## EXERCISE 2.1

(1) (i) If $(a / 3+1, b-2 / 3)=(5 / 3,1 / 3)$, find the values of $a$ and $b$.
(ii) If $(x+1,1)=(3 y, y-1)$, find the values of $x$ and $y$.

## Solution:

Given:
$(\mathrm{a} / 3+1, \mathrm{~b}-2 / 3)=(5 / 3,1 / 3)$
By the definition of equality of ordered pairs,
Let us solve for a and b
$\mathrm{a} / 3+1=5 / 3$ and $\mathrm{b}-2 / 3=1 / 3$
$\mathrm{a} / 3=5 / 3-1$ and $\mathrm{b}=1 / 3+2 / 3$
$\mathrm{a} / 3=(5-3) / 3$ and $\mathrm{b}=(1+2) / 3$
$\mathrm{a} / 3=2 / 3$ and $\mathrm{b}=3 / 3$
$\mathrm{a}=2(3) / 3$ and $\mathrm{b}=1$
$\mathrm{a}=2$ and $\mathrm{b}=1$
$\therefore$ Values of a and b are, $\mathrm{a}=2$ and $\mathrm{b}=1$
(ii) If $(\mathrm{x}+1,1)=(3 \mathrm{y}, \mathrm{y}-1)$, find the values of x and y .

Given:
$(\mathrm{x}+1,1)=(3 \mathrm{y}, \mathrm{y}-1)$
By the definition of equality of ordered pairs,
Let us solve for x and y
$x+1=3 y$ and $1=y-1$
$x=3 y-1$ and $y=1+1$
$x=3 y-1$ and $y=2$
Since, $y=2$ we can substitute in

$$
\begin{aligned}
\mathrm{x} & =3 y-1 \\
& =3(2)-1 \\
& =6-1 \\
& =5
\end{aligned}
$$

$\therefore$ Values of x and y are, $\mathrm{x}=5$ and $\mathrm{y}=2$
2. If the ordered pairs $(x,-1)$ and $(5, y)$ belong to the set $\{(a, b): b=2 a-3\}$, find the values of $x$ and $y$.

## Solution:

Given:
The ordered pairs $(\mathrm{x},-1)$ and $(5, \mathrm{y})$ belong to the set $\{(\mathrm{a}, \mathrm{b})$ : $\mathrm{b}=2 \mathrm{a}-3\}$
Solving for first order pair
$(\mathrm{x},-1)=\{(\mathrm{a}, \mathrm{b}): \mathrm{b}=2 \mathrm{a}-3\}$
$\mathrm{x}=\mathrm{a}$ and $-1=\mathrm{b}$
By taking $b=2 a-3$
If $b=-1$ then $2 a=-1+3$

$$
\begin{aligned}
& =2 \\
\mathrm{a} & =2 / 2 \\
& =1
\end{aligned}
$$

So, $\mathrm{a}=1$
Since $\mathrm{x}=\mathrm{a}, \mathrm{x}=1$
Similarly, solving for second order pair
$(5, y)=\{(a, b): b=2 a-3\}$
$5=\mathrm{a}$ and $\mathrm{y}=\mathrm{b}$
By taking $\mathrm{b}=2 \mathrm{a}-3$
If $\mathrm{a}=5$ then $\mathrm{b}=2 \times 5-3$

$$
\begin{aligned}
& =10-3 \\
& =7
\end{aligned}
$$

So, $\mathrm{b}=7$
Since $y=b, y=7$
$\therefore$ Values of x and y are, $\mathrm{x}=1$ and $\mathrm{y}=7$
3. If $a \in\{-1,2,3,4,5\}$ and $b \in\{0,3,6\}$, write the set of all ordered pairs (a,b) such that $a+b=5$.
Solution:
Given: $\mathrm{a} \in\{-1,2,3,4,5\}$ and $\mathrm{b} \in\{0,3,6\}$,
To find: the ordered pair $(a, b)$ such that $a+b=5$
Then the ordered pair $(a, b)$ such that $\mathrm{a}+\mathrm{b}=5$ are as follows
$(\mathrm{a}, \mathrm{b}) \in\{(-1,6),(2,3),(5,0)\}$
4. If $a \in\{2,4,6,9\}$ and $b \in\{4,6,18,27\}$, then form the set of all ordered pairs (a,b) such that a divides $b$ and $a<b$.

## Solution:

Given:
$a \in\{2,4,6,9\}$ and $b \in\{4,6,18,27\}$
Here,
2 divides 4, 6, 18 and is also less than all of them
4 divides 4 and is also less than none of them
6 divides 6,18 and is less than 18 only
9 divides 18,27 and is less than all of them
$\therefore$ Ordered pairs $(\mathrm{a}, \mathrm{b})$ are $(2,4),(2,6),(2,18),(6,18),(9,18)$ and $(9,27)$
5. If $A=\{1,2\}$ and $B=\{1,3\}$, find $A \times B$ and $B \times A$.

## Solution:

Given:

$$
\begin{aligned}
& A=\{1,2\} \text { and } B=\{1,3\} \\
& A \times B=\{1,2\} \times\{1,3\} \\
&=\{(1,1),(1,3),(2,1),(2,3)\} \\
& B \times A=\{1,3\} \times\{1,2\} \\
&=\{(1,1),(1,2),(3,1),(3,2)\}
\end{aligned}
$$

## 6. Let $A=\{1,2,3\}$ and $B=\{3,4\}$. Find $A \times B$ and show it graphically

## Solution:

Given:
$A=\{1,2,3\}$ and $B=\{3,4\}$
$\mathrm{A} \times \mathrm{B}=\{1,2,3\} \times\{3,4\}$

$$
=\{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}
$$

Steps to follow to represent $\mathrm{A} \times \mathrm{B}$ graphically,
Step 1: One horizontal and one vertical axis should be drawn
Step 2: Element of set A should be represented in horizontal axis and on vertical axis elements of set B should be represented
Step 3: Draw dotted lines perpendicular to horizontal and vertical axes through the elements of set A and B
Step 4: Point of intersection of these perpendicular represents $A \times B$

7. If $A=\{1,2,3\}$ and $B=\{2,4\}$, what are $A \times B, B \times A, A \times A, B \times B$, and $(A \times$ B) $\cap(\mathbf{B} \times \mathbf{A})$ ?

## Solution:

Given:
$\mathrm{A}=\{1,2,3\}$ and $\mathrm{B}=\{2,4\}$
Now let us find: $\mathrm{A} \times \mathrm{B}, \mathrm{B} \times \mathrm{A}, \mathrm{A} \times \mathrm{A},(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{B} \times \mathrm{A})$
$\mathrm{A} \times \mathrm{B}=\{1,2,3\} \times\{2,4\}$
$=\{(1,2),(1,4),(2,2),(2,4),(3,2),(3,4)\}$
$B \times A=\{2,4\} \times\{1,2,3\}$
$=\{(2,1),(2,2),(2,3),(4,1),(4,2),(4,3)\}$

$$
\begin{aligned}
\mathrm{A} \times \mathrm{A} & =\{1,2,3\} \times\{1,2,3\} \\
& =\{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3),(3,1),(3,2),(3,3)\}
\end{aligned}
$$

$$
\begin{aligned}
B \times B & =\{2,4\} \times\{2,4\} \\
& =\{(2,2),(2,4),(4,2),(4,4)\}
\end{aligned}
$$

Intersection of two sets represents common elements of both the sets So,
$(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{B} \times \mathrm{A})=\{(2,2)\}$

1. Given $A=\{1,2,3\}, B=\{3,4\}, C=\{4,5,6\}$, find $(A \times B) \cap(B \times C)$. Solution:
Given:
$A=\{1,2,3\}, B=\{3,4\}$ and $C=\{4,5,6\}$
Let us find: $(A \times B) \cap(B \times C)$

$$
\begin{aligned}
&(A \times B)=\{1,2,3\} \times\{3,4\} \\
&=\{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\} \\
&(B \times C)=\{3,4\} \times\{4,5,6\} \\
&=\{(3,4),(3,5),(3,6),(4,4),(4,5),(4,6)\} \\
& \therefore(A \times B) \cap(B \times C)=\{(3,4)\}
\end{aligned}
$$

2. If $A=\{2,3\}, B=\{4,5\}, C=\{5,6\}$ find $A \times(B \cup C),(A \times B) \cup(A \times C)$.

Solution:
Given: $\mathrm{A}=\{2,3\}, \mathrm{B}=\{4,5\}$ and $\mathrm{C}=\{5,6\}$
Let us find: $\mathrm{A} \times(\mathrm{B} \cup \mathrm{C})$ and $(\mathrm{A} \times \mathrm{B}) \cup(\mathrm{A} \times \mathrm{C})$
$(B \cup C)=\{4,5,6\}$
$A \times(B \cup C)=\{2,3\} \times\{4,5,6\}$

$$
=\{(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}
$$

$(A \times B)=\{2,3\} \times\{4,5\}$
$=\{(2,4),(2,5),(3,4),(3,5)\}$
$(A \times C)=\{2,3\} \times\{5,6\}$
$=\{(2,5),(2,6),(3,5),(3,6)\}$
$\therefore(\mathrm{A} \times \mathrm{B}) \cup(\mathrm{A} \times \mathrm{C})=\{(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$
$A \times(B \cup C)=\{(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$
3. If $A=\{1,2,3\}, B=\{4\}, C=\{5\}$, then verify that:
(i) $\mathbf{A x}(\mathbf{B} \cup \mathbf{C})=(\mathbf{A x B}) \cup(\mathbf{A x C})$
(ii) $\mathbf{A x}(\mathbf{B} \cap \mathbf{C})=(\mathbf{A} \times B) \cap(\mathbf{A} \times \mathbf{C})$
(iii) $\mathbf{A x}(\mathbf{B}-\mathbf{C})=(\mathbf{A} \times \mathbf{B})-(\mathbf{A} \times \mathbf{C})$

## Solution:

Given:
$A=\{1,2,3\}, B=\{4\}$ and $C=\{5\}$
(i) $\mathrm{A} \times(\mathrm{B} \cup \mathrm{C})=(\mathrm{A} \times \mathrm{B}) \cup(\mathrm{A} \times \mathrm{C})$

Let us consider LHS: ( $\mathrm{B} \cup \mathrm{C}$ )
$(B \cup C)=\{4,5\}$
$A \times(B \cup C)=\{1,2,3\} \times\{4,5\}$
$=\{(1,4),(1,5),(2,4),(2,5),(3,4),(3,5)\}$
Now, RHS

$$
\begin{aligned}
(\mathrm{A} \times \mathrm{B}) & =\{1,2,3\} \times\{4\} \\
& =\{(1,4),(2,4),(3,4)\} \\
(\mathrm{A} \times \mathrm{C}) & =\{1,2,3\} \times\{5\} \\
& =\{(1,5),(2,5),(3,5)\}
\end{aligned}
$$

$(\mathrm{A} \times \mathrm{B}) \cup(\mathrm{A} \times \mathrm{C})=\{(1,4),(2,4),(3,4),(1,5),(2,5),(3,5)\}$
$\therefore$ LHS $=$ RHS
(ii) $\mathrm{A} \times(\mathrm{B} \cap \mathrm{C})=(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{A} \times \mathrm{C})$

Let us consider LHS: ( $\mathrm{B} \cap \mathrm{C}$ )
$(\mathrm{B} \cap \mathrm{C})=\emptyset($ No common element)
$A \times(B \cap C)=\{1,2,3\} \times \emptyset$

$$
=\varnothing
$$

Now, RHS

$$
\begin{aligned}
(\mathrm{A} \times \mathrm{B}) & =\{1,2,3\} \times\{4\} \\
& =\{(1,4),(2,4),(3,4)\} \\
(\mathrm{A} \times \mathrm{C}) & =\{1,2,3\} \times\{5\} \\
& =\{(1,5),(2,5),(3,5)\}
\end{aligned}
$$

$(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{A} \times \mathrm{C})=\varnothing$
$\therefore$ LHS $=$ RHS
(iii) $\mathrm{A} \times(\mathrm{B}-\mathrm{C})=(\mathrm{A} \times \mathrm{B})-(\mathrm{A} \times \mathrm{C})$

Let us consider LHS: $(\mathrm{B}-\mathrm{C})$
$(\mathrm{B}-\mathrm{C})=\varnothing$
$\mathrm{A} \times(\mathrm{B}-\mathrm{C})=\{1,2,3\} \times \varnothing$

$$
=\varnothing
$$

Now, RHS

$$
\begin{aligned}
(\mathrm{A} \times \mathrm{B}) & =\{1,2,3\} \times\{4\} \\
& =\{(1,4),(2,4),(3,4)\} \\
(\mathrm{A} \times \mathrm{C}) & =\{1,2,3\} \times\{5\} \\
& =\{(1,5),(2,5),(3,5)\}
\end{aligned}
$$

$(\mathrm{A} \times \mathrm{B})-(\mathrm{A} \times \mathrm{C})=\varnothing$
$\therefore$ LHS $=$ RHS
4. Let $A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ and $D=\{5,6,7,8\}$. Verify that:
(i) $\mathbf{A} \times \mathbf{C} \subset \mathbf{B x D}$
(ii) $\mathbf{A x}(\mathbf{B} \cap \mathbf{C})=(\mathbf{A} \times \mathbf{B}) \cap(\mathbf{A} \times \mathbf{C})$

## Solution:

Given:
$A=\{1,2\}, B=\{1,2,3,4\}, C=\{5,6\}$ and $D=\{5,6,7,8\}$
(i) $\mathrm{AxC} \subset \mathrm{B} \times \mathrm{D}$

Let us consider LHS A x C
$A \times C=\{1,2\} \times\{5,6\}$

$$
=\{(1,5),(1,6),(2,5),(2,6)\}
$$

Now, RHS

$$
\begin{aligned}
& \mathrm{B} \times \mathrm{D}=\{1,2,3,4\} \times\{5,6,7,8\} \\
&=\{(1,5),(1,6),(1,7),(1,8),(2,5),(2,6),(2,7),(2,8),(3,5),(3,6),(3,7),(3,8), \\
&(4,5),(4,6),(4,7),(4,8)\}
\end{aligned}
$$

Since, all elements of $\mathrm{A} \times \mathrm{C}$ is in $\mathrm{B} \times \mathrm{D}$.
$\therefore$ We can say $\mathrm{A} \times \mathrm{C} \subset \mathrm{B} \times \mathrm{D}$
(ii) $\mathrm{A} \times(\mathrm{B} \cap \mathrm{C})=(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{A} \times \mathrm{C})$

Let us consider LHS A $\times(\mathrm{B} \cap \mathrm{C})$
$(\mathrm{B} \cap \mathrm{C})=\varnothing$
$A \times(B \cap C)=\{1,2\} \times \varnothing$

$$
=\emptyset
$$

Now, RHS

$$
\begin{aligned}
(\mathrm{A} \times \mathrm{B}) & =\{1,2\} \times\{1,2,3,4\} \\
& =\{(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4)\}
\end{aligned}
$$

$$
\begin{aligned}
(A \times C) & =\{1,2\} \times\{5,6\} \\
& =\{(1,5),(1,6),(2,5),(2,6)\}
\end{aligned}
$$

Since, there is no common element between $\mathrm{A} \times \mathrm{B}$ and $\mathrm{A} \times \mathrm{C}$
$(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{A} \times \mathrm{C})=\varnothing$
$\therefore \mathrm{A} \times(\mathrm{B} \cap \mathrm{C})=(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{A} \times \mathrm{C})$

## 5. If $A=\{1,2,3\}, B=\{3,4\}$ and $C=\{4,5,6\}$, find

(i) $\mathbf{A x}(\mathrm{B} \cap \mathrm{C})$
(ii) $(\mathbf{A} \times \mathbf{B}) \cap(\mathbf{A} \times \mathbf{C})$
(iii) $A x(B \cup C)$
(iv) $(\mathbf{A} \times B) \cup(A \times C)$

## Solution:

Given:
$\mathrm{A}=\{1,2,3\}, \mathrm{B}=\{3,4\}$ and $\mathrm{C}=\{4,5,6\}$
(i) $\mathrm{A} \times(\mathrm{B} \cap \mathrm{C})$
$(B \cap C)=\{4\}$
$A \times(B \cap C)=\{1,2,3\} \times\{4\}$

$$
=\{(1,4),(2,4),(3,4)\}
$$

(ii) $(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{A} \times \mathrm{C})$

$$
\begin{aligned}
(\mathrm{A} \times \mathrm{B}) & =\{1,2,3\} \times\{3,4\} \\
& =\{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\}
\end{aligned}
$$

$(\mathrm{A} \times \mathrm{C})=\{1,2,3\} \times\{4,5,6\}$
$=\{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$
$(\mathrm{A} \times \mathrm{B}) \cap(\mathrm{A} \times \mathrm{C})=\{(1,4),(2,4),(3,4)\}$
(iii) $\mathrm{A} \times(\mathrm{B} \cup \mathrm{C})$
$(B \cup C)=\{3,4,5,6\}$
$A \times(B \cup C)=\{1,2,3\} \times\{3,4,5,6\}$

$$
=\{(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4),(3,5),
$$

$(3,6)\}$
(iv) $(\mathrm{A} \times \mathrm{B}) \cup(\mathrm{A} \times \mathrm{C})$

$$
\begin{aligned}
(\mathrm{A} \times \mathrm{B}) & =\{1,2,3\} \times\{3,4\} \\
& =\{(1,3),(1,4),(2,3),(2,4),(3,3),(3,4)\} \\
(\mathrm{A} \times \mathrm{C}) & =\{1,2,3\} \times\{4,5,6\} \\
& =\{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}
\end{aligned}
$$

$(A \times B) \cup(A \times C)=\{(1,3),(1,4),(1,5),(1,6),(2,3),(2,4),(2,5),(2,6),(3,3),(3,4)$, $(3,5),(3,6)\}$

## 6. Prove that:

(i) $(\mathbf{A} \cup B) \times C=(A \times C)=(A \times C) \cup(B \times C)$
(ii) $(\mathbf{A} \cap B) \times C=(A \times C) \cap(B \times C)$

## Solution:

(i) $(\mathrm{A} \cup \mathrm{B}) \times \mathrm{C}=(\mathrm{A} \times \mathrm{C})=(\mathrm{A} \times \mathrm{C}) \cup(\mathrm{B} \times \mathrm{C})$

Let $(x, y)$ be an arbitrary element of $(A \cup B) \times C$
$(\mathrm{x}, \mathrm{y}) \in(\mathrm{A} \cup \mathrm{B}) \mathrm{C}$
Since, $(x, y)$ are elements of Cartesian product of $(A \cup B) \times C$
$x \in(A \cup B)$ and $y \in C$
$(x \in A$ or $x \in B)$ and $y \in C$
( $x \in A$ and $y \in C$ ) or ( $x \in$ Band $y \in C$ )
$(\mathrm{x}, \mathrm{y}) \in \mathrm{A} \times \mathrm{C}$ or $(\mathrm{x}, \mathrm{y}) \in \mathrm{B} \times \mathrm{C}$
$(\mathrm{x}, \mathrm{y}) \in(\mathrm{A} \times \mathrm{C}) \cup(\mathrm{B} \times \mathrm{C}) \ldots(1)$
Let $(\mathrm{x}, \mathrm{y})$ be an arbitrary element of $(\mathrm{A} \times \mathrm{C}) \cup(\mathrm{B} \times \mathrm{C})$.
$(\mathrm{x}, \mathrm{y}) \in(\mathrm{A} \times \mathrm{C}) \cup(\mathrm{B} \times \mathrm{C})$
$(\mathrm{x}, \mathrm{y}) \in(\mathrm{A} \times \mathrm{C})$ or $(\mathrm{x}, \mathrm{y}) \in(\mathrm{B} \times \mathrm{C})$
$(x \in A$ and $y \in C)$ or $(x \in B$ and $y \in C)$
$(x \in A$ or $x \in B)$ and $y \in C$
$x \in(A \cup B)$ and $y \in C$
$(\mathrm{x}, \mathrm{y}) \in(\mathrm{A} \cup \mathrm{B}) \times \mathrm{C} \ldots$ (2)
From 1 and 2, we get: $(A \cup B) \times C=(A \times C) \cup(B \times C)$
(ii) $(\mathrm{A} \cap \mathrm{B}) \times \mathrm{C}=(\mathrm{A} \times \mathrm{C}) \cap(\mathrm{B} \times \mathrm{C})$

Let $(x, y)$ be an arbitrary element of $(A \cap B) \times C$.
$(\mathrm{x}, \mathrm{y}) \in(\mathrm{A} \cap \mathrm{B}) \times \mathrm{C}$
Since, $(x, y)$ are elements of Cartesian product of $(A \cap B) \times C$
$x \in(A \cap B)$ and $y \in C$
( $x \in A$ and $x \in B$ ) and $y \in C$
( $x \in A$ and $y \in C$ ) and $(x \in$ Band $y \in C)$
$(\mathrm{x}, \mathrm{y}) \in \mathrm{A} \times \mathrm{C}$ and $(\mathrm{x}, \mathrm{y}) \in \mathrm{B} \times \mathrm{C}$
$(x, y) \in(A \times C) \cap(B \times C) \ldots(1)$
Let $(\mathrm{x}, \mathrm{y})$ be an arbitrary element of $(\mathrm{A} \times \mathrm{C}) \cap(\mathrm{B} \times \mathrm{C})$.
$(\mathrm{x}, \mathrm{y}) \in(\mathrm{A} \times \mathrm{C}) \cap(\mathrm{B} \times \mathrm{C})$
$(\mathrm{x}, \mathrm{y}) \in(\mathrm{A} \times \mathrm{C})$ and $(\mathrm{x}, \mathrm{y}) \in(\mathrm{B} \times \mathrm{C})$
( $x \in A$ and $y \in C)$ and $(x \in$ Band $y \in C)$
( $x \in A$ and $x \in B$ ) and $y \in C$
$x \in(A \cap B)$ and $y \in C$
$(x, y) \in(A \cap B) \times C \ldots(2)$
From 1 and 2, we get: $(A \cap B) \times C=(A \times C) \cap(B \times C)$

## 7. If $A \times B \subseteq C \times D$ and $A \cap B \in \emptyset$, Prove that $A \subseteq C$ and $B \subseteq D$. <br> Solution:

Given:
$A \times B \subseteq C \times D$ and $A \cap B \in \emptyset$
$A \times B \subseteq C \times D$ denotes $A \times B$ is subset of $C \times D$ that is every element $A \times B$ is in $C \times D$. And $\mathrm{A} \cap \mathrm{B} \in \emptyset$ denotes A and B does not have any common element between them.
$A \times B=\{(a, b): a \in A$ and $b \in B\}$
$\therefore$ We can say $(\mathrm{a}, \mathrm{b}) \subseteq \mathrm{C} \times \mathrm{D} \quad$ [Since, $\mathrm{A} \times \mathrm{B} \subseteq \mathrm{C} \times \mathrm{D}$ is given]
$a \in C$ and $b \in D$
$a \in A=a \in C$
$\mathrm{A} \subseteq \mathrm{C}$
And
$b \in B=b \in D$
$\mathrm{B} \subseteq \mathrm{D}$
Hence proved.

## EXERCISE 2.3

1. If $A=\{1,2,3\}, B=\{4,5,6\}$, which of the following are relations from $A$ to $B$ ? Give reasons in support of your answer.
(i) $\{(1,6),(3,4),(5,2)\}$
(ii) $\{(1,5),(2,6),(3,4),(3,6)\}$
(iii) $\{(4,2),(4,3),(5,1)\}$
(iv) $\mathbf{A} \times \mathbf{B}$

Solution:
Given,
$A=\{1,2,3\}, B=\{4,5,6\}$
A relation from $A$ to $B$ can be defined as:
$A \times B=\{1,2,3\} \times\{4,5,6\}$ $=\{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$
(i) $\{(1,6),(3,4),(5,2)\}$

No, it is not a relation from $A$ to $B$. The given set is not a subset of $A \times B$ as $(5,2)$ is not a part of the relation from A to B.
(ii) $\{(1,5),(2,6),(3,4),(3,6)\}$

Yes, it is a relation from A to $B$. The given set is a subset of $A \times B$.
(iii) $\{(4,2),(4,3),(5,1)\}$

No, it is not a relation from A to B . The given set is not a subset of $\mathrm{A} \times \mathrm{B}$.
(iv) $\mathrm{A} \times \mathrm{B}$
$\mathrm{A} \times \mathrm{B}$ is a relation from A to B and can be defined as:
$\{(1,4),(1,5),(1,6),(2,4),(2,5),(2,6),(3,4),(3,5),(3,6)\}$
2. A relation $R$ is defined from a set $A=\{2,3,4,5\}$ to a set $B=\{3,6,7,10\}$ as follows: ( $x, y$ ) $R x$ is relatively prime to $y$. Express $R$ as a set of ordered pairs and determine its domain and range.
Solution:
Relatively prime numbers are also known as co-prime numbers. If there is no integer greater than one that divides both (that is, their greatest common divisor is one).

Given: $(\mathrm{x}, \mathrm{y}) \in \mathrm{R}=\mathrm{x}$ is relatively prime to y
Here,
2 is co-prime to 3 and 7 .

3 is co-prime to 7 and 10.
4 is co-prime to 3 and 7 .
5 is co-prime to 3,6 and 7 .
$\therefore \mathrm{R}=\{(2,3),(2,7),(3,7),(3,10),(4,3),(4,7),(5,3),(5,6),(5,7)\}$
Domain of relation $\mathrm{R}=\{2,3,4,5\}$
Range of relation $R=\{3,6,7,10\}$

## 3. Let $A$ be the set of first five natural and let $R$ be a relation on $A$ defined as follows: ( $\mathbf{x}, \mathrm{y}$ ) $\mathbf{R} \mathbf{x} \leq \mathrm{y}$ <br> Express $\mathbf{R}$ and $\mathbf{R}^{-1}$ as sets of ordered pairs. Determine also

(i) the domain of $R^{-1}$
(ii) The Range of $R$.

## Solution:

A is set of first five natural numbers.
So, $A=\{1,2,3,4,5\}$
Given: ( $\mathrm{x}, \mathrm{y}$ ) $\mathrm{R} \mathrm{x} \leq \mathrm{y}$
1 is less than 2, 3, 4 and 5 .
2 is less than 3,4 and 5 .
3 is less than 4 and 5 .
4 is less than 5 .
5 is not less than any number A
$\therefore \mathrm{R}=\{(1,1),(1,2),(1,3),(1,4),(1,5),(2,2),(2,3),(2,4),(2,5),(3,3),(3,4),(3,5)$, $(4,4),(4,5),(5,5)\}$
"An inverse relation is the set of ordered pairs obtained by interchanging the first and second elements of each pair in the original relation. If the graph of a function contains a point ( $a, b$ ), then the graph of the inverse relation of this function contains the point ( $b$, a)".
$\therefore \mathrm{R}^{-1}=\{(1,1),(2,1),(3,1),(4,1),(5,1),(2,2),(3,2),(4,2),(5,2),(3,3),(4,3),(5,3)$, $(4,4),(5,4)(5,5)\}$
(i) Domain of $\mathrm{R}^{-1}=\{1,2,3,4,5\}$
(ii) Range of $R=\{1,2,3,4,5\}$
4. Find the inverse relation $R^{-1}$ in each of the following cases:
(i) $\mathrm{R}=\{(1,2),(1,3),(2,3),(3,2),(5,6)\}$
(ii) $R=\{(x, y): x, y \in N ; x+2 y=8\}$
(iii) $R$ is a relation from $\{11,12,13\}$ to $(8,10,12\}$ defined by $y=x-3$

## Solution:

(i) Given:
$\mathrm{R}=\{(1,2),(1,3),(2,3),(3,2),(5,6)\}$
So, $\mathrm{R}^{-1}=\{(2,1),(3,1),(3,2),(2,3),(6,5)\}$
(ii) Given,
$R=\{(x, y): x, y \in N ; x+2 y=8\}$
Here, $x+2 y=8$
$\mathrm{x}=8-2 \mathrm{y}$
As $y \in N$, Put the values of $y=1,2,3, \ldots \ldots$ till $x \in N$
When, $y=1, x=8-2(1)=8-2=6$
When, $y=2, x=8-2(2)=8-4=4$
When, $y=3, x=8-2(3)=8-6=2$
When, $\mathrm{y}=4, \mathrm{x}=8-2(4)=8-8=0$
Now, y cannot hold value 4 because $\mathrm{x}=0$ for $\mathrm{y}=4$ which is not a natural number.
$\therefore \mathrm{R}=\{(2,3),(4,2),(6,1)\}$
$\mathrm{R}^{-1}=\{(3,2),(2,4),(1,6)\}$
(iii) Given,

R is a relation from $\{11,12,13\}$ to $(8,10,12\}$ defined by $\mathrm{y}=\mathrm{x}-3$
Here,
$\mathrm{x}=\{11,12,13\}$ and $\mathrm{y}=(8,10,12\}$
$y=x-3$
When, $x=11, y=11-3=8 \in(8,10,12\}$
When, $x=12, y=12-3=9 \notin(8,10,12\}$
When, $x=13, y=13-3=10 \in(8,10,12\}$
$\therefore \mathrm{R}=\{(11,8),(13,10)\}$
$R^{-1}=\{(8,11),(10,13)\}$

## 5. Write the following relations as the sets of ordered pairs:

(i) A relation $R$ from the set $\{2,3,4,5,6\}$ to the set $\{1,2,3\}$ defined by $x=2 y$.
(ii) A relation $R$ on the set $\{1,2,3,4,5,6,7\}$ defined by $(x, y) \in R \Leftrightarrow x$ is relatively prime to $y$.
(iii) A relation $R$ on the set $\{0,1,2, \ldots, 10\}$ defined by $2 x+3 y=12$.
(iv) A relation $R$ form a set $A=\{5,6,7,8\}$ to the set $B=\{10,12,15,16,18\}$ defined by ( $\mathbf{x}, \mathrm{y}$ ) $\mathrm{R} x$ divides y .

## Solution:

(i) A relation $R$ from the set $\{2,3,4,5,6\}$ to the set $\{1,2,3\}$ defined by $\mathrm{x}=2 \mathrm{y}$.

Let $\mathrm{A}=\{2,3,4,5,6\}$ and $\mathrm{B}=\{1,2,3\}$

Given, $\mathrm{x}=2 \mathrm{y}$ where $\mathrm{y}=\{1,2,3\}$
When, $\mathrm{y}=1, \mathrm{x}=2(1)=2$
When, $\mathrm{y}=2, \mathrm{x}=2(2)=4$
When, $\mathrm{y}=3, \mathrm{x}=2(3)=6$
$\therefore \mathrm{R}=\{(2,1),(4,2),(6,3)\}$
(ii) A relation R on the set $\{1,2,3,4,5,6,7\}$ defined by $(\mathrm{x}, \mathrm{y}) \in \mathrm{R} \Leftrightarrow \mathrm{x}$ is relatively prime to $y$.
Given:
( $\mathrm{x}, \mathrm{y}$ ) R x is relatively prime to y
Here,
2 is co-prime to 3,5 and 7 .
3 is co-prime to $2,4,5$ and 7 .
4 is co-prime to 3,5 and 7 .
5 is co-prime to $2,3,4,6$ and 7 .
6 is co-prime to 5 and 7 .
7 is co-prime to $2,3,4,5$ and 6 .
$\therefore \mathrm{R}=\{(2,3),(2,5),(2,7),(3,2),(3,4),(3,5),(3,7),(4,3),(4,5),(4,7),(5,2),(5,3),(5$, $4),(5,6),(5,7),(6,5),(6,7),(7,2),(7,3),(7,4),(7,5),(7,6),(7,7)\}$
(iii) A relation R on the set $\{0,1,2, \ldots, 10\}$ defined by $2 \mathrm{x}+3 \mathrm{y}=12$.

Given,
$(x, y) R 2 x+3 y=12$
Where x and $\mathrm{y}=\{0,1,2, \ldots, 10\}$
$2 x+3 y=12$
$2 \mathrm{x}=12-3 \mathrm{y}$
$\mathrm{x}=(12-3 \mathrm{y}) / 2$
When, $\mathrm{y}=0, \mathrm{x}=(12-3(0)) / 2=12 / 2=6$
When, $\mathrm{y}=2, \mathrm{x}=(12-3(2)) / 2=(12-6) / 2=6 / 2=3$
When, $\mathrm{y}=4, \mathrm{x}=(12-3(4)) / 2=(12-12) / 2=0 / 2=0$
$\therefore \mathrm{R}=\{(0,4),(3,2),(6,0)\}$
(iv) A relation R form a set $\mathrm{A}=\{5,6,7,8\}$ to the set $\mathrm{B}=\{10,12,15,16,18\}$ defined by ( $\mathrm{x}, \mathrm{y}$ ) $\in \mathrm{R} \Leftrightarrow \mathrm{x}$ divides y .
Given,
( $\mathrm{x}, \mathrm{y}$ ) R x divides y
Where, $x=\{5,6,7,8\}$ and $y=\{10,12,15,16,18\}$
Here,
5 divides 10 and 15.

6 divides 12 and 18 .
7 divides none of the value of set B.
8 divides 16.
$\therefore \mathrm{R}=\{(5,10),(5,15),(6,12),(6,18),(8,16)\}$
6. Let $R$ be a relation in $N$ defined by $(x, y) \in R \Leftrightarrow x+2 y=8$. Express $R$ and $R^{-1}$ as sets of ordered pairs.
Solution:
Given,
( $\mathrm{x}, \mathrm{y}$ ) $\mathrm{R} \mathrm{x}+2 \mathrm{y}=8$ where $\mathrm{x} \in \mathrm{N}$ and $\mathrm{y} \in \mathrm{N}$
$x+2 y=8$
$x=8-2 y$
Putting the values $\mathrm{y}=1,2,3, \ldots \ldots$ till $\mathrm{x} \in \mathrm{N}$
When, $y=1, x=8-2(1)=8-2=6$
When, $y=2, x=8-2(2)=8-4=4$
When, $y=3, x=8-2(3)=8-6=2$
When, $y=4, x=8-2(4)=8-8=0$
Now, y cannot hold value 4 because $\mathrm{x}=0$ for $\mathrm{y}=4$ which is not a natural number.
$\therefore \mathrm{R}=\{(2,3),(4,2),(6,1)\}$
$\mathrm{R}^{-1}=\{(3,2),(2,4),(1,6)\}$
7. Let $A=\{3,5\}$ and $B=\{7,11\}$. Let $R=\{(a, b): a \in A, b \in B, a-b$ is odd $\}$. Show that $R$ is an empty relation from $A$ into $B$.

## Solution:

Given,
$A=\{3,5\}$ and $B=\{7,11\}$
$R=\{(a, b): a \in A, b \in B, a-b$ is odd $\}$
On putting $a=3$ and $b=7$,
$a-b=3-7=-4$ which is not odd
On putting $\mathrm{a}=3$ and $\mathrm{b}=11$,
$\mathrm{a}-\mathrm{b}=3-11=-8$ which is not odd
On putting $\mathrm{a}=5$ and $\mathrm{b}=7$ :
$\mathrm{a}-\mathrm{b}=5-7=-2$ which is not odd
On putting $\mathrm{a}=5$ and $\mathrm{b}=11$ :
$a-b=5-11=-6$ which is not odd
$\therefore \mathrm{R}=\{ \}=\Phi$
$R$ is an empty relation from $A$ into $B$.
Hence proved.
8. Let $A=\{1,2\}$ and $B=\{3,4\}$. Find the total number of relations from $A$ into $B$. Solution:
Given,
$\mathrm{A}=\{1,2\}, \mathrm{B}=\{3,4\}$
$\mathrm{n}(\mathrm{A})=2$ (Number of elements in set A$)$.
$n(B)=2($ Number of elements in set B).
We know,

$$
\begin{aligned}
\mathrm{n}(\mathrm{~A} \times \mathrm{B}) & =\mathrm{n}(\mathrm{~A}) \times \mathrm{n}(\mathrm{~B}) \\
& =2 \times 2
\end{aligned}
$$

$$
=4 \quad\left[\text { since, } n(x)=a, n(y)=b \text {. total number of relations }=2^{a b}\right]
$$

$\therefore$ Number of relations from A to B are $2^{4}=16$.
9. Determine the domain and range of the relation $R$ defined by
(i) $R=\{(x, x+5): x \in\{0,1,2,3,4,5\}$
(ii) $R=\left\{\left(x, x^{3}\right): x\right.$ is a prime number less than 10$\}$

Solution:
(i) $R=\{(x, x+5): x \in\{0,1,2,3,4,5\}$

Given,
$R=\{(x, x+5): x \in\{0,1,2,3,4,5\}$
$\therefore \mathrm{R}=\{(0,0+5),(1,1+5),(2,2+5),(3,3+5),(4,4+5),(5,5+5)\}$
$R=\{(0,5),(1,6),(2,7),(3,8),(4,9),(5,10)\}$
So,
Domain of relation $\mathrm{R}=\{0,1,2,3,4,5\}$
Range of relation $R=\{5,6,7,8,9,10\}$
(ii) $R=\left\{\left(x, x^{3}\right): x\right.$ is a prime number less than 10$\}$

Given,
$R=\left\{\left(x, x^{3}\right): x\right.$ is a prime number less than 10$\}$
Prime numbers less than 10 are 2,3,5 and 7
$\therefore \mathrm{R}=\left\{\left(2,2^{3}\right),\left(3,3^{3}\right),\left(5,5^{3}\right),\left(7,7^{3}\right)\right\}$
$R=\{(2,8),(3,27),(5,125),(7,343)\}$
So,
Domain of relation $R=\{2,3,5,7\}$
Range of relation $R=\{8,27,125,343\}$
10. Determine the domain and range of the following relations:
(i) $R=\{a, b): a \in N, a<5, b=4\}$
(ii) $S=\{a, b): b=|a-1|, a \in Z$ and $|a| \leq 3\}$

## Solution:

(i) $\mathrm{R}=\{\mathrm{a}, \mathrm{b}): \mathrm{a} \in \mathrm{N}, \mathrm{a}<5, \mathrm{~b}=4\}$

Given,
$\mathrm{R}=\{\mathrm{a}, \mathrm{b}): \mathrm{a} \in \mathrm{N}, \mathrm{a}<5, \mathrm{~b}=4\}$
Natural numbers less than 5 are 1,2,3 and 4
$\mathrm{a}=\{1,2,3,4\}$ and $\mathrm{b}=\{4\}$
$\mathrm{R}=\{(1,4),(2,4),(3,4),(4,4)\}$
So,
Domain of relation $\mathrm{R}=\{1,2,3,4\}$
Range of relation $R=\{4\}$
(ii) $\mathrm{S}=\{\mathrm{a}, \mathrm{b}): \mathrm{b}=|\mathrm{a}-1|, \mathrm{a} \in \mathrm{Z}$ and $|\mathrm{a}| \leq 3\}$

Given,
$\mathrm{S}=\{\mathrm{a}, \mathrm{b}): \mathrm{b}=|\mathrm{a}-1|, \mathrm{a} \in \mathrm{Z}$ and $|\mathrm{a}| \leq 3\}$
Z denotes integer which can be positive as well as negative
Now, $|a| \leq 3$ and $b=|a-1|$
$\therefore \mathrm{a}=\{-3,-2,-1,0,1,2,3\}$
For, $\mathrm{a}=-3,-2,-1,0,1,2,3$ we get,
$\mathrm{S}=\{(-3,|-3-1|),(-2,|-2-1|),(-1,|-1-1|),(0,|0-1|),(1,|1-1|),(2,|2-1|),(3, \mid 3-$

1) $\}$
$S=\{(-3,|-4|),(-2,|-3|),(-1,|-2|),(0,|-1|),(1,|0|),(2,|1|),(3,|2|)\}$
$\mathrm{S}=\{(-3,4),(-2,3),(-1,2),(0,1),(1,0),(2,1),(3,2)\}$
$\mathrm{b}=4,3,2,1,0,1,2$
So,
Domain of relation $S=\{0,-1,-2,-3,1,2,3\}$
Range of relation $S=\{0,1,2,3,4\}$

## 11. Let $A=\{a, b\}$. List all relations on $A$ and find their number. <br> Solution:

The total number of relations that can be defined from a set $A$ to a set $B$ is the number of possible subsets of $A \times B$. If $n(A)=p$ and $n(B)=q$, then $n(A \times B)=p q$.
So, the total number of relations is $2^{\mathrm{pq}}$.
Now,
$\mathrm{A} \times \mathrm{A}=\{(\mathrm{a}, \mathrm{a}),(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{b})\}$
Total number of relations are all possible subsets of $\mathrm{A} \times \mathrm{A}$ :
$[\{(\mathrm{a}, \mathrm{a}),(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{b})\},\{(\mathrm{a}, \mathrm{a}),(\mathrm{a}, \mathrm{b})\},\{(\mathrm{a}, \mathrm{a}),(\mathrm{b}, \mathrm{a})\},\{(\mathrm{a}, \mathrm{a}),(\mathrm{b}, \mathrm{b})\},\{(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{a})\}$, $\{(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{b})\},\{(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{b})\},\{(\mathrm{a}, \mathrm{a}),(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{a})\},\{(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{b})\},\{(\mathrm{a}, \mathrm{a}),(\mathrm{b}, \mathrm{a})$, (b, b) $\},\{(\mathrm{a}, \mathrm{a}),(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{b})\},\{(\mathrm{a}, \mathrm{a}),(\mathrm{a}, \mathrm{b}),(\mathrm{b}, \mathrm{a}),(\mathrm{b}, \mathrm{b})\}]$
$\mathrm{n}(\mathrm{A})=2 \Rightarrow \mathrm{n}(\mathrm{A} \times \mathrm{A})=2 \times 2=4$
$\therefore$ Total number of relations $=2^{4}=16$

RD Sharma Solutions for Class 11 Maths Chapter 2 Relations

