EXERCISE 11.1 PAGE NO: 11.21

1. Find the general solutions of the following equations:
(i) sin x = 1/2
(ii) cos x = - √3/2
(iii) cosex x = - √2
(iv) sec x = √2
(v) tan x = -1/√3
(vi) √3 sec x = 2
Solution:
The general solution of any trigonometric equation is given as:
sin x = sin y, implies $x = n\pi + (-1)^n y$, where n ∈ Z.
cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.
tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) sin x = 1/2
We know sin 30^o = sin π/6 = ½
So,
Sin x = sin π/6
∴ the general solution is
$x = n\pi + (-1)^n \pi/6$, where n ∈ Z. [since, sin x = sin A => $x = n\pi + (-1)^n A$]

(ii) cos x = - √3/2
We know, cos 150^o = (- √3/2) = cos 5π/6
So,
Cos x = cos 5π/6
∴ the general solution is
x = 2nπ ± 5π/6, where n ϵ Z.

(iii) cosex x = - √2
Let us simplify,
1/sin x = - √2 [since, cosec x = 1/sin x]
Sin x = -1/√2
= sin [π + π/4]
= sin 5π/4 or sin (-π/4)
∴ the general solution is
$x = n\pi + (-1)^{n+1} \pi/4$, where n ϵ Z.

(iv) sec x = √2

Let us simplify,
1/cos x = √2 [since, sec x = 1/cos x]
Cos x = 1/√2
= cos π/4
∴ the general solution is
x = 2nπ ± π/4, where n ϵ Z.

(v) tan x = -1/√3
Let us simplify,
tan x = -1/√3
tan x = tan (π/6)
= tan (-π/6) [since, tan (-x) = -tan x]
∴ the general solution is
x = nπ + (-π/6), where n ϵ Z.
or x = nπ - π/6, where n ϵ Z.

(vi) √3 sec x = 2
Let us simplify,
sec x = 2/√3
1/cos x = 2/√3
Cos x = √3/2
= cos (π/6)
∴ the general solution is
x = 2nπ ± π/6, where n ϵ Z.

2. Find the general solutions of the following equations:
(i) sin 2x = √3/2
(ii) cos 3x = 1/2
(iii) sin 9x = sin x
(iv) sin 2x = cos 3x
(v) tan x + cot 2x = 0
(vi) tan 3x = cot x
(vii) tan 2x tan x = 1
(viii) tan mx + cot nx = 0
(ix) tan px = cot qx
(x) sin 2x + cos x = 0
(xi) sin x = tan x
(xii) sin 3x + cos 2x = 0
Solution:

The general solution of any trigonometric equation is given as:
sin x = sin y, implies $x = n\pi + (-1)^n y$, where $n \in Z$.
cos x = cos y, implies $x = 2n\pi \pm y$, where $n \in Z$.
tan x = tan y, implies $x = n\pi + y$, where $n \in Z$.

(i) sin 2x = √3/2
Let us simplify,
sin 2x = √3/2
= sin (π/3)
∴ the general solution is
$2x = n\pi + (-1)^n \pi/3$, where n ϵ Z.
$x = n\pi/2 + (-1)^n \pi/6$, where n ϵ Z.

(ii) cos 3x = 1/2
Let us simplify,
cos 3x = 1/2
= cos (π/3)
∴ the general solution is
3x = 2nπ ± π/3, where n ϵ Z.
x = 2nπ/3 ± π/9, where n ϵ Z.

(iii) sin 9x = sin x
Let us simplify,
Sin 9x – sin x = 0
Using transformation formula,
Sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2
So,
= 2 cos (9x+x)/2 sin (9x-x)/2
=> cos 5x sin 4x = 0
Cos 5x = 0 or sin 4x = 0
Let us verify both the expressions,
Cos 5x = 0
Cos 5x = cos π/2
5x = (2n + 1)π/2
x = (2n + 1)π/10, where n ϵ Z.

sin 4x = 0
sin 4x = sin 0
4x = nπ

$x = n\pi/4$, where $n \in Z$.
∴ the general solution is
$x = (2n + 1)\pi/10$ or $n\pi/4$, where $n \in Z$.

(iv) $\sin 2x = \cos 3x$
Let us simplify,
$\sin 2x = \cos 3x$
$\cos (\pi/2 – 2x) = \cos 3x$ [since, $\sin A = \cos (\pi/2 – A)$]
$\pi/2 – 2x = 2n\pi \pm 3x$
$\pi/2 – 2x = 2n\pi + 3x$ [or] $\pi/2 – 2x = 2n\pi - 3x$
$5x = \pi/2 + 2n\pi$ [or] $x = 2n\pi - \pi/2$
$5x = \pi/2 (1 + 4n)$ [or] $x = \pi/2 (4n - 1)$
$x = \pi/10 (1 + 4n)$ [or] $x = \pi/2 (4n - 1)$
∴ the general solution is
$x = \pi/10 (4n + 1)$ [or] $x = \pi/2 (4n - 1)$, where $n \in Z$.

(v) $\tan x + \cot 2x = 0$
Let us simplify,
$\tan x = - \cot 2x$
$\tan x = - \tan (\pi/2 – 2x)$ [since, $\cot A = \tan (\pi/2 - A)$]
$\tan x = \tan (2x - \pi/2)$ [since, $- \tan A = \tan -A$]
$x = n\pi + 2x - \pi/2$
$x = n\pi - \pi/2$
∴ the general solution is
$x = n\pi - \pi/2$, where $n \in Z$.

(vi) $\tan 3x = \cot x$
Let us simplify,
$\tan 3x = \cot x$
$\tan 3x = \tan (\pi/2 - x)$ [since, $\cot A = \tan (\pi/2 - A)$]
$3x = n\pi + \pi/2 – x$
$4x = n\pi + \pi/2$
$x = n\pi/4 + \pi/8$
∴ the general solution is
$x = n\pi/4 + \pi/8$, where $n \in Z$.

(vii) $\tan 2x \tan x = 1$
Let us simplify,
$\tan 2x \tan x = 1$

tan 2x = 1/tan x
= cot x
tan 2x = tan (π/2 - x) [since, cot A = tan (π/2 - A)]
2x = nπ + π/2 – x
3x = nπ + π/2
x = nπ/3 + π/6
∴ the general solution is
x = nπ/3 + π/6, where n ϵ Z.

(viii) tan mx + cot nx = 0
Let us simplify,
tan mx + cot nx = 0
tan mx = - cot nx
= - tan (π/2 - nx) [since, cot A = tan (π/2 - A)]
tan mx = tan (nx + π/2) [since, - tan A = tan -A]
mx = kπ + nx + π/2
(m - n) x = kπ + π/2
(m - n) x = π (2k + 1)/2
x = π (2k + 1)/2(m - n)
∴ the general solution is
x = π (2k + 1)/2(m - n), where m, n, k ϵ Z.

(ix) tan px = cot qx
Let us simplify,
tan px = cot qx
tan px = tan (π/2 - qx) [since, cot A = tan (π/2 - A)]
px = nπ ± (π/2 – qx)
(p + q) x = nπ + π/2
x = nπ/(p+q) + π/2(p+q)
= π (2n +1)/ 2(p+q)
∴ the general solution is
x = π (2n +1)/ 2(p+q), where n ϵ Z.

(x) sin 2x + cos x = 0
Let us simplify,
sin 2x + cos x = 0
cos x = - sin 2x
cos x = - cos (π/2 – 2x) [since, sin A = cos (π/2 - A)]
= cos (π – (π/2 – 2x)) [since, -cos A = cos (π - A)]

= cos (π/2 + 2x)
x = 2nπ ± (π/2 + 2x)
So,
x = 2nπ + (π/2 + 2x) [or] x = 2nπ - (π/2 + 2x)
x = - π/2 - 2nπ [or] 3x = 2nπ - π/2
x = - π/2 (1 + 4n) [or] x = π/6 (4n - 1)
∴ the general solution is
x = - π/2 (1 + 4n), where n ϵ Z. [or] x = π/6 (4n - 1)
x = π/2 (4n - 1), where n ϵ Z. [or] x = π/6 (4n - 1), where n ϵ Z.

(xi) sin x = tan x
Let us simplify,
sin x = tan x
sin x = sin x/cos x
sin x cos x = sin x
sin x (cos x - 1) = 0
So,
Sin x = 0 or cos x – 1 = 0
Sin x = sin 0 [or] cos x = 1
Sin x = sin 0 [or] cos x = cos 0
x = nπ [or] x = 2mπ
∴ the general solution is
x = nπ [or] 2mπ, where n, m ϵ Z.

(xii) sin 3x + cos 2x = 0
Let us simplify,
sin 3x + cos 2x = 0
cos 2x = - sin 3x
cos 2x = - cos (π/2 – 3x) [since, sin A = cos (π/2 - A)]
cos 2x = cos (π – (π/2 – 3x)) [since, -cos A = cos (π - A)]
cos 2x = cos (π/2 + 3x)
2x = 2nπ ± (π/2 + 3x)
So,
2x = 2nπ + (π/2 + 3x) [or] 2x = 2nπ - (π/2 + 3x)
x = -π/2 – 2nπ [or] 5x = 2nπ – π/2
x = -π/2 (1 + 4n) [or] x = π/10 (4n - 1)
x = - π/2 (4n + 1) [or] π/10 (4n - 1)
∴ the general solution is
x = - π/2 (4n + 1) [or] π/10 (4n - 1)

x = π/2 (4n - 1) [or] π/10 (4n - 1), where n ϵ Z.

3. Solve the following equations:
(i) $\sin^2 x – \cos x = 1/4$
(ii) $2 \cos^2 x – 5 \cos x + 2 = 0$
(iii) $2 \sin^2 x + \sqrt{3} \cos x + 1 = 0$
(iv) $4 \sin^2 x – 8 \cos x + 1 = 0$
(v) $\tan^2 x + (1 - \sqrt{3}) \tan x - \sqrt{3} = 0$
(vi) $3 \cos^2 x - 2\sqrt{3} \sin x \cos x – 3 \sin^2 x = 0$
(vii) cos 4x = cos 2x
Solution:
The general solution of any trigonometric equation is given as:
sin x = sin y, implies $x = n\pi + (– 1)^n y$, where n ∈ Z.
cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.
tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) $\sin^2 x – \cos x = 1/4$
Let us simplify,
$\sin^2 x – \cos x = ¼$
$1 – \cos^2 x – \cos x = 1/4$ [since, $\sin^2 x = 1 – \cos^2 x$]
$4 – 4 \cos^2 x – 4 \cos x = 1$
$4\cos^2 x + 4\cos x – 3 = 0$
Let cos x be ‘k’
So,
$4k^2 + 4k – 3 = 0$
$4k^2 – 2k + 6k – 3 = 0$
2k (2k - 1) + 3 (2k - 1) = 0
(2k - 1) + (2k + 3) = 0
(2k - 1) = 0 or (2k + 3) = 0
k = 1/2 or k = -3/2
cos x = 1/2 or cos x = -3/2
we shall consider only cos x = 1/2. cos x = -3/2 is not possible.
so,
$\cos x = \cos 60^o = \cos \pi/3$
x = 2nπ ± π/3
∴ the general solution is
x = 2nπ ± π/3, where n ϵ Z.

(ii) $2 \cos^2 x – 5 \cos x + 2 = 0$

Let us simplify,
$2\cos^2 x - 5\cos x + 2 = 0$
Let cos x be ‘k’
$2k^2 - 5k + 2 = 0$
$2k^2 - 4k - k + 2 = 0$
$2k(k - 2) - 1(k - 2) = 0$
$(k - 2)(2k - 1) = 0$
k = 2 or k = 1/2
cos x = 2 or cos x = 1/2
we shall consider only cos x = 1/2. cos x = 2 is not possible.
so,
$\cos x = \cos 60^\circ = \cos \pi/3$
$x = 2n\pi \pm \pi/3$
∴ the general solution is
$x = 2n\pi \pm \pi/3$, where n ϵ Z.

(iii) $2\sin^2 x + \sqrt{3}\cos x + 1 = 0$
Let us simplify,
$2\sin^2 x + \sqrt{3}\cos x + 1 = 0$
$2(1 - \cos^2 x) + \sqrt{3}\cos x + 1 = 0$ [since, $\sin^2 x = 1 - \cos^2 x$]
$2 - 2\cos^2 x + \sqrt{3}\cos x + 1 = 0$
$2\cos^2 x - \sqrt{3}\cos x - 3 = 0$
Let cos x be ‘k’
$2k^2 - \sqrt{3}k - 3 = 0$
$2k^2 - 2\sqrt{3}k + \sqrt{3}k - 3 = 0$
$2k(k - \sqrt{3}) + \sqrt{3}(k - \sqrt{3}) = 0$
$(2k + \sqrt{3})(k - \sqrt{3}) = 0$
$k = \sqrt{3}$ or $k = -\sqrt{3}/2$
$\cos x = \sqrt{3}$ or $\cos x = -\sqrt{3}/2$
we shall consider only $\cos x = -\sqrt{3}/2$. $\cos x = \sqrt{3}$ is not possible.
so,
$\cos x = -\sqrt{3}/2$
$\cos x = \cos 150^\circ = \cos 5\pi/6$
$x = 2n\pi \pm 5\pi/6$, where n ϵ Z.

(iv) $4\sin^2 x - 8\cos x + 1 = 0$
Let us simplify,
$4\sin^2 x - 8\cos x + 1 = 0$
$4(1 - \cos^2 x) - 8\cos x + 1 = 0$ [since, $\sin^2 x = 1 - \cos^2 x$]

$4 - 4\cos^2 x - 8\cos x + 1 = 0$
$4\cos^2 x + 8\cos x - 5 = 0$
Let cos x be 'k'
$4k^2 + 8k - 5 = 0$
$4k^2 - 2k + 10k - 5 = 0$
$2k(2k - 1) + 5(2k - 1) = 0$
$(2k + 5)(2k - 1) = 0$
$k = -5/2 = -2.5$ or $k = 1/2$
$\cos x = -2.5$ or $\cos x = 1/2$
we shall consider only $\cos x = 1/2$. $\cos x = -2.5$ is not possible.
so,
$\cos x = \cos 60^\circ = \cos \pi/3$
$x = 2n\pi \pm \pi/3$
∴ the general solution is
$x = 2n\pi \pm \pi/3$, where $n \in Z$.

(v) $\tan^2 x + (1 - \sqrt{3})\tan x - \sqrt{3} = 0$
Let us simplify,
$\tan^2 x + (1 - \sqrt{3})\tan x - \sqrt{3} = 0$
$\tan^2 x + \tan x - \sqrt{3}\tan x - \sqrt{3} = 0$
$\tan x(\tan x + 1) - \sqrt{3}(\tan x + 1) = 0$
$(\tan x + 1)(\tan x - \sqrt{3}) = 0$
$\tan x = -1$ or $\tan x = \sqrt{3}$
As, $\tan x \in (-\infty, \infty)$ so both values are valid and acceptable.
$\tan x = \tan(-\pi/4)$ or $\tan x = \tan(\pi/3)$
$x = m\pi - \pi/4$ or $x = n\pi + \pi/3$
∴ the general solution is
$x = m\pi - \pi/4$ or $n\pi + \pi/3$, where $m, n \in Z$.

(vi) $3\cos^2 x - 2\sqrt{3}\sin x\cos x - 3\sin^2 x = 0$
Let us simplify,
$3\cos^2 x - 2\sqrt{3}\sin x\cos x - 3\sin^2 x = 0$
$3\cos^2 x - 3\sqrt{3}\sin x\cos x + \sqrt{3}\sin x\cos x - 3\sin^2 x = 0$
$3\cos x(\cos x - \sqrt{3}\sin x) + \sqrt{3}\sin x(\cos x - \sqrt{3}\sin x) = 0$
$\sqrt{3}(\cos x - \sqrt{3}\sin x)(\sqrt{3}\cos x + \sin x) = 0$
$\cos x - \sqrt{3}\sin x = 0$ or $\sin x + \sqrt{3}\cos x = 0$
$\cos x = \sqrt{3}\sin x$ or $\sin x = -\sqrt{3}\cos x$
$\tan x = 1/\sqrt{3}$ or $\tan x = -\sqrt{3}$
As, $\tan x \in (-\infty, \infty)$ so both values are valid and acceptable.

tan x = tan (π/6) or tan x = tan (-π/3)
x = mπ + π/6 or x = nπ – π/3
∴ the general solution is
x = mπ + π/6 or nπ – π/3, where m, n ϵ Z.

(vii) cos 4x = cos 2x
Let us simplify,
cos 4x = cos 2x
4x = 2nπ ± 2x
So,
4x = 2nπ + 2x [or] 4x = 2nπ - 2x
2x = 2nπ [or] 6x = 2nπ
x = nπ [or] x = nπ/3
∴ the general solution is
x = nπ [or] nπ/3, where n ϵ Z.

4. Solve the following equations:
(i) cos x + cos 2x + cos 3x = 0
(ii) cos x + cos 3x – cos 2x = 0
(iii) sin x + sin 5x = sin 3x
(iv) cos x cos 2x cos 3x = 1/4
(v) cos x + sin x = cos 2x + sin 2x
(vi) sin x + sin 2x + sin 3x = 0
(vii) sin x + sin 2x + sin 3x + sin 4x = 0
(viii) sin 3x – sin x = 4 $\cos^2$ x - 2
(ix) sin 2x – sin 4x + sin 6x = 0
Solution:
The general solution of any trigonometric equation is given as:
sin x = sin y, implies x = nπ + $(-1)^n$ y, where n ∈ Z.
cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.
tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) cos x + cos 2x + cos 3x = 0
Let us simplify,
cos x + cos 2x + cos 3x = 0
we shall rearrange and use transformation formula
cos 2x + (cos x + cos 3x) = 0
by using the formula, cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2
cos 2x + 2 cos (3x+x)/2 cos (3x-x)/2 = 0

cos 2x + 2cos 2x cos x = 0
cos 2x (1 + 2 cos x) = 0
cos 2x = 0 or 1 + 2cos x = 0
cos 2x = cos 0 or cos x = -1/2
cos 2x = cos π/2 or cos x = cos (π - π/3)
cos 2x = cos π/2 or cos x = cos (2π/3)
2x = (2n + 1) π/2 or x = 2mπ ± 2π/3
x = (2n + 1) π/4 or x = 2mπ ± 2π/3
∴ the general solution is
x = (2n + 1) π/4 or 2mπ ± 2π/3, where m, n ϵ Z.

(ii) cos x + cos 3x – cos 2x = 0
Let us simplify,
cos x + cos 3x – cos 2x = 0
we shall rearrange and use transformation formula
cos x – cos 2x + cos 3x = 0
- cos 2x + (cos x + cos 3x) = 0
By using the formula, cos A + cos B = 2 cos (A+B)/2 cos (A-B)/2
- cos 2x + 2 cos (3x+x)/2 cos (3x-x)/2 = 0
- cos 2x + 2cos 2x cos x = 0
cos 2x (-1 + 2 cos x) = 0
cos 2x = 0 or -1 + 2cos x = 0
cos 2x = cos 0 or cos x = 1/2
cos 2x = cos π/2 or cos x = cos (π/3)
2x = (2n + 1) π/2 or x = 2mπ ± π/3
x = (2n + 1) π/4 or x = 2mπ ± π/3
∴ the general solution is
x = (2n + 1) π/4 or 2mπ ± π/3, where m, n ϵ Z.

(iii) sin x + sin 5x = sin 3x
Let us simplify,
sin x + sin 5x = sin 3x
sin x + sin 5x - sin 3x = 0
we shall rearrange and use transformation formula
- sin 3x + sin x + sin 5x = 0
- sin 3x + (sin 5x + sin x) = 0
By using the formula, sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2
- sin 3x + 2 sin (5x+x)/2 cos (5x-x)/2 = 0
2sin 3x cos 2x – sin 3x = 0

sin 3x (2cos 2x – 1) = 0
sin 3x = 0 or 2cos 2x – 1 = 0
sin 3x = sin 0 or cos 2x = 1/2
sin 3x = sin 0 or cos 2x = cos π/3
3x = nπ or 2x = 2mπ ± π/3
x = nπ/3 or x = mπ ± π/6
∴ the general solution is
x = nπ/3 or mπ ± π/6, where m, n ϵ Z.

(iv) cos x cos 2x cos 3x = 1/4
Let us simplify,
cos x cos 2x cos 3x = 1/4
4 cos x cos 2x cos 3x – 1 = 0
By using the formula,
2 cos A cos B = cos (A + B) + cos (A – B)
2(2cos x cos 3x) cos 2x – 1 = 0
2(cos 4x + cos 2x) cos2x – 1 = 0
2(2cos^2 2x – 1 + cos 2x) cos 2x – 1 = 0 [using cos 2A = 2cos^{2}A – 1]
4cos^3 2x – 2cos 2x + 2cos^2 2x – 1 = 0
2cos^2 2x (2cos 2x + 1) -1(2cos 2x + 1) = 0
(2cos^2 2x – 1) (2 cos 2x + 1) = 0
So,
2cos 2x + 1 = 0 or (2cos^2 2x – 1) = 0
cos 2x = -1/2 or cos 4x = 0 [using cos 2θ = 2cos^{2}θ – 1]
cos 2x = cos (π - π/3) or cos 4x = cos π/2
cos 2x = cos 2π/3 or cos 4x = cos π/2
2x = 2mπ ± 2π/3 or 4x = (2n + 1) π/2
x = mπ ± π/3 or x = (2n + 1) π/8
∴ the general solution is
x = mπ ± π/3 or (2n + 1) π/8, where m, n ϵ Z.

(v) cos x + sin x = cos 2x + sin 2x
Let us simplify,
cos x + sin x = cos 2x + sin 2x
upon rearranging we get,
cos x – cos 2x = sin 2x – sin x
By using the formula,
sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2
cos A – cos B = - 2 sin (A+B)/2 sin (A-B)/2

So,
-2 sin (2x+x)/2 sin (2x-x)/2 = 2 cos (2x+x)/2 sin (2x-x)/2
2 sin 3x/2 sin x/2 = 2 cos 3x/2 sin x/2
Sin x/2 (sin 3x/2 – cos 3x/2) = 0
So,
Sin x/2 = 0 or sin 3x/2 = cos 3x/2
Sin x/2 = sin mπ or sin 3x/2 / cos 3x/2 = 0
Sin x/2 = sin mπ or tan 3x/2 = 1
Sin x/2 = sin mπ or tan 3x/2 = tan π/4
x/2 = mπ or 3x/2 = nπ + π/4
x = 2mπ or x = 2nπ/3 + π/6
∴ the general solution is
x = 2mπ or 2nπ/3 + π/6, where m, n ϵ Z.

(vi) sin x + sin 2x + sin 3x = 0
Let us simplify,
sin x + sin 2x + sin 3x = 0
we shall rearrange and use transformation formula
sin 2x + sin x + sin 3x = 0
By using the formula,
sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2
So,
Sin 2x + 2 sin (3x+x)/2 cos (3x-x)/2 = 0
Sin 2x + 2sin 2x cos x = 0
Sin 2x (2 cos x + 1) = 0
Sin 2x = 0 or 2cos x + 1 = 0
Sin 2x = sin 0 or cos x = -1/2
Sin 2x = sin 0 or cos x = cos (π – π/3)
Sin 2x = sin 0 or cos x = cos 2π/3
2x = nπ or x = 2mπ ± 2π/3
x = nπ/2 or x = 2mπ ± 2π/3
∴ the general solution is
x = nπ/2 or 2mπ ± 2π/3, where m, n ϵ Z.

(vii) sin x + sin 2x + sin 3x + sin 4x = 0
Let us simplify,
sin x + sin 2x + sin 3x + sin 4x = 0
we shall rearrange and use transformation formula
sin x + sin 3x + sin 2x + sin 4x = 0

By using the formula,
sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2
So,
2 sin (3x+x)/2 cos (3x-x)/2 + 2 sin (4x+2x)/2 cos (4x-2x)/2 = 0
2 sin 2x cos x + 2 sin 3x cos x = 0
2cos x (sin 2x + sin 3x) = 0
Again by using the formula,
sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2
we get,
2cos x (2 sin (3x+2x)/2 cos (3x-2x)/2) = 0
2cos x (2 sin 5x/2 cos x/2) = 0
4 cos x sin 5x/2 cos x/2 = 0
So,
Cos x = 0 or sin 5x/2 = 0 or cos x/2 = 0
Cos x = cos 0 or sin 5x/2 = sin 0 or cos x/2 = cos 0
Cos x = cos π/2 or sin 5x/2 = kπ or cos x/2 = cos (2p + 1) π/2
x = (2n + 1) π/2 or 5x/2 = kπ or x/2 = (2p + 1) π/2
x = (2n + 1) π/2 or x = 2kπ/5 or x = (2p + 1)
x = nπ + π/2 or x = 2kπ/5 or x = (2p + 1)
∴ the general solution is
x = nπ + π/2 or x = 2kπ/5 or x = (2p + 1), where n, k, p ϵ Z.

(viii) sin 3x – sin x = 4 $\cos^2$ x - 2
Let us simplify,
sin 3x – sin x = 4 $\cos^2$ x - 2
sin 3x – sin x = 2(2 $\cos^2$ x – 1)
sin 3x – sin x = 2 cos 2x [since, cos 2A = 2$\cos^2$ A – 1]
By using the formula,
Sin A – sin B = 2 cos (A+B)/2 sin (A-B)/2
So,
2 cos (3x+x)/2 sin (3x-x)/2 = 2 cos 2x
2 cos 2x sin x – 2 cos 2x = 0
2 cos 2x (sin x - 1) = 0
Then,
2 cos 2x = 0 or sin x – 1 = 0
Cos 2x = 0 or sin x = 1
Cos 2x = cos 0 or sin x = sin 1
Cos 2x = cos 0 or sin x = sin π/2
2x = (2n + 1) π/2 or x = mπ + $(-1)^m$ π/2

$x = (2n + 1)\pi/4$ or $x = m\pi + (-1)^m \pi/2$
∴ the general solution is
$x = (2n + 1)\pi/4$ or $m\pi + (-1)^m \pi/2$, where m, n ϵ Z.

(ix) sin 2x – sin 4x + sin 6x = 0
Let us simplify,
sin 2x – sin 4x + sin 6x = 0
we shall rearrange and use transformation formula
- sin 4x + sin 6x + sin 2x = 0
By using the formula,
sin A + sin B = 2 sin (A+B)/2 cos (A-B)/2
we get,
- sin 4x + 2 sin (6x+2x)/2 cos (6x-2x)/2 = 0
- sin 4x + 2 sin 4x cos 2x = 0
Sin 4x (2 cos 2x - 1) = 0
So,
Sin 4x = 0 or 2 cos 2x – 1 = 0
Sin 4x = sin 0 or cos 2x = 1/2
Sin 4x = sin 0 or cos 2x = π/3
4x = nπ or 2x = 2mπ ± π/3
x = nπ/4 or x = mπ ± π/6
∴ the general solution is
x = nπ/4 or mπ ± π/6, where m, n ϵ Z.

5. Solve the following equations:
(i) tan x + tan 2x + tan 3x = 0
(ii) tan x + tan 2x = tan 3x
(iii) tan 3x + tan x = 2 tan 2x
Solution:
The general solution of any trigonometric equation is given as:
sin x = sin y, implies $x = n\pi + (-1)^n y$, where n ∈ Z.
cos x = cos y, implies x = 2nπ ± y, where n ∈ Z.
tan x = tan y, implies x = nπ + y, where n ∈ Z.

(i) tan x + tan 2x + tan 3x = 0
Let us simplify,
tan x + tan 2x + tan 3x = 0
tan x + tan 2x + tan (x + 2x) = 0
By using the formula,

tan (A+B) = [tan A + tan B] / [1 – tan A tan B]
So,
tan x + tan 2x + [[tan x + tan 2x]/[1- tan x tan 2x]] = 0
(tan x + tan 2x) (1 + 1/(1- tan x tan 2x)) = 0
(tan x + tan 2x) ([2 – tan x tan 2x] / [1 – tan x tan 2x]) = 0
Then,
(tan x + tan 2x) = 0 or ([2 – tan x tan 2x] / [1 – tan x tan 2x]) = 0
(tan x + tan 2x) = 0 or [2 – tan x tan 2x] = 0
tan x = tan (-2x) or tan x tan 2x = 2
x = nπ + (-2x) or tax x [2tan x/(1 – $\tan^2$ x)] = 2 [Using, tan 2x = 2 tan x / 1-$\tan^2$ x]
3x = nπ or 2 $\tan^2$ x / (1-$\tan^2$ x) = 2
3x = nπ or 2 $\tan^2$ x = 2(1 – $\tan^2$ x)
3x = nπ or 2 $\tan^2$ x = 2 - 2$\tan^2$ x
3x = nπ or 4 $\tan^2$ x = 2
x = nπ/3 or $\tan^2$ x = 2/4
x = nπ/3 or $\tan^2$ x = 1/2
x = nπ/3 or tan x = 1/√2
x = nπ/3 or x = tan α [let 1/√2 be 'α']
x = nπ/3 or x = mπ + α
∴ the general solution is
x = nπ/3 or mπ + α, where α = $\tan^{-1}$1/√2, m, n ∈ Z.

(ii) tan x + tan 2x = tan 3x
Let us simplify,
tan x + tan 2x = tan 3x
tan x + tan 2x – tan 3x = 0
tan x + tan 2x – tan (x + 2x) = 0
By using the formula,
tan (A+B) = [tan A + tan B] / [1 – tan A tan B]
So,
tan x + tan 2x - [[tan x + tan 2x]/[1- tan x tan 2x]] = 0
(tan x + tan 2x) (1 - 1/(1- tan x tan 2x)) = 0
(tan x + tan 2x) ([– tan x tan 2x] / [1 – tan x tan 2x]) = 0
Then,
(tan x + tan 2x) = 0 or ([– tan x tan 2x] / [1 – tan x tan 2x]) = 0
(tan x + tan 2x) = 0 or [– tan x tan 2x] = 0
tan x = tan (-2x) or -tan x tan 2x = 0
tan x = tan (-2x) or 2$\tan^2$ x / (1 – $\tan^2$ x) = 0 [Using, tan 2x = 2 tan x / 1-$\tan^2$ x]
x = nπ + (-2x) or x = mπ + 0

3x = nπ or x = mπ
x = nπ/3 or x = mπ
∴ the general solution is
x = nπ/3 or mπ, where m, n ∈ Z.

(iii) tan 3x + tan x = 2 tan 2x
Let us simplify,
tan 3x + tan x = 2 tan 2x
tan 3x + tan x = tan 2x + tan 2x
upon rearranging we get,
tan 3x - tan 2x = tan 2x - tan x
By using the formula,
tan (A-B) = [tan A – tan B] / [1 + tan A tan B]
so,
[(tan 3x – tan 2x) (1+tan 3x tan 2x)] / [1 + tan 3x tan 2x] = [(tan 2x-tan x) (1+tan x tan 2x)] / [1 + tan 2x tan x]
tan (3x – 2x) (1 + tan 3x tan 2x) = tan (2x - x) (1 + tan x tan 2x)
tan x [1 + tan 3x tan 2x – 1 – tan 2x tan x] = 0
tan x tan 2x (tan 3x – tan x) = 0
so,
tan x = 0 or tan 2x = 0 or (tan 3x – tan x) = 0
tan x = 0 or tan 2x = 0 or tan 3x = tan x
x = nπ or 2x = mπ or 3x = kπ + x
x = nπ or x = mπ/2 or 2x = kπ
x = nπ or x = mπ/2 or x = kπ/2
∴ the general solution is
x = nπ or mπ/2 or kπ/2, where, m, n, k ∈ Z.