EXERCISE 2.2 PAGE NO: 2.12

1. Given A = {1, 2, 3}, B = {3, 4}, C = {4, 5, 6}, find (A x B) ∩ (B x C).

Solution:

Given:

A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}

Let us find: (A × B) ∩ (B × C)

(A × B) = {1, 2, 3} × {3, 4}

= {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

(B × C) = {3, 4} × {4, 5, 6}

= {(3, 4), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6)}

∴ (A × B) ∩ (B × C) = {(3, 4)}

2. If A = {2, 3}, B = {4, 5}, C = {5, 6} find A x (B ∪ C), (A x B) ∪ (A x C).

Solution:

Given: A = {2, 3}, B = {4, 5} and C = {5, 6}

Let us find: A x (B ∪ C) and (A x B) ∪ (A x C)

(B ∪ C) = {4, 5, 6}

A × (B ∪ C) = {2, 3} × {4, 5, 6}

= {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

(A × B) = {2, 3} × {4, 5}

= {(2, 4), (2, 5), (3, 4), (3, 5)}

(A × C) = {2, 3} × {5, 6}

= {(2, 5), (2, 6), (3, 5), (3, 6)}

∴ (A × B) ∪ (A × C) = {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

A × (B ∪ C) = {(2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

3. If A = {1, 2, 3}, B = {4}, C = {5}, then verify that:

(i) A x (B ∪ C) = (A x B) ∪ (A x C)

(ii) A x (B ∩ C) = (A x B) ∩ (A x C)

(iii) A x (B – C) = (A x B) – (A x C)

Solution:

Given:

A = {1, 2, 3}, B = {4} and C = {5}

(i) A × (B ∪ C) = (A × B) ∪ (A × C)
Let us consider LHS: (B ∪ C)
(B ∪ C) = {4, 5}
A × (B ∪ C) = {1, 2, 3} × {4, 5}
= {(1, 4), (1, 5), (2, 4), (2, 5), (3, 4), (3, 5)}

Now, RHS
(A × B) = {1, 2, 3} × {4}
= {(1, 4), (2, 4), (3, 4)}

(A × C) = {1, 2, 3} × {5}
= {(1, 5), (2, 5), (3, 5)}

(A × B) ∪ (A × C) = {(1, 4), (2, 4), (3, 4), (1, 5), (2, 5), (3, 5)}
∴ LHS = RHS

(ii) A × (B ∩ C) = (A × B) ∩ (A × C)
Let us consider LHS: (B ∩ C)
(B ∩ C) = ∅ (No common element)
A × (B ∩ C) = {1, 2, 3} × ∅
= ∅

Now, RHS
(A × B) = {1, 2, 3} × {4}
= {(1, 4), (2, 4), (3, 4)}

(A × C) = {1, 2, 3} × {5}
= {(1, 5), (2, 5), (3, 5)}

(A × B) ∩ (A × C) = ∅
∴ LHS = RHS

(iii) A × (B − C) = (A × B) − (A × C)
Let us consider LHS: (B − C)
(B − C) = ∅
A × (B − C) = {1, 2, 3} × ∅
= ∅

Now, RHS
(A × B) = {1, 2, 3} × {4}
= {(1, 4), (2, 4), (3, 4)}

(A × C) = {1, 2, 3} × {5}
= {(1, 5), (2, 5), (3, 5)}

(A × B) − (A × C) = ∅
∴ LHS = RHS

4. Let A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}. Verify that:
(i) A x C ⊂ B x D
(ii) A x (B ∩ C) = (A x B) ∩ (A x C)
Solution:
Given:
A = {1, 2}, B = {1, 2, 3, 4}, C = {5, 6} and D = {5, 6, 7, 8}

(i) A x C ⊂ B x D
Let us consider LHS A x C
A × C = {1, 2} × {5, 6}
= {(1, 5), (1, 6), (2, 5), (2, 6)}

Now, RHS
B × D = {1, 2, 3, 4} × {5, 6, 7, 8}
= {(1, 5), (1, 6), (1, 7), (1, 8), (2, 5), (2, 6), (2, 7), (2, 8), (3, 5), (3, 6), (3, 7), (3, 8), (4, 5), (4, 6), (4, 7), (4, 8)}

Since, all elements of A × C is in B × D.
∴We can say A × C ⊂ B × D

(ii) A × (B ∩ C) = (A × B) ∩ (A × C)
Let us consider LHS A × (B ∩ C)
(B ∩ C) = ∅
A × (B ∩ C) = {1, 2} × ∅
= ∅

Now, RHS
(A × B) = {1, 2} × {1, 2, 3, 4}
= {(1, 1), (1, 2), (1, 3), (1, 4), (2, 1), (2, 2), (2, 3), (2, 4)}

(A × C) = {1, 2} × {5, 6}
= {(1, 5), (1, 6), (2, 5), (2, 6)}
Since, there is no common element between A × B and A × C
(A × B) ∩ (A × C) = ∅
∴ A × (B ∩ C) = (A × B) ∩ (A × C)

5. If A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}, find
(i) A x (B ∩ C)
(ii) (A x B) ∩ (A x C)
(iii) A x (B ∪ C)
(iv) (A x B) ∪ (A x C)
Solution:
Given:
A = {1, 2, 3}, B = {3, 4} and C = {4, 5, 6}
(i) A × (B ∩ C)
(B ∩ C) = {4}
A × (B ∩ C) = {1, 2, 3} × {4}
= {(1, 4), (2, 4), (3, 4)}

(ii) (A × B) ∩ (A × C)
(A × B) = {1, 2, 3} × {3, 4}
= {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

(A × C) = {1, 2, 3} × {4, 5, 6}
= {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}
(A × B) ∩ (A × C) = {(1, 4), (2, 4), (3, 4)}

(iii) A × (B ∪ C)
(B ∪ C) = {3, 4, 5, 6}
A × (B ∪ C) = {1, 2, 3} × {3, 4, 5, 6}
= {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)}

(iv) (A × B) ∪ (A × C)
(A × B) = {1, 2, 3} × {3, 4}
= {(1, 3), (1, 4), (2, 3), (2, 4), (3, 3), (3, 4)}

(A × C) = {1, 2, 3} × {4, 5, 6}
= {(1, 4), (1, 5), (1, 6), (2, 4), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6)}

(A × B) ∪ (A × C) = {(1, 3), (1, 4), (1, 5), (1, 6), (2, 3), (2, 4), (2, 5), (2, 6), (3, 3), (3, 4), (3, 5), (3, 6)}

6. Prove that:
(i) (A ∪ B) x C = (A x C) = (A x C) ∪ (B x C)
(ii) (A ∩ B) x C = (A x C) ∩ (B x C)
Solution:
(i) (A ∪ B) x C = (A x C) = (A x C) ∪ (B x C)
Let (x, y) be an arbitrary element of (A ∪ B) × C
(x, y) ∈ (A ∪ B) C
Since, (x, y) are elements of Cartesian product of (A ∪ B) × C
x ∈ (A ∪ B) and y ∈ C
(x ∈ A or x ∈ B) and y ∈ C
(x ∈ A and y ∈ C) or (x ∈ Band y ∈ C)
(x, y) ∈ A × C or (x, y) ∈ B × C
(x, y) ∈ (A × C) ∪ (B × C) … (1)

Let (x, y) be an arbitrary element of (A × C) ∪ (B × C).
(x, y) ∈ (A × C) ∪ (B × C)
(x, y) ∈ (A × C) or (x, y) ∈ (B × C)
(x ∈ A and y ∈ C) or (x ∈ B and y ∈ C)
(x ∈ A or x ∈ B) and y ∈ C
x ∈ (A ∪ B) and y ∈ C
(x, y) ∈ (A ∪ B) × C … (2)
From 1 and 2, we get: (A ∪ B) × C = (A × C) ∪ (B × C)

(ii) (A ∩ B) x C = (A x C) ∩ (B x C)
Let (x, y) be an arbitrary element of (A ∩ B) × C.
(x, y) ∈ (A ∩ B) × C
Since, (x, y) are elements of Cartesian product of (A ∩ B) × C
x ∈ (A ∩ B) and y ∈ C
(x ∈ A and x ∈ B) and y ∈ C
(x ∈ A and y ∈ C) and (x ∈ Band y ∈ C)
(x, y) ∈ A × C and (x, y) ∈ B × C
(x, y) ∈ (A × C) ∩ (B × C) … (1)

Let (x, y) be an arbitrary element of (A × C) ∩ (B × C).
(x, y) ∈ (A × C) ∩ (B × C)
(x, y) ∈ (A × C) and (x, y) ∈ (B × C)

(x ∈A and y ∈ C) and (x ∈ Band y ∈ C)
(x ∈A and x ∈ B) and y ∈ C
x ∈ (A ∩ B) and y ∈ C
(x, y) ∈ (A ∩ B) × C … (2)
From 1 and 2, we get: (A ∩ B) × C = (A × C) ∩ (B × C)

7. If A x B ⊆ C x D and A ∩ B ∈ ∅, Prove that A ⊆ C and B ⊆ D.
Solution:
Given:
A × B ⊆ C x D and A ∩ B ∈ ∅
A × B ⊆ C x D denotes A × B is subset of C × D that is every element A × B is in C × D.
And A ∩ B ∈ ∅ denotes A and B does not have any common element between them.
A × B = {(a, b): a ∈ A and b ∈ B}
∴We can say (a, b) ⊆ C × D [Since, A × B ⊆ C x D is given]
a ∈ C and b ∈ D
a ∈ A = a ∈ C
A ⊆ C
And
b ∈ B = b ∈ D
B ⊆ D
Hence proved.