EXERCISE 10.2 PAGE NO: 10.25

In any ΔABC, prove the following:

1. In a ΔABC, if a = 5, b = 6 and C = 60°, show that its area is (15√3)/2 sq. units.
Solution:
Given:
In a ΔABC, a = 5, b = 6 and C = 60°
By using the formula,
Area of ΔABC = 1/2 ab sin θ where, a and b are the lengths of the sides of a triangle and θ is the angle between sides.
So,
Area of ΔABC = 1/2 ab sin θ
= 1/2 × 5 × 6 × sin 60°
= 30/2 × √3/2
= (15√3)/2 sq. units
Hence proved.

2. In a ΔABC, if a = √2, b = √3 and c = √5 show that its area is1/2 √6 sq. units.
Solution:
Given:
In a ΔABC, a = √2, b = √3 and c = √5
By using the formulas,
We know, cos A = $(b^2 + c^2 – a^2)/2bc$
By substituting the values we get,
= $[(\sqrt{3})^2 + (\sqrt{5})^2 – (\sqrt{2})^2] / [2 \times \sqrt{3} \times \sqrt{5}]$
= 3/√15

We know, Area of ΔABC = 1/2 bc sin A
To find sin A:
Sin A = $\sqrt{(1 – \cos^2 A)}$ [by using trigonometric identity]
= $\sqrt{(1 – (3/\sqrt{15})^2)}$
= √(1- (9/15))
= √(6/15)
Now,
Area of ΔABC = 1/2 bc sin A
= 1/2 × √3 × √5 × √(6/15)
= 1/2 √6 sq. units
Hence proved.

3. The sides of a triangle are a = 4, b = 6 and c = 8, show that: 8 cos A + 16 cos B + 4 cos C = 17.

Solution:

Given:

Sides of a triangle are a = 4, b = 6 and c = 8

By using the formulas,

Cos A = $(b^2 + c^2 – a^2)/2bc$

Cos B = $(a^2 + c^2 – b^2)/2ac$

Cos C = $(a^2 + b^2 – c^2)/2ab$

So now let us substitute the values of a, b and c we get,

Cos A = $(b^2 + c^2 – a^2)/2bc$

$= (6^2 + 8^2 - 4^2)/2×6×8$

$= (36 + 64 - 16)/96$

$= 84/96$

$= 7/8$

Cos B = $(a^2 + c^2 – b^2)/2ac$

$= (4^2 + 8^2 - 6^2)/2×4×8$

$= (16 + 64 - 36)/64$

$= 44/64$

Cos C = $(a^2 + b^2 – c^2)/2ab$

$= (4^2 + 6^2 - 8^2)/2×4×6$

$= (16 + 36 - 64)/48$

$= -12/48$

$= -1/4$

Now considering LHS:

8 cos A + 16 cos B + 4 cos C = 8 × 7/8 + 16 × 44/64 + 4 × (-1/4)

= 7 + 11 – 1

= 17

Hence proved.

4. In a ΔABC, if a = 18, b = 24, c = 30, find cos A, cos B and cos C

Solution:

Given:

Sides of a triangle are a = 18, b = 24 and c = 30

By using the formulas,

Cos A = $(b^2 + c^2 – a^2)/2bc$

Cos B = $(a^2 + c^2 – b^2)/2ac$
Cos C = $(a^2 + b^2 – c^2)/2ab$
So now let us substitute the values of a, b and c we get,
Cos A = $(b^2 + c^2 – a^2)/2bc$
= $(24^2 + 30^2 - 18^2)/2×24×30$
= 1152/1440
= 4/5

Cos B = $(a^2 + c^2 – b^2)/2ac$
= $(18^2 + 30^2 - 24^2)/2×18×30$
= 648/1080
= 3/5

Cos C = $(a^2 + b^2 – c^2)/2ab$
= $(18^2 + 24^2 - 30^2)/2×18×24$
= 0/864
= 0
∴ cos A = 4/5, cos B = 3/5, cos C = 0

5. For any ΔABC, show that b (c cos A – a cos C) = $c^2 – a^2$
Solution:
Let us consider LHS:
b (c cos A – a cos C)
As LHS contain bc cos A and ab cos C which can be obtained from cosine formulae.
From cosine formula we have:
Cos A = $(b^2 + c^2 – a^2)/2bc$
bc cos A = $(b^2 + c^2 – a^2)/2$ … (i)

Cos C = $(a^2 + b^2 – c^2)/2ab$
ab cos C = $(a^2 + b^2 – c^2)/2$ … (ii)
Now let us subtract equation (i) and (ii) we get,
bc cos A - ab cos C = $(b^2 + c^2 – a^2)/2 - (a^2 + b^2 – c^2)/2$
= $c^2 – a^2$
∴ b (c cos A – a cos C) = $c^2 – a^2$
Hence proved.

6. For any Δ ABC show that c (a cos B – b cos A) = $a^2 – b^2$
Solution:
Let us consider LHS:

c (a cos B – b cos A)
As LHS contain ca cos B and cb cos A which can be obtained from cosine formulae.
From cosine formula we have:
Cos A = $(b^2 + c^2 – a^2)/2bc$
bc cos A = $(b^2 + c^2 – a^2)/2$ … (i)

Cos B = $(a^2 + c^2 – b^2)/2ac$
ac cos B = $(a^2 + c^2 – b^2)/2$ … (ii)
Now let us subtract equation (ii) from (i) we get,
ac cos B - bc cos A = $(a^2 + c^2 – b^2)/2 - (b^2 + c^2 – a^2)/2$
= $a^2 – b^2$
∴ c (a cos B – b cos A) = $a^2 – b^2$
Hence proved.

7. For any Δ ABC show that
2 (bc cos A + ca cos B + ab cos C) = $a^2 + b^2 + c^2$
Solution:
Let us consider LHS:
2 (bc cos A + ca cos B + ab cos C)
As LHS contain 2ca cos B, 2ab cos C and 2cb cos A, which can be obtained from cosine formulae.
From cosine formula we have:
Cos A = $(b^2 + c^2 – a^2)/2bc$
2bc cos A = $(b^2 + c^2 – a^2)$ … (i)

Cos B = $(a^2 + c^2 – b^2)/2ac$
2ac cos B = $(a^2 + c^2 – b^2)$… (ii)

Cos C = $(a^2 + b^2 – c^2)/2ab$
2ab cos C = $(a^2 + b^2 – c^2)$ … (iii)
Now let us add equation (i), (ii) and (ii) we get,
2bc cos A + 2ac cos B + 2ab cos C = $(b^2 + c^2 – a^2) + (a^2 + c^2 – b^2) + (a^2 + b^2 – c^2)$
Upon simplification we get,
= $c^2 + b^2 + a^2$
2 (bc cos A + ac cos B + ab cos C) = $a^2 + b^2 + c^2$
Hence proved.

8. For any Δ ABC show that
$(c^2 – a^2 + b^2)$ tan A = $(a^2 – b^2 + c^2)$ tan B = $(b^2 – c^2 + a^2)$ tan C

Solution:
Let us consider LHS:
$(c^2 – a^2 + b^2)$, $(a^2 – b^2 + c^2)$, $(b^2 – c^2 + a^2)$
We know sine rule in Δ ABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

As LHS contain $(c^2 – a^2 + b^2)$, $(a^2 – b^2 + c^2)$ and $(b^2 – c^2 + a^2)$, which can be obtained from cosine formulae.
From cosine formula we have:
Cos A = $(b^2 + c^2 – a^2)/2bc$
2bc cos A = $(b^2 + c^2 – a^2)$
Let us multiply both the sides by tan A we get,
2bc cos A tan A = $(b^2 + c^2 – a^2)$ tan A
2bc cos A (sin A/cos A) = $(b^2 + c^2 – a^2)$ tan A
2bc sin A = $(b^2 + c^2 – a^2)$ tan A … (i)

Cos B = $(a^2 + c^2 – b^2)/2ac$
2ac cos B = $(a^2 + c^2 – b^2)$
Let us multiply both the sides by tan B we get,
2ac cos B tan B = $(a^2 + c^2 – b^2)$ tan B
2ac cos B (sin B/cos B) = $(a^2 + c^2 – b^2)$ tan B
2ac sin B = $(a^2 + c^2 – b^2)$ tan B … (ii)

Cos C = $(a^2 + b^2 – c^2)/2ab$
2ab cos C = $(a^2 + b^2 – c^2)$
Let us multiply both the sides by tan C we get,
2ab cos C tan C = $(a^2 + b^2 – c^2)$ tan C
2ab cos C (sin C/cos C) = $(a^2 + b^2 – c^2)$ tan C
2ab sin C = $(a^2 + b^2 – c^2)$ tan C … (iii)
As we are observing that sin terms are being involved so let's use sine formula.
From sine formula we have,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \Rightarrow \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Let us multiply abc to each of the expression we get,

$$\frac{abc \sin A}{a} = \frac{abc \sin B}{b} = \frac{abc \sin C}{c}$$

bc sin A = ac sin B = ab sin C
2bc sin A = 2ac sin B = 2ab sin C

∴ From equation (i), (ii) and (iii) we have,
$(c^2 – a^2 + b^2) \tan A = (a^2 – b^2 + c^2) \tan B = (b^2 – c^2 + a^2) \tan C$
Hence proved.

9. For any Δ ABC show that:

$$\frac{c - b\cos A}{b - c\cos A} = \frac{\cos B}{\cos C}$$

Solution:

Let us consider LHS:

$$\frac{c - b\cos A}{b - c\cos A}$$

We can observe that we can get terms c – b cos A and b – c cos A from projection formula

From projection formula we get,

c = a cos B + b cos A

c – b cos A = a cos B …. (i)

And,

b = c cos A + a cos C

b – c cos A = a cos C …. (ii)

Dividing equation (i) by (ii), we get,

$$\frac{c - b\cos A}{b - c\cos A} = \frac{a\cos B}{a\cos C}$$

$$= \frac{\cos B}{\cos C}$$

= RHS

Hence proved.