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1. For any two sets A and B, prove that: A' – B' = B – A
Solution:
To prove, A' – B' = B – A
Firstly we need to show
A' – B' ⊆ B – A
Let, x ∈ A' – B'
⇒ x ∈ A' and x ∉ B'
⇒ x ∉ A and x ∈ B (since, A ∩ A' = ϕ)
⇒ x ∈ B – A
It is true for all x ∈ A' – B'
∴ A' – B' = B – A
Hence Proved.

2. For any two sets A and B, prove the following:
(i) A ∩ (A' ∪ B) = A ∩ B
(ii) A – (A – B) = A ∩ B
(iii) A ∩ (A ∪ B') = ϕ
(iv) A – B = A Δ (A ∩ B)
Solution:
(i) A ∩ (A' ∪ B) = A ∩ B
Let us consider LHS A ∩ (A' ∪ B)
Expanding
(A ∩ A') ∪ (A ∩ B)
We know, (A ∩ A') =ϕ
⇒ ϕ ∪ (A∩ B)
⇒ (A ∩ B)
∴ LHS = RHS
Hence proved.

(ii) A – (A – B) = A ∩ B
For any sets A and B we have De-Morgan's law
(A ∪ B)' = A' ∩ B', (A ∩ B) ' = A' ∪ B'
Consider LHS
= A – (A–B)
= A ∩ (A–B)'
= A ∩ (A∩B')'

$= A \cap (A' \cup B')')$ (since, $(B')' = B$)
$= A \cap (A' \cup B)$
$= (A \cap A') \cup (A \cap B)$
$= \phi \cup (A \cap B)$ (since, $A \cap A' = \phi$)
$= (A \cap B)$ (since, $\phi \cup x = x$, for any set)
= RHS
∴ LHS=RHS
Hence proved.

(iii) $A \cap (A \cup B') = \phi$
Let us consider LHS $A \cap (A \cup B')$
$= A \cap (A \cup B')$
$= A \cap (A' \cap B')$ (By De–Morgan's law)
$= (A \cap A') \cap B'$ (since, $A \cap A' = \phi$)
$= \phi \cap B'$
$= \phi$ (since, $\phi \cap B' = \phi$)
= RHS
∴ LHS=RHS
Hence proved.

(iv) $A – B = A \Delta (A \cap B)$
Let us consider RHS $A \Delta (A \cap B)$
$A \Delta (A \cap B)$ (since, $E \Delta F = (E–F) \cup (F–E)$)
$= (A – (A \cap B)) \cup (A \cap B –A)$ (since, $E – F = E \cap F'$)
$= (A \cap (A \cap B)') \cup (A \cap B \cap A')$
$= (A \cap (A' \cup B')) \cup (A \cap A' \cap B)$ (by using De-Morgan's law and associative law)
$= (A \cap A') \cup (A \cap B') \cup (\phi \cap B)$ (by using distributive law)
$= \phi \cup (A \cap B') \cup \phi$
$= A \cap B'$ (since, $A \cap B' = A–B$)
$= A – B$
= LHS
∴ LHS=RHS
Hence Proved

3. If A, B, C are three sets such that $A \subset B$, then prove that $C – B \subset C – A$.
Solution:
Given, ACB
To prove: $C – B \subset C – A$
Let us consider, $x \in C–B$

⇒ $x \in C$ and $x \notin B$
⇒ $x \in C$ and $x \notin A$
⇒ $x \in C - A$
Thus, $x \in C–B \Rightarrow x \in C - A$
This is true for all $x \in C–B$
∴ $C - B \subset C - A$
Hence proved.

4. For any two sets A and B, prove that
(i) $(A \cup B) - B = A - B$
(ii) $A - (A \cap B) = A - B$
(iii) $A - (A - B) = A \cap B$
(iv) $A \cup (B - A) = A \cup B$
(v) $(A - B) \cup (A \cap B) = A$
Solution:
(i) $(A \cup B) - B = A - B$
Let us consider LHS $(A \cup B) - B$
$= (A–B) \cup (B–B)$
$= (A–B) \cup \phi$ (since, $B–B = \phi$)
$= A–B$ (since, $x \cup \phi = x$ for any set)
= RHS
Hence proved.

(ii) $A - (A \cap B) = A - B$
Let us consider LHS $A - (A \cap B)$
$= (A–A) \cap (A–B)$
$= \phi \cap (A - B)$ (since, $A\text{-}A = \phi$)
$= A - B$
= RHS
Hence proved.

(iii) $A - (A - B) = A \cap B$
Let us consider LHS $A - (A - B)$
Let, $x \in A - (A–B) = x \in A$ and $x \notin (A–B)$
$x \in A$ and $x \notin (A \cap B)$
$= x \in A \cap (A \cap B)$
$= x \in (A \cap B)$
$= (A \cap B)$
= RHS

Hence proved.

(iv) $A \cup (B - A) = A \cup B$
Let us consider LHS $A \cup (B - A)$
Let, $x \in A \cup (B - A) \Rightarrow x \in A$ or $x \in (B - A)$
$\Rightarrow x \in A$ or $x \in B$ and $x \notin A$
$\Rightarrow x \in B$
$\Rightarrow x \in (A \cup B)$ (since, $B \subset (A \cup B)$)
This is true for all $x \in A \cup (B–A)$
$\therefore A \cup (B–A) \subset (A \cup B)$…… (1)

Conversely,
Let $x \in (A \cup B) \Rightarrow x \in A$ or $x \in B$
$\Rightarrow x \in A$ or $x \in (B–A)$ (since, $B \subset (A \cup B)$)
$\Rightarrow x \in A \cup (B–A)$
$\therefore (A \cup B) \subset A \cup (B–A)$…… (2)
From 1 and 2 we get,
$A \cup (B - A) = A \cup B$
Hence proved.

(v) $(A - B) \cup (A \cap B) = A$
Let us consider LHS $(A - B) \cup (A \cap B)$
Let, $x \in A$
Then either $x \in (A–B)$ or $x \in (A \cap B)$
$\Rightarrow x \in (A–B) \cup (A \cap B)$
$\therefore A \subset (A - B) \cup (A \cap B)$…. (1)

Conversely,
Let $x \in (A–B) \cup (A \cap B)$
$\Rightarrow x \in (A–B)$ or $x \in (A \cap B)$
$\Rightarrow x \in A$ and $x \notin B$ or $x \in B$
$\Rightarrow x \in A$
$(A–B) \cup (A \cap B) \subset A$………. (2)
$\therefore$ From (1) and (2), We get
$(A–B) \cup (A \cap B) = A$
Hence proved.