EXERCISE 12.1 PAGE NO: 12.3

1. If P (n) is the statement "n (n + 1) is even", then what is P (3)?

Solution:

Given:

P (n) = n (n + 1) is even.

So,

P (3) = 3 (3 + 1)

= 3 (4)

= 12

Hence, P (3) = 12, P (3) is also even.

2. If P (n) is the statement "$n^3 + n$ is divisible by 3", prove that P (3) is true but P (4) is not true.

Solution:

Given:

P (n) = $n^3 + n$ is divisible by 3

We have P (n) = $n^3 + n$

So,

P (3) = $3^3 + 3$

= 27 + 3

= 30

P (3) = 30, So it is divisible by 3

Now, let's check with P (4)

P (4) = $4^3 + 4$

= 64 + 4

= 68

P (4) = 68, so it is not divisible by 3

Hence, P (3) is true and P (4) is not true.

3. If P (n) is the statement "$2^n \geq 3n$", and if P (r) is true, prove that P (r + 1) is true.

Solution:

Given:

P (n) = "$2^n \geq 3n$" and p(r) is true.

We have, P (n) = $2^n \geq 3n$

Since, P (r) is true

So,

$2^r \geq 3r$

Now, let's multiply both sides by 2
$2\times2^r \geq 3r\times2$
$2^{r+1} \geq 6r$
$2^{r+1} \geq 3r + 3r$ [since $3r>3 = 3r + 3r \geq 3 + 3r$]
$\therefore 2^{r+1} \geq 3(r + 1)$
Hence, P (r + 1) is true.

4. If P (n) is the statement "$n^2 + n$" is even", and if P (r) is true, then P (r + 1) is true
Solution:
Given:
P (n) = $n^2 + n$ is even and P (r) is true, then $r^2 + r$ is even
Let us consider $r^2 + r = 2k$ … (i)
Now, $(r + 1)^2 + (r + 1)$
$r^2 + 1 + 2r + r + 1$
$(r^2 + r) + 2r + 2$
$2k + 2r + 2$ [from equation (i)]
$2(k + r + 1)$
2μ
$\therefore (r + 1)^2 + (r + 1)$ is Even.
Hence, P (r + 1) is true.

5. Given an example of a statement P (n) such that it is true for all n ϵ N.
Solution:
Let us consider
P (n) = 1 + 2 + 3 + - - - - - + n = n(n+1)/2
So,
P (n) is true for all natural numbers.
Hence, P (n) is true for all n ∈ N.

6. If P (n) is the statement "$n^2 – n + 41$ is prime", prove that P (1), P (2) and P (3) are true. Prove also that P (41) is not true.
Solution:
Given:
P(n) = $n^2 - n + 41$ is prime.
P(n) = $n^2 - n + 41$
P (1) = 1 - 1 + 41
= 41
P (1) is Prime.
Similarly,

$P(2) = 2^2 – 2 + 41$
$= 4 - 2 + 41$
$= 43$
P (2) is prime.
Similarly,
$P(3) = 3^2 – 3 + 41$
$= 9 – 3 + 41$
$= 47$
P (3) is prime
Now,
$P(41) = (41)^2 - 41 + 41$
$= 1681$
P (41) is not prime
Hence, P (1), P(2), P (3) are true but P (41) is not true.

EXERCISE 12.2 PAGE NO: 12.27

Prove the following by the principle of mathematical induction:

1. 1 + 2 + 3 + … + n = n (n +1)/2 i.e., the sum of the first n natural numbers is n (n + 1)/2.
Solution:
Let us consider P (n) = 1 + 2 + 3 + ….. + n = n (n +1)/2
For, n = 1
LHS of P (n) = 1
RHS of P (n) = 1 (1+1)/2 = 1
So, LHS = RHS
Since, P (n) is true for n = 1
Let us consider P (n) be the true for n = k, so
1 + 2 + 3 + …. + k = k (k+1)/2 … (i)
Now,
(1 + 2 + 3 + … + k) + (k + 1) = k (k+1)/2 + (k+1)
= (k + 1) (k/2 + 1)
= [(k + 1) (k + 2)] / 2
= [(k+1) [(k+1) + 1]] / 2
P (n) is true for n = k + 1
P (n) is true for all n ∈ N
So, by the principle of Mathematical Induction
Hence, P (n) = 1 + 2 + 3 + ….. + n = n (n +1)/2 is true for all n ∈ N.

2. $1^2 + 2^2 + 3^2 + … + n^2$ = [n (n+1) (2n+1)]/6
Solution:
Let us consider P (n) = $1^2 + 2^2 + 3^2 + … + n^2$ = [n (n+1) (2n+1)]/6
For, n = 1
P (1) = [1 (1+1) (2+1)]/6
1 = 1
P (n) is true for n = 1

Let P (n) is true for n = k, so
P (k): $1^2 + 2^2 + 3^2 + … + k^2$ = [k (k+1) (2k+1)]/6
Let's check for P (n) = k + 1, so
P (k) = $1^2 + 2^2 + 3^2 + - - - - - - + k^2 + (k + 1)^2$ = [k + 1 (k+2) (2k+3)] /6
= $1^2 + 2^2 + 3^2 + - - - - - - + k^2 + (k + 1)^2$
= [k + 1 (k+2) (2k+3)] /6 + $(k + 1)^2$

$= (k+1)\ [(2k^2 + k)/6 + (k + 1)/1]$
$= (k+1)\ [2k^2 + k + 6k + 6]/6$
$= (k+1)\ [2k^2 + 7k + 6]/6$
$= (k+1)\ [2k^2 + 4k + 3k + 6]/6$
$= (k+1)\ [2k(k + 2) + 3(k + 2)]/6$
$= [(k+1)\ (2k + 3)\ (k + 2)]\ /\ 6$
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

3. $1 + 3 + 3^2 + \ldots + 3^{n-1} = (3^n - 1)/2$
Solution:
Let P (n) = $1 + 3 + 3^2 + \text{- - - -} + 3^{n-1} = (3^n - 1)/2$
Now, For n = 1
P (1) = $1 = (3^1 - 1)/2 = 2/2 = 1$
P (n) is true for n = 1
Now, let's check for P (n) is true for n = k
P (k) = $1 + 3 + 3^2 + \text{- - - -} + 3^{k-1} = (3^k - 1)/2$ … (i)
Now, we have to show P (n) is true for n = k + 1
P (k + 1) = $1 + 3 + 3^2 + \text{- - - -} + 3^k = (3^{k+1} - 1)/2$
Then, $\{1 + 3 + 3^2 + \text{- - - -} + 3^{k-1}\} + 3^{k+1-1}$
$= (3k - 1)/2 + 3^k$ using equation (i)
$= (3k – 1 + 2\times3^k)/2$
$= (3\times3\ k - 1)/2$
$= (3^{k+1} - 1)/2$
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

4. 1/1.2 + 1/2.3 + 1/3.4 + … + 1/n(n+1) = n/(n+1)
Solution:
Let P (n) = 1/1.2 + 1/2.3 + 1/3.4 + … + 1/n(n+1) = n/(n+1)
For, n = 1
P (n) = 1/1.2 = 1/1+1
1/2 = 1/2
P (n) is true for n = 1
Let's check for P (n) is true for n = k,
1/1.2 + 1/2.3 + 1/3.4 + … + 1/k(k+1) + k/(k+1) (k+2) = (k+1)/(k+2)
Then,
1/1.2 + 1/2.3 + 1/3.4 + … + 1/k(k+1) + k/(k+1) (k+2)
= 1/(k+1)/(k+2) + k/(k+1)

= 1/(k+1) [k(k+2)+1]/(k+2)
= 1/(k+1) [k^2 + 2k + 1]/(k+2)
=1/(k+1) [(k+1) (k+1)]/(k+2)
= (k+1) / (k+2)
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

5. 1 + 3 + 5 + … + (2n - 1) = n^2 i.e., the sum of first n odd natural numbers is n^2.
Solution:
Let P (n): 1 + 3 + 5 + … + (2n - 1) = n^2
Let us check P (n) is true for n = 1
P (1) = 1 =1^2
1 = 1
P (n) is true for n = 1

Now, Let's check P (n) is true for n = k
P (k) = 1 + 3 + 5 + … + (2k - 1) = k^2 … (i)
We have to show that
1 + 3 + 5 + … + (2k - 1) + 2(k + 1) - 1 = $(k + 1)^2$
Now,
1 + 3 + 5 + … + (2k - 1) + 2(k + 1) - 1
= k^2 + (2k + 1)
= k^2 + 2k + 1
= $(k + 1)^2$
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

6. 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3n-1) (3n+2) = n/(6n+4)
Solution:
Let P (n) = 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3n-1) (3n+2) = n/(6n+4)
Let us check P (n) is true for n = 1
P (1): 1/2.5 = 1/6.1+4 => 1/10 = 1/10
P (1) is true.
Now,
Let us check for P (k) is true, and have to prove that P (k + 1) is true.
P (k): 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3k-1) (3k+2) = k/(6k+4)
P (k +1): 1/2.5 + 1/5.8 + 1/8.11 + … + 1/(3k-1)(3k+2) + 1/(3k+3-1)(3k+3+2)
: k/(6k+4) + 1/(3k+2)(3k+5)
: [k(3k+5)+2] / [2(3k+2)(3k+5)]

: (k+1) / (6(k+1)+4)
P (k + 1) is true.
Hence proved by mathematical induction.

7. 1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3n-2)(3n+1) = n/3n+1
Solution:
Let P (n) = 1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3n-2)(3n+1) = n/3n+1
Let us check for n = 1,
P (1): 1/1.4 = 1/4
1/4 = 1/4
P (n) is true for n = 1.
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.
P (k) = 1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3k-2)(3k+1) = k/3k+1 … (i)
So,
[1/1.4 + 1/4.7 + 1/7.10 + … + 1/(3k-2)(3k+1)]+ 1/(3k+1)(3k+4)
= k/(3k+1) + 1/(3k+1)(3k+4)
= 1/(3k+1) [k/1 + 1/(3k+4)]
= 1/(3k+1) [k(3k+4)+1]/(3k+4)
= 1/(3k+1) [3k^2 + 4k + 1]/ (3k+4)
= 1/(3k+1) [3k^2 + 3k+k+1]/(3k+4)
= [3k(k+1) + (k+1)] / [(3k+4) (3k+1)]
= [(3k+1)(k+1)] / [(3k+4) (3k+1)]
= (k+1) / (3k+4)
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

8. 1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2n+1)(2n+3) = n/3(2n+3)
Solution:
Let P (n) = 1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2n+1)(2n+3) = n/3(2n+3)
Let us check for n = 1,
P (1): 1/3.5 = 1/3(2.1+3)
: 1/15 = 1/15
P (n) is true for n = 1.
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.
P (k) = 1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2k+1)(2k+3) = k/3(2k+3) … (i)
So,
1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2k+1)(2k+3) + 1/[2(k+1)+1][2(k+1)+3]
1/3.5 + 1/5.7 + 1/7.9 + … + 1/(2k+1)(2k+3) + 1/(2k+3)(2k+5)
Now substituting the value of P (k) we get,

= k/3(2k+3) + 1/(2k+3)(2k+5)
= [k(2k+5)+3] / [3(2k+3)(2k+5)]
= (k+1) / [3(2(k+1)+3)]
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

9. 1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4n-1)(4n+3) = n/3(4n+3)
Solution:
Let P (n) = 1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4n-1)(4n+3) = n/3(4n+3)
Let us check for n = 1,
P (1): 1/3.7 = 1/(4.1-1)(4+3)
: 1/21 = 1/21
P (n) is true for n =1.
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.
P (k): 1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4k-1)(4k+3) = k/3(4k+3) …. (i)
So,
1/3.7 + 1/7.11 + 1/11.15 + … + 1/(4k-1)(4k+3) + 1/(4k+3)(4k+7)
Substituting the value of P (k) we get,
= k/(4k+3) + 1/(4k+3)(4k+7)
= 1/(4k+3) [k(4k+7)+3] / [3(4k+7)]
= 1/(4k+3) [$4k^2$ + 7k +3]/ [3(4k+7)]
= 1/(4k+3) [$4k^2$ + 3k+4k+3] / [3(4k+7)]
= 1/(4k+3) [4k(k+1)+3(k+1)]/ [3(4k+7)]
= 1/(4k+3) [(4k+3)(k+1)] / [3(4k+7)]
= (k+1) / [3(4k+7)]
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

10. $1.2 + 2.2^2 + 3.2^3 + \ldots + n.2^n = (n–1)\ 2^{n+1} + 2$
Solution:
Let P (n) = $1.2 + 2.2^2 + 3.2^3 + \ldots + n.2^n = (n–1)\ 2^{n+1} + 2$
Let us check for n = 1,
P (1):$1.2 = 0.2^0 + 2$
: 2 = 2
P (n) is true for n = 1.
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.
P (k): $1.2 + 2.2^2 + 3.2^3 + \ldots + k.2^k = (k–1)\ 2^{k+1} + 2$ …. (i)
So,
$\{1.2 + 2.2^2 + 3.2^3 + \ldots + k.2^k\} + (k + 1)2^{k+1}$

Now, substituting the value of P (k) we get,

$= [(k - 1)2^{k+1} + 2] + (k + 1)2^{k+1}$ using equation (i)

$= (k - 1)2^{k+1} + 2 + (k + 1)2^{k+1}$

$= 2^{k+1}(k - 1 + k + 1) + 2$

$= 2^{k+1} \times 2k + 2$

$= k \times 2^{k+2} + 2$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

11. 2 + 5 + 8 + 11 + … + (3n – 1) = 1/2 n (3n + 1)

Solution:

Let P (n) = 2 + 5 + 8 + 11 + … + (3n – 1) = 1/2 n (3n + 1)

Let us check for n = 1,

P (1): 2 = 1/2 × 1 × 4

: 2 = 2

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k) = 2 + 5 + 8 + 11 + … + (3k – 1) = 1/2 k (3k + 1) … (i)

So,

2 + 5 + 8 + 11 + … + (3k – 1) + (3k + 2)

Now, substituting the value of P (k) we get,

= 1/2 × k (3k + 1) + (3k + 2) by using equation (i)

$= [3k^2 + k + 2 (3k + 2)] / 2$

$= [3k^2 + k + 6k + 2] / 2$

$= [3k^2 + 7k + 2] / 2$

$= [3k^2 + 4k + 3k + 2] / 2$

= [3k (k+1) + 4(k+1)] / 2

= [(k+1) (3k+4)] /2

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

12. 1.3 + 2.4 + 3.5 + … + n. (n+2) = 1/6 n (n+1) (2n+7)

Solution:

Let P (n): 1.3 + 2.4 + 3.5 + … + n. (n+2) = 1/6 n (n+1) (2n+7)

Let us check for n = 1,

P (1): 1.3 = 1/6 × 1 × 2 × 9

: 3 = 3

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 1.3 + 2.4 + 3.5 + … + k. (k+2) = 1/6 k (k+1) (2k+7) … (i)
So,
1.3 + 2.4 + 3.5 + … + k. (k+2) + (k+1) (k+3)
Now, substituting the value of P (k) we get,
= 1/6 k (k+1) (2k+7) + (k+1) (k+3) by using equation (i)
= (k+1) [{k(2k+7)/6} + {(k+3)/1}]
= (k+1) [($2k^2$ + 7k + 6k + 18)] / 6
= (k+1) [$2k^2$ + 13k + 18] / 6
= (k+1) [$2k^2$ + 9k + 4k + 18] / 6
= (k+1) [2k(k+2) + 9(k+2)] / 6
= (k+1) [(2k+9) (k+2)] / 6
= 1/6 (k+1) (k+2) (2k+9)
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

13. 1.3 + 3.5 + 5.7 + … + (2n - 1) (2n + 1) = n($4n^2$ + 6n - 1)/3
Solution:
Let P (n): 1.3 + 3.5 + 5.7 + … + (2n – 1) (2n + 1) = n($4n^2$ + 6n - 1)/3
Let us check for n = 1,
P (1): (2.1 - 1) (2.1 + 1) = 1(4.1^2 + 6.1 -1)/3
: 1×3 = 1(4+6-1)/3
: 3 = 3
P (n) is true for n = 1.
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.
P (k): 1.3 + 3.5 + 5.7 + … + (2k – 1) (2k + 1) = k($4k^2$ + 6k - 1)/3 … (i)
So,
1.3 + 3.5 + 5.7 + … + (2k – 1) (2k + 1) + (2k + 1) (2k + 3)
Now, substituting the value of P (k) we get,
= k($4k^2$ + 6k - 1)/3 + (2k + 1) (2k + 3) by using equation (i)
= [k($4k^2$ + 6k-1) + 3 ($4k^2$ + 6k + 2k + 3)] / 3
= [$4k^3$ + $6k^2$ – k + $12k^2$ + 18k + 6k + 9] /3
= [$4k^3$ + $18k^2$ + 23k + 9] /3
= [$4k^3$ + $4k^2$ + $14k^2$ + 14k +9k + 9] /3
= [(k+1) ($4k^2$ + 8k +4 + 6k + 6 - 1)] / 3
= [(k+1) 4[$(k+1)^2$ + 6(k+1) -1]] /3
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

14. 1.2 + 2.3 + 3.4 + … + n(n+1) = [n (n+1) (n+2)] / 3

Solution:

Let P (n): 1.2 + 2.3 + 3.4 + … + n(n+1) = [n (n+1) (n+2)] / 3

Let us check for n = 1,

P (1): 1(1+1) = [1(1+1) (1+2)] /3

: 2 = 2

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 1.2 + 2.3 + 3.4 + … + k(k+1) = [k (k+1) (k+2)] / 3 … (i)

So,

1.2 + 2.3 + 3.4 + … + k(k+1) + (k+1) (k+2)

Now, substituting the value of P (k) we get,

= [k (k+1) (k+2)] / 3 + (k+1) (k+2) by using equation (i)

= (k+2) (k+1) [k/2 + 1]

= [(k+1) (k+2) (k+3)] /3

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

15. $1/2 + 1/4 + 1/8 + … + 1/2^n = 1 – 1/2^n$

Solution:

Let P (n): $1/2 + 1/4 + 1/8 + … + 1/2^n = 1 – 1/2^n$

Let us check for n = 1,

P (1): $1/2^1 = 1 – 1/2^1$

: 1/2 = 1/2

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

Let P (k): $1/2 + 1/4 + 1/8 + … + 1/2^k = 1 – 1/2^k$ … (i)

So,

$1/2 + 1/4 + 1/8 + … + 1/2^k + 1/2^{k+1}$

Now, substituting the value of P (k) we get,

$= 1 – 1/2^k + 1/2^{k+1}$ by using equation (i)

$= 1 – ((2-1)/2^{k+1})$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

16. $1^2 + 3^2 + 5^2 + … + (2n – 1)^2 = 1/3\ n (4n^2 - 1)$

Solution:

Let P (n): $1^2 + 3^2 + 5^2 + … + (2n – 1)^2 = 1/3\ n (4n^2 - 1)$

Let us check for n = 1,

P (1): $(2.1 - 1)^2 = 1/3 × 1 × (4 - 1)$

: 1 = 1
P (n) is true for n = 1.
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.
P (k): $1^2 + 3^2 + 5^2 + \ldots + (2k – 1)^2 = 1/3\ k\ (4k^2 - 1)$ … (i)
So,
$1^2 + 3^2 + 5^2 + \ldots + (2k – 1)^2 + (2k + 1)^2$
Now, substituting the value of P (k) we get,
$= 1/3\ k\ (4k^2 - 1) + (2k + 1)^2$ by using equation (i)
$= 1/3\ k\ (2k + 1)\ (2k - 1) + (2k + 1)^2$
$= (2k + 1)\ [\{k(2k-1)/3\} + (2k+1)]$
$= (2k + 1)\ [2k^2 – k + 3(2k+1)] / 3$
$= (2k + 1)\ [2k^2 – k + 6k + 3] / 3$
$= [(2k+1)\ 2k^2 + 5k + 3] /3$
$= [(2k+1)\ (2k(k+1)) + 3\ (k+1)] /3$
$= [(2k+1)\ (2k+3)\ (k+1)] /3$
$= (k+2)/2\ [4k^2 + 6k + 2k + 3]$
$= (k+2)/2\ [4k^2 + 8k - 1]$
$= (k+2)/2\ [4(k+1)^2 - 1]$
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

17. $a + ar + ar^2 + \ldots + ar^{n-1} = a\ [(r^n - 1)/(r - 1)],\ r \neq 1$
Solution:
Let P (n): $a + ar + ar^2 + \ldots + ar^{n-1} = a\ [(r^n - 1)/(r - 1)]$
Let us check for n = 1,
P (1): $a = a\ (r^1 - 1)/(r-1)$
: a = a
P (n) is true for n = 1.
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.
P (k): $a + ar + ar^2 + \ldots + ar^{k-1} = a\ [(r^k - 1)/(r - 1)]$ … (i)
So,
$a + ar + ar^2 + \ldots + ar^{k-1} + ar^k$
Now, substituting the value of P (k) we get,
$= a\ [(r^k - 1)/(r - 1)] + ar^k$ by using equation (i)
$= a[r^k – 1 + r^k(r-1)] / (r-1)$
$= a[r^k – 1 + r^{k+1} – r^{-k}] / (r-1)$
$= a[r^{k+1} - 1] / (r-1)$
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

18. a + (a + d) + (a + 2d) + … + (a + (n-1)d) = n/2 [2a + (n-1)d]

Solution:

Let P (n): a + (a + d) + (a + 2d) + … + (a + (n-1)d) = n/2 [2a + (n-1)d]

Let us check for n = 1,

P (1): a = ½ [2a + (1-1)d]

: a = a

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): a + (a + d) + (a + 2d) + … + (a + (k-1)d) = k/2 [2a + (k-1)d] … (i)

So,

a + (a + d) + (a + 2d) + … + (a + (k-1)d) + (a + (k)d)

Now, substituting the value of P (k) we get,

= k/2 [2a + (k-1)d] + (a + kd) by using equation (i)

= [2ka + k(k-1)d + 2(a+kd)] / 2

= [2ka + k^2d – kd + 2a + 2kd] / 2

= [2ka + 2a + k^2d + kd] / 2

= [2a(k+1) + d(k^2 + k)] / 2

= (k+1)/2 [2a + kd]

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

19. 5^{2n} – 1 is divisible by 24 for all n ϵ N

Solution:

Let P (n): 5^{2n} – 1 is divisible by 24

Let us check for n = 1,

P (1): 5^2 – 1 = 25 – 1 = 24

P (n) is true for n = 1. Where, P (n) is divisible by 24

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): 5^{2k} – 1 is divisible by 24

: 5^{2k} – 1 = 24λ … (i)

We have to prove,

$5^{2k + 1}$ - 1 is divisible by 24

$5^{2(k + 1)}$ - 1 = 24μ

So,

= $5^{2(k + 1)}$ - 1

= $5^{2k}.5^2$ - 1

= 25.5^{2k} - 1

= 25.(24λ + 1) - 1 by using equation (1)

= 25.24λ + 24
= 24λ
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

20. 3^{2n} + 7 is divisible by 8 for all n ϵ N
Solution:
Let P (n): 3^{2n} + 7 is divisible by 8
Let us check for n = 1,
P (1): $3^2 + 7 = 9 + 7 = 16$
P (n) is true for n = 1. Where, P (n) is divisible by 8
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.
P (k): 3^{2k} + 7 is divisible by 8
: $3^{2k} + 7 = 8\lambda$
: $3^{2k} = 8\lambda - 7$ … (i)
We have to prove,
$3^{2(k+1)}$ + 7 is divisible by 8
$3^{2k+2} + 7 = 8\mu$
So,
= $3^{2(k+1)} + 7$
= $3^{2k}.3^2 + 7$
= $9.3^{2k} + 7$
= $9.(8\lambda - 7) + 7$ by using equation (i)
= $72\lambda - 63 + 7$
= $72\lambda - 56$
= $8(9\lambda - 7)$
= 8μ
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

21. 5^{2n+2} – 24n – 25 is divisible by 576 for all n ϵ N
Solution:
Let P (n): 5^{2n+2} – 24n – 25 is divisible by 576
Let us check for n = 1,
P (1): $5^{2.1+2}$ – 24.1 – 25
: 625 – 49
: 576
P (n) is true for n = 1. Where, P (n) is divisible by 576
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $5^{2k+2} - 24k - 25$ is divisible by 576
: $5^{2k+2} - 24k - 25 = 576\lambda$ …. (i)
We have to prove,
$5^{2k+4} - 24(k+1) - 25$ is divisible by 576
$5^{(2k+2)+2} - 24(k+1) - 25 = 576\mu$
So,
$= 5^{(2k+2)+2} - 24(k+1) - 25$
$= 5^{(2k+2)}.5^2 - 24k - 24 - 25$
$= (576\lambda + 24k + 25)25 - 24k - 49$ by using equation (i)
$= 25.\ 576\lambda + 576k + 576$
$= 576(25\lambda + k + 1)$
$= 576\mu$
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

22. $3^{2n+2} - 8n - 9$ is divisible by 8 for all n ϵ N
Solution:
Let P (n): $3^{2n+2} - 8n - 9$ is divisible by 8
Let us check for n = 1,
P (1): $3^{2.1+2} - 8.1 - 9$
: 81 – 17
: 64
P (n) is true for n = 1. Where, P (n) is divisible by 8
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.
P (k): $3^{2k+2} - 8k - 9$ is divisible by 8
: $3^{2k+2} - 8k - 9 = 8\lambda$ … (i)
We have to prove,
$3^{2k+4} - 8(k+1) - 9$ is divisible by 8
$3^{(2k+2)+2} - 8(k+1) - 9 = 8\mu$
So,
$= 3^{2(k+1)}.3^2 - 8(k+1) - 9$
$= (8\lambda + 8k + 9)9 - 8k - 8 - 9$
$= 72\lambda + 72k + 81 - 8k - 17$ using equation (1)
$= 72\lambda + 64k + 64$
$= 8(9\lambda + 8k + 8)$
$= 8\mu$
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

23. $(ab)^n = a^n b^n$ for all n ϵ N

Solution:

Let P (n): $(ab)^n = a^n b^n$

Let us check for n = 1,

P (1): $(ab)^1 = a^1 b^1$

: ab = ab

P (n) is true for n = 1.

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): $(ab)^k = a^k b^k$ … (i)

We have to prove,

$(ab)^{k+1} = a^{k+1}.b^{k+1}$

So,

$= (ab)^{k+1}$

$= (ab)^k (ab)$

$= (a^k b^k) (ab)$ using equation (1)

$= (a^{k+1}) (b^{k+1})$

P (n) is true for n = k + 1

Hence, P (n) is true for all n ∈ N.

24. n (n + 1) (n + 5) is a multiple of 3 for all n ϵ N.

Solution:

Let P (n): n (n + 1) (n + 5) is a multiple of 3

Let us check for n = 1,

P (1): 1 (1 + 1) (1 + 5)

: 2 × 6

: 12

P (n) is true for n = 1. Where, P (n) is a multiple of 3

Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.

P (k): k (k + 1) (k + 5) is a multiple of 3

: $k(k + 1) (k + 5) = 3\lambda$ … (i)

We have to prove,

(k + 1)[(k + 1) + 1][(k + 1) + 5] is a multiple of 3

$(k + 1)[(k + 1) + 1][(k + 1) + 5] = 3\mu$

So,

$= (k + 1) [(k + 1) + 1] [(k + 1) + 5]$

$= (k + 1) (k + 2) [(k + 1) + 5]$

$= [k (k + 1) + 2(k + 1)] [(k + 5) + 1]$

$= k (k + 1) (k + 5) + k(k + 1) + 2(k + 1) (k + 5) + 2(k + 1)$

$= 3\lambda + k^2 + k + 2(k^2 + 6k + 5) + 2k + 2$

$= 3\lambda + k^2 + k + 2k^2 + 12k + 10 + 2k + 2$
$= 3\lambda + 3k^2 + 15k + 12$
$= 3(\lambda + k^2 + 5k + 4)$
$= 3\mu$
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.

25. $7^{2n} + 2^{3n-3}$. 3n – 1 is divisible by 25 for all n ϵ N
Solution:
Let P (n): $7^{2n} + 2^{3n-3}$. 3n – 1 is divisible by 25
Let us check for n = 1,
P (1): $7^2 + 2^0.3^0$
: 49 + 1
: 50
P (n) is true for n = 1. Where, P (n) is divisible by 25
Now, let us check for P (n) is true for n = k, and have to prove that P (k + 1) is true.
P (k): $7^{2k} + 2^{3k-3}$. 3k – 1 is divisible by 25
: $7^{2k} + 2^{3k-3}. 3^{k-1} = 25\lambda$ … (i)
We have to prove that:
$7^{2k+1} + 2^{3k}. 3^k$ is divisible by 25
$7^{2k+2} + 2^{3k}. 3^k = 25\mu$
So,
$= 7^{2(k+1)} + 2^{3k}. 3^k$
$= 7^{2k}.7^1 + 2^{3k}. 3^k$
$= (25\lambda – 2^{3k-3}. 3^{k-1})\ 49 + 2^{3k}$. 3k by using equation (i)
$= 25\lambda. 49 – 2^{3k}/8. 3^k/3. 49 + 2^{3k}. 3^k$
$= 24\times25\times49\lambda - 2^{3k} . 3^k . 49 + 24 . 2^{3k}.3^k$
$= 24\times25\times49\lambda – 25 . 2^{3k}. 3^k$
$= 25(24 . 49\lambda - 2^{3k}. 3^k)$
$= 25\mu$
P (n) is true for n = k + 1
Hence, P (n) is true for all n ∈ N.