EXERCISE 3.2 PAGE NO: 3.11

1. If f (x) = x^2 – 3x + 4, then find the values of x satisfying the equation f (x) = f (2x + 1).

Solution:

Given:

f(x) = $x^2 - 3x + 4$.

Let us find x satisfying f (x) = f (2x + 1).

We have,

$f(2x + 1) = (2x + 1)^2 - 3(2x + 1) + 4$

$= (2x)^2 + 2(2x)(1) + 1^2 - 6x - 3 + 4$

$= 4x^2 + 4x + 1 - 6x + 1$

$= 4x^2 - 2x + 2$

Now, f (x) = f (2x + 1)

$x^2 - 3x + 4 = 4x^2 - 2x + 2$

$4x^2 - 2x + 2 - x^2 + 3x - 4 = 0$

$3x^2 + x - 2 = 0$

$3x^2 + 3x - 2x - 2 = 0$

$3x(x + 1) - 2(x + 1) = 0$

$(x + 1)(3x - 2) = 0$

x + 1 = 0 or 3x – 2 = 0

x = –1 or 3x = 2

x = –1 or 2/3

∴ The values of x are –1 and 2/3.

2. If f (x) = $(x - a)^2 (x - b)^2$, find f (a + b).

Solution:

Given:

F (x) = $(x - a)^2(x - b)^2$

Let us find f (a + b).

We have,

$f(a + b) = (a + b - a)^2 (a + b - b)^2$

$f(a + b) = (b)^2 (a)^2$

∴ $f(a + b) = a^2b^2$

3. If y = f (x) = (ax - b) / (bx - a), show that x = f (y).

Solution:

Given:

y = f (x) = (ax - b) / (bx - a) ⇒ f (y) = (ay - b) / (by - a)
Let us prove that x = f (y).
We have,
y = (ax - b) / (bx - a)
By cross-multiplying,
y(bx – a) = ax – b
bxy – ay = ax – b
bxy – ax = ay – b
x(by – a) = ay – b
x = (ay - b) / (by - a) = f (y)
∴ x = f (y)
Hence proved.

4. If f (x) = 1 / (1 - x), show that f [f {f (x)}] = x.
Solution:
Given:
f (x) = 1 / (1 - x)
Let us prove that f [f {f (x)}] = x.
Firstly, let us solve for f {f (x)}.
f {f (x)} = f {1/(1 - x)}
= 1 / 1 - (1/(1 - x))
= 1 / [(1 – x - 1)/(1 - x)]
= 1 / (-x/(1 - x))
= (1 - x) / -x
= (x – 1) / x
∴ f {f (x)} = (x – 1) / x

Now, we shall solve for f [f {f (x)}]
f [f {f (x)}] = f [(x-1)/x]
= 1 / [1 – (x-1)/x]
= 1 / [(x – (x-1))/x]
= 1 / [(x - x + 1)/x]
= 1 / (1/x)
∴ f [f {f (x)}] = x
Hence proved.

5. If f (x) = (x + 1) / (x - 1), show that f [f (x)] = x.
Solution:
Given:

f (x) = (x + 1) / (x - 1)
Let us prove that f [f (x)] = x.
f [f (x)] = f [(x+1)/(x-1)]
= [(x+1)/(x-1) + 1] / [(x+1)/(x-1) - 1]
= [[(x+1) + (x-1)]/(x-1)] / [[(x+1) – (x-1)]/(x-1)]
= [(x+1) + (x-1)] / [(x+1) – (x-1)]
= (x+1+x-1)/(x+1-x+1)
= 2x/2
= x
∴ f [f (x)] = x
Hence proved.

6. If

$$f(x) = \begin{cases} x^2, \text{when } x < 0 \\ x, \text{when } 0 \le x < 1 \\ \frac{1}{x}, \text{when } x \ge 1 \end{cases}$$

Find:
(i) f (1/2)
(ii) f (-2)
(iii) f (1)
(iv) f (√3)
(v) f (√-3)
Solution:
(i) f (1/2)
When, 0 ≤ x ≤ 1, f(x) = x
∴ f (1/2) = ½

(ii) f (-2)
When, x < 0, f(x) = x^2
f (–2) = $(–2)^2$
= 4
∴ f (–2) = 4

(iii) f (1)
When, x ≥ 1, f (x) = 1/x
f (1) = 1/1
∴ f(1) = 1

(iv) f (√3)
We have √3 = 1.732 > 1
When, x ≥ 1, f (x) = 1/x
∴ f (√3) = 1/√3

(v) f (√-3)
We know √-3 is not a real number and the function f(x) is defined only when x ∈ R.
∴ f (√-3) does not exist.