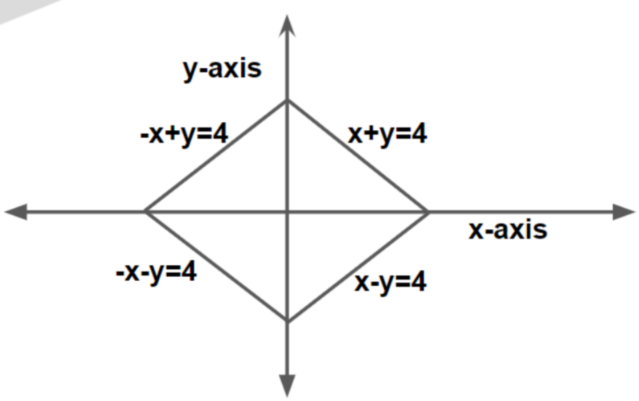
Answer: (a)

Solution:

$|\mathrm{Re}(z)| + |\mathrm{Im}(z)| = 4$

Let $z = x + iy$

$\Rightarrow |x| + |y| = 4$



$\therefore$ z lies on the rhombus.

Maximum value of $|z| = 4$ when $z = 4, -4, 4i, -4i$

Minimum value of $|z| = 2\sqrt{2}$ when $z = 2 \pm 2i,\ \pm 2 + 2i$

$|z| \in [2\sqrt{2}, 4]$

$|z| \in [\sqrt{8}, \sqrt{16}]$

$|z| \neq \sqrt{7}$

15. If $x = \sum_{n=0}^{\infty}(-1)^n \tan^{2n}\theta$ and $y = \sum_{n=0}^{\infty}\cos^{2n}\theta$, where $0 < \theta < \frac{\pi}{4}$, then:

a. $y(1 + x) = 1$

b. $x(1 - y) = 1$

c. $y(1 - x) = 1$

d. $x(1 + y) = 1$

Answer: (c)

Solution:

$y = 1 + \cos^2\theta + \cos^4\theta + \cdots$

$\Rightarrow y = \dfrac{1}{1 - \cos^2\theta} \Rightarrow \dfrac{1}{y} = \sin^2\theta$

$x = 1 - \tan^2\theta + \tan^4\theta - \cdots$

$\Rightarrow x = \dfrac{1}{1 - (-\tan^2\theta)} = \cos^2\theta$

$\therefore x + \dfrac{1}{y} = 1 \Rightarrow y(1 - x) = 1$

16. If $\frac{dy}{dx} = \frac{xy}{x^2+y^2}$; $y(1) = 1$; then a value of x satisfying $y(x) = e$ is:

a. $\sqrt{3}e$

b. $\frac{1}{2}\sqrt{3}e$

c. $\sqrt{2}e$

d. $\frac{e}{\sqrt{2}}$

Answer: (a)

Solution:

Let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{vx^2}{x^2(1 + v^2)} = \frac{v}{1 + v^2}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-v^3}{1 + v^2}$$

$$\Rightarrow \frac{1}{x}dx = \left(-\frac{1}{v^3} - \frac{1}{v}\right)dv$$

$$\Rightarrow \log x = \frac{1}{2v^2} - \log v + \log c$$

$$\Rightarrow \log x = \frac{x^2}{2y^2} - \log y + \log x + \log c$$

$$\log c + \frac{x^2}{2y^2} - \log y = 0$$

$$y(1) = 1 \Rightarrow \log c + \frac{1}{2} - 0 = 0$$

$$\log c = -\frac{1}{2}$$

$$y(x) = e$$

$$\Rightarrow -\frac{1}{2} + \frac{x^2}{2e^2} - 1 = 0$$

$$\Rightarrow \frac{x^2}{e^2} = 3$$

$$\Rightarrow x = \pm\sqrt{3}\text{e}$$

17. If one end of focal chord AB of the parabola $y^2 = 8x$ is at $A\left(\frac{1}{2}, -2\right)$, then the equation of tangent to it at B is

a. $x + 2y + 8 = 0$
b. $2x - y - 24 = 0$
c. $x - 2y + 8 = 0$
d. $2x + y - 24 = 0$

Answer: (c)

Solution:

Let PQ be the focal chord of the parabola $y^2 = 8x$

$$\Rightarrow P(t_1) = (2t_1^2, 4t_1) \ \& \ Q(t_2) = (2t_2^2, 4t_2)$$

$$\Rightarrow t_1 t_2 = -1$$

$\because \left(\frac{1}{2}, -2\right)$ is one of the ends of the focal chord of the parabola

Let $\left(\frac{1}{2}, -2\right) = (2t_2^2, 4t_2)$

$$\Rightarrow t_2 = -\frac{1}{2}$$

$\Rightarrow$ Other end of focal chord will have parameter $t_1 = 2$

$\Rightarrow$ The co-ordinate of the other end of the focal chord will be $(8, 8)$

$\therefore$ The equation of the tangent will be given as $\rightarrow\ 8y = 4(x+8)$

$\Rightarrow 2y - x = 8$

18. Let a_n be the n^{th} term of a G.P. of positive terms. If $\sum_{n=1}^{100} a_{2n+1} = 200$ and $\sum_{n=1}^{100} a_{2n} = 100$ then $\sum_{n=1}^{200} a_n$ is equal to:

a. 300
b. 175
c. 225
d. 150

Answer: (d)

Solution:

a_n is a positive term of GP.

Let GP be $a, ar, ar^2, \ldots..$

$\sum_{n=1}^{100} a_{2n+1} = a_3 + a_5 + \ldots\ldots. + a_{201}$

$200 = ar^2 + ar^4 + \ldots\ldots. + ar^{201} \Rightarrow 200 = \frac{ar^2(r^{200}-1)}{r^2-1}$ ... (1)

Also, $\sum_{n=1}^{100} a_{2n} = 100$

$100 = a_2 + a_4 + \ldots\ \ldots.. + a_{200} \Rightarrow\ 100 = ar + ar^3 + \ldots\ldots.. + ar^{199}$

$100 = \frac{ar(r^{200}-1)}{r^2-1}$ ... (2)

From (1) and (2), $r = 2$

And $\sum_{n=1}^{100} a_{2n+1} + \sum_{n=1}^{100} a_{2n} = 300$

$\Rightarrow\ a_2 + a_3 + a_4 \ldots\ \ldots.. + a_{200} + a_{201} = 300$

$\Rightarrow\ ar + ar^2 + ar^3 + \ldots\ldots. + ar^{200} = 300 \Rightarrow r(a + ar + ar^2 + \ldots.. + ar^{199}) = 300$

$\Rightarrow 2(a_1 + a_2 + a_3 + \ldots\ldots + a_{200}) = 300$

$\sum_{n=1}^{200} a_n = 150$

19. A random variable X has the following probability distribution:

X	1	2	3	4	5
P(X)	K^2	$2K$	K	$2K$	$5K^2$

Then $P(X > 2)$ is equal to:

a. $\frac{7}{12}$
b. $\frac{23}{36}$
c. $\frac{1}{36}$
d. $\frac{1}{6}$

Answer: (b)

Solution:

We know that $\sum_{X=1}^{5} P(X) = 1$

$\Rightarrow K^2 + 2K + K + 2K + 5K^2 = 1$

$\Rightarrow K = -1, \frac{1}{6} \Rightarrow K = \frac{1}{6}$

$P(X > 2) = P(X = 3) + P(X = 4) + P(X = 5)$

$= K + 2K + 5K^2 = \frac{23}{36}$

20. If $\int \frac{d\theta}{\cos^2\theta \ (\tan 2\theta + \sec 2\theta)} = \lambda \tan\theta + 2\log_e|f(\theta)| + C$ where C is constant if integration, then the ordered pair $(\lambda,\ f(\theta))$ is equal to:

a. $(-1, 1 - \tan\theta)$

b. $(-1, 1 + \tan\theta)$

c. $(1, 1 + \tan\theta)$

d. $(1, 1 - \tan\theta)$

Answer: (b)

Solution:

Let $I = \int \frac{d\theta}{\cos^2\theta(\sec 2\theta + \tan 2\theta)}$

$I = \int \frac{\sec^2\theta d\theta}{\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right) + \left(\frac{2\tan\theta}{1-\tan^2\theta}\right)}$

$I = \int \frac{(1-\tan^2\theta)(\sec^2\theta)d\theta}{(1+\tan\theta)^2}$

Let $\tan\theta = k \Rightarrow \sec^2\theta \ d\theta = \mathrm{d}k$

$I = \int \frac{(1-k^2)}{(1+k)^2} dk = \int \frac{(1-k)}{(1+k)} dk$

$I = \left(\frac{2}{1+k} - 1\right) dk$

$I = 2\ln|1 + k| - k + c$

$I = 2\ln|1 + \tan\theta| - \tan\theta + c$

Given $I = \lambda\tan\theta + 2\log f(\theta) + c$

$\therefore \lambda = -1, f(\theta) = |1 + tan\theta|$

21. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{3}, |\vec{b}| = 5, \vec{b}.\vec{c} = 10$ and the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$. If $\vec{a}$ is perpendicular to vector $\vec{b} \times \vec{c}$, then $|\vec{a} \times (\vec{b} \times \vec{c})|$ is equal to ______

Answer : (30)

Solution:

$|\vec{a} \times (\vec{b} \times \vec{c})| = |\vec{a}||\vec{b} \times \vec{c}| \sin\theta$ where θ is the angle between $\vec{a}$ and $\vec{b} \times \vec{c}$

$\theta = \frac{\pi}{2}$ given

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3}|\vec{b} \times \vec{c}| = \sqrt{3}|\vec{b}||\vec{c}| \sin\frac{\pi}{3}$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \sqrt{3} \times 5 \times |\vec{c}| \times \frac{\sqrt{3}}{2}$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = \frac{15}{2}|\vec{c}|$

Now, $|\vec{b}||\vec{c}| \cos\theta = 10$

$5|\vec{c}|\frac{1}{2} = 10$

$|\vec{c}| = 4$

$\Rightarrow |\vec{a} \times (\vec{b} \times \vec{c})| = 30$

22. If $C_r = {}^{25}C_r$ and $C_0 + 5 \cdot C_1 + 9 \cdot C_2 + \cdots + 101 \cdot C_{25} = 2^{25} \cdot k$ then k is equal to________.

Answer: (51)

Solution:

$S = {}^{25}C_0 + 5\,{}^{25}C_1 + 9\,{}^{25}C_2 + \cdots + 97\,{}^{25}C_{24} + 101\,{}^{25}C_{25} = 2^{25}k$ (1)

Reverse and apply property ${}^{n}C_r = {}^{n}C_{n-r}$ in all coefficients

$S = 101\,{}^{25}C_0 + 97\,{}^{25}C_1 + \cdots + 5\,{}^{25}C_{24} + {}^{25}C_{25}$ (2)

Adding (1) and (2), we get

$2S = 102[{}^{25}C_0 + {}^{25}C_1 + \cdots + {}^{25}C_{25}]$

$S = 51 \times 2^{25}$

$\Rightarrow k = 51$

23. If the curves $x^2 - 6x + y^2 + 8 = 0$ and $x^2 - 8y + y^2 + 16 - k = 0, (k > 0)$ touch each other at a point, then the largest value of k is ________.

Answer: (36)

Solution:

Two circles touch each other if $C_1C_2 = |r_1 \pm r_2|$

$\sqrt{k} + 1 = 5$ or $\left|\sqrt{k} - 1\right| = 5$

$\Rightarrow k = 16$ or 36

Maximum value of k is 36

24. The number of terms common to the A.P.'s 3, 7, 11, ... 407 and 2, 9, 16, ... 709 is ______.

Answer: (14)

Solution:

First common term is 23

Common difference $= \text{LCM}(7, 4) = 28$

$23 + (n-1)28 \leq 407$

$n - 1 \leq 13.71$

$n = 14$

25. If the distance between the plane, $23x - 10y - 2z + 48 = 0$ and the plane containing the lines $\frac{x+1}{2} = \frac{y-3}{4} = \frac{z+1}{3}$ and $\frac{x+3}{2} = \frac{y+2}{6} = \frac{z-1}{\lambda}$, $(\lambda \in R)$ is equal to $\frac{k}{\sqrt{633}}$, then k is equal to ______

Answer: (3)

Solution:

We find the point of intersection of the two lines, and the distance of given plane from the two lines is the distance of plane from the point of intersection.

$\therefore (2p-1,\ 4p+3,\ 3p-1) = (2q-3, 6q-2,\ \lambda q+1)$

$p = -\frac{1}{2}$ and $q = \frac{1}{2}$

$\lambda = -7$

Point of intersection is $\left(-2, 1, -\frac{5}{2}\right)$

$\therefore \frac{k}{\sqrt{633}} = \left|\frac{-46-10+5+48}{\sqrt{633}}\right| \Rightarrow k = 3$