

**EXERCISE 3.1****PAGE NO: 3.7****1. Define a function as a set of ordered pairs.****Solution:**

Let A and B be two non-empty sets. A relation from A to B, i.e., a subset of  $A \times B$ , is called a function (or a mapping) from A to B, if

- (i) for each  $a \in A$  there exists  $b \in B$  such that  $(a, b) \in f$
- (ii)  $(a, b) \in f$  and  $(a, c) \in f \Rightarrow b = c$

**2. Define a function as a correspondence between two sets.****Solution:**

Let A and B be two non-empty sets. Then a function 'f' from set A to B is a rule or method or correspondence which associates elements of set A to elements of set B such that:

- (i) all elements of set A are associated to elements in set B.
- (ii) an element of set A is associated to a unique element in set B.

**3. What is the fundamental difference between a relation and a function? Is every relation a function?****Solution:**

Let 'f' be a function and R be a relation defined from set X to set Y.

The domain of the relation R might be a subset of the set X, but the domain of the function f must be equal to X. This is because each element of the domain of a function must have an element associated with it, whereas this is not necessary for a relation.

In relation, one element of X might be associated with one or more elements of Y, while it must be associated with only one element of Y in a function.

Thus, not every relation is a function. However, every function is necessarily a relation.

**4. Let  $A = \{-2, -1, 0, 1, 2\}$  and  $f: A \rightarrow Z$  be a function defined by  $f(x) = x^2 - 2x - 3$ .****Find:**

- (i) range of f i.e.  $f(A)$
- (ii) pre-images of 6, -3 and 5

**Solution:**

Given:

$$A = \{-2, -1, 0, 1, 2\}$$

$$f: A \rightarrow Z \text{ such that } f(x) = x^2 - 2x - 3$$

(i) Range of  $f$  i.e.  $f(A)$

$A$  is the domain of the function  $f$ . Hence, range is the set of elements  $f(x)$  for all  $x \in A$ .

Substituting  $x = -2$  in  $f(x)$ , we get

$$\begin{aligned}f(-2) &= (-2)^2 - 2(-2) - 3 \\&= 4 + 4 - 3 \\&= 5\end{aligned}$$

Substituting  $x = -1$  in  $f(x)$ , we get

$$\begin{aligned}f(-1) &= (-1)^2 - 2(-1) - 3 \\&= 1 + 2 - 3 \\&= 0\end{aligned}$$

Substituting  $x = 0$  in  $f(x)$ , we get

$$\begin{aligned}f(0) &= (0)^2 - 2(0) - 3 \\&= 0 - 0 - 3 \\&= -3\end{aligned}$$

Substituting  $x = 1$  in  $f(x)$ , we get

$$\begin{aligned}f(1) &= 1^2 - 2(1) - 3 \\&= 1 - 2 - 3 \\&= -4\end{aligned}$$

Substituting  $x = 2$  in  $f(x)$ , we get

$$\begin{aligned}f(2) &= 2^2 - 2(2) - 3 \\&= 4 - 4 - 3 \\&= -3\end{aligned}$$

Thus, the range of  $f$  is  $\{-4, -3, 0, 5\}$ .

(ii) pre-images of 6,  $-3$  and 5

Let  $x$  be the pre-image of 6  $\Rightarrow f(x) = 6$

$$x^2 - 2x - 3 = 6$$

$$x^2 - 2x - 9 = 0$$

$$\begin{aligned}x &= \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-9)}}{2(1)} \\&= \frac{2 \pm \sqrt{4+36}}{2} \\&= \frac{2 \pm \sqrt{40}}{2} \\&= 1 \pm \sqrt{10}\end{aligned}$$

However,  $1 \pm \sqrt{10} \notin A$

Thus, there exists no pre-image of 6.

Now, let  $x$  be the pre-image of  $-3 \Rightarrow f(x) = -3$

$$x^2 - 2x - 3 = -3$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$x = 0 \text{ or } 2$$

Clearly, both 0 and 2 are elements of A.

Thus, 0 and 2 are the pre-images of -3.

Now, let x be the pre-image of 5  $\Rightarrow f(x) = 5$

$$x^2 - 2x - 3 = 5$$

$$x^2 - 2x - 8 = 0$$

$$x^2 - 4x + 2x - 8 = 0$$

$$x(x - 4) + 2(x - 4) = 0$$

$$(x + 2)(x - 4) = 0$$

$$x = -2 \text{ or } 4$$

However,  $4 \notin A$  but  $-2 \in A$

Thus, -2 is the pre-images of 5.

$\therefore \emptyset, \{0, 2\}, -2$  are the pre-images of 6, -3, 5

**5. If a function  $f: \mathbb{R} \rightarrow \mathbb{R}$  be defined by**

$$f(x) = \begin{cases} 3x - 2, & x < 0 \\ 1, & x = 0 \\ 4x + 1, & x > 0 \end{cases}$$

**Find:  $f(1), f(-1), f(0), f(2)$ .**

**Solution:**

Given:

Let us find  $f(1), f(-1), f(0)$  and  $f(2)$ .

When  $x > 0$ ,  $f(x) = 4x + 1$

Substituting  $x = 1$  in the above equation, we get

$$\begin{aligned} f(1) &= 4(1) + 1 \\ &= 4 + 1 \\ &= 5 \end{aligned}$$

When  $x < 0$ ,  $f(x) = 3x - 2$

Substituting  $x = -1$  in the above equation, we get

$$\begin{aligned} f(-1) &= 3(-1) - 2 \\ &= -3 - 2 \\ &= -5 \end{aligned}$$

When  $x = 0$ ,  $f(x) = 1$

Substituting  $x = 0$  in the above equation, we get

$$f(0) = 1$$

When  $x > 0$ ,  $f(x) = 4x + 1$

Substituting  $x = 2$  in the above equation, we get

$$f(2) = 4(2) + 1$$

$$= 8 + 1$$

$$= 9$$

$\therefore f(1) = 5$ ,  $f(-1) = -5$ ,  $f(0) = 1$  and  $f(2) = 9$ .

**6. A function  $f: \mathbf{R} \rightarrow \mathbf{R}$  is defined by  $f(x) = x^2$ . Determine**

**(i) range of  $f$**

**(ii)  $\{x: f(x) = 4\}$**

**(iii)  $\{y: f(y) = -1\}$**

**Solution:**

Given:

$$f: \mathbf{R} \rightarrow \mathbf{R} \text{ and } f(x) = x^2.$$

**(i) range of  $f$**

Domain of  $f = \mathbf{R}$  (set of real numbers)

We know that the square of a real number is always positive or equal to zero.

$$\therefore \text{range of } f = \mathbf{R}^+ \cup \{0\}$$

**(ii)  $\{x: f(x) = 4\}$**

Given:

$$f(x) = 4$$

$$\text{we know, } x^2 = 4$$

$$x^2 - 4 = 0$$

$$(x - 2)(x + 2) = 0$$

$$\therefore x = \pm 2$$

$$\therefore \{x: f(x) = 4\} = \{-2, 2\}$$

**(iii)  $\{y: f(y) = -1\}$**

Given:

$$f(y) = -1$$

$$y^2 = -1$$

However, the domain of  $f$  is  $\mathbf{R}$ , and for every real number  $y$ , the value of  $y^2$  is non-negative.

Hence, there exists no real  $y$  for which  $y^2 = -1$ .

$$\therefore \{y: f(y) = -1\} = \emptyset$$

**7. Let  $f: \mathbf{R}^+ \rightarrow \mathbf{R}$ , where  $\mathbf{R}^+$  is the set of all positive real numbers, be such that  $f(x) =$**

**$\log_e x$ . Determine**

**(i) the image set of the domain of  $f$**

**(ii)  $\{x: f(x) = -2\}$**

**(iii) whether  $f(xy) = f(x) + f(y)$  holds.**

**Solution:**

Given  $f: \mathbb{R}^+ \rightarrow \mathbb{R}$  and  $f(x) = \log_e x$ .

**(i) the image set of the domain of  $f$**

Domain of  $f = \mathbb{R}^+$  (set of positive real numbers)

We know the value of logarithm to the base  $e$  (natural logarithm) can take all possible real values.

$\therefore$  The image set of  $f = \mathbb{R}$

**(ii)  $\{x: f(x) = -2\}$**

Given  $f(x) = -2$

$\log_e x = -2$

$\therefore x = e^{-2}$  [since,  $\log_b a = c \Rightarrow a = b^c$ ]

$\therefore \{x: f(x) = -2\} = \{e^{-2}\}$

**(iii) Whether  $f(xy) = f(x) + f(y)$  holds.**

We have  $f(x) = \log_e x \Rightarrow f(y) = \log_e y$

Now, let us consider  $f(xy)$

$f(xy) = \log_e(xy)$

$f(xy) = \log_e(x \times y)$  [since,  $\log_b(a \times c) = \log_b a + \log_b c$ ]

$f(xy) = \log_e x + \log_e y$

$f(xy) = f(x) + f(y)$

$\therefore$  the equation  $f(xy) = f(x) + f(y)$  holds.

**8. Write the following relations as sets of ordered pairs and find which of them are functions:**

**(i)  $\{(x, y): y = 3x, x \in \{1, 2, 3\}, y \in \{3, 6, 9, 12\}\}$**

**(ii)  $\{(x, y): y > x + 1, x = 1, 2 \text{ and } y = 2, 4, 6\}$**

**(iii)  $\{(x, y): x + y = 3, x, y \in \{0, 1, 2, 3\}\}$**

**Solution:**

**(i)  $\{(x, y): y = 3x, x \in \{1, 2, 3\}, y \in \{3, 6, 9, 12\}\}$**

When  $x = 1$ ,  $y = 3(1) = 3$

When  $x = 2$ ,  $y = 3(2) = 6$

When  $x = 3$ ,  $y = 3(3) = 9$

$\therefore R = \{(1, 3), (2, 6), (3, 9)\}$

Hence, the given relation  $R$  is a function.

(ii)  $\{(x, y): y > x + 1, x = 1, 2 \text{ and } y = 2, 4, 6\}$

When  $x = 1$ ,  $y > 1 + 1$  or  $y > 2 \Rightarrow y = \{4, 6\}$

When  $x = 2$ ,  $y > 2 + 1$  or  $y > 3 \Rightarrow y = \{4, 6\}$

$\therefore R = \{(1, 4), (1, 6), (2, 4), (2, 6)\}$

Hence, the given relation  $R$  is not a function.

(iii)  $\{(x, y): x + y = 3, x, y \in \{0, 1, 2, 3\}\}$

When  $x = 0$ ,  $0 + y = 3 \Rightarrow y = 3$

When  $x = 1$ ,  $1 + y = 3 \Rightarrow y = 2$

When  $x = 2$ ,  $2 + y = 3 \Rightarrow y = 1$

When  $x = 3$ ,  $3 + y = 3 \Rightarrow y = 0$

$\therefore R = \{(0, 3), (1, 2), (2, 1), (3, 0)\}$

Hence, the given relation  $R$  is a function.

**9. Let  $f: R \rightarrow R$  and  $g: C \rightarrow C$  be two functions defined as  $f(x) = x^2$  and  $g(x) = x^2$ . Are they equal functions?**

**Solution:**

Given:

$f: R \rightarrow R \in f(x) = x^2$  and  $g: R \rightarrow R \in g(x) = x^2$

$f$  is defined from  $R$  to  $R$ , the domain of  $f = R$ .

$g$  is defined from  $C$  to  $C$ , the domain of  $g = C$ .

Two functions are equal only when the domain and codomain of both the functions are equal.

In this case, the domain of  $f \neq$  domain of  $g$ .

$\therefore f$  and  $g$  are not equal functions.

## EXERCISE 3.2

PAGE NO: 3.11

1. If  $f(x) = x^2 - 3x + 4$ , then find the values of  $x$  satisfying the equation  $f(x) = f(2x + 1)$ .

**Solution:**

Given:

$$f(x) = x^2 - 3x + 4.$$

Let us find  $x$  satisfying  $f(x) = f(2x + 1)$ .

We have,

$$\begin{aligned} f(2x + 1) &= (2x + 1)^2 - 3(2x + 1) + 4 \\ &= (2x)^2 + 2(2x)(1) + 1^2 - 6x - 3 + 4 \\ &= 4x^2 + 4x + 1 - 6x + 1 \\ &= 4x^2 - 2x + 2 \end{aligned}$$

$$\text{Now, } f(x) = f(2x + 1)$$

$$x^2 - 3x + 4 = 4x^2 - 2x + 2$$

$$4x^2 - 2x + 2 - x^2 + 3x - 4 = 0$$

$$3x^2 + x - 2 = 0$$

$$3x^2 + 3x - 2x - 2 = 0$$

$$3x(x + 1) - 2(x + 1) = 0$$

$$(x + 1)(3x - 2) = 0$$

$$x + 1 = 0 \text{ or } 3x - 2 = 0$$

$$x = -1 \text{ or } 3x = 2$$

$$x = -1 \text{ or } 2/3$$

$\therefore$  The values of  $x$  are  $-1$  and  $2/3$ .

2. If  $f(x) = (x - a)^2(x - b)^2$ , find  $f(a + b)$ .

**Solution:**

Given:

$$F(x) = (x - a)^2(x - b)^2$$

Let us find  $f(a + b)$ .

We have,

$$f(a + b) = (a + b - a)^2(a + b - b)^2$$

$$f(a + b) = (b)^2(a)^2$$

$$\therefore f(a + b) = a^2b^2$$

3. If  $y = f(x) = (ax - b) / (bx - a)$ , show that  $x = f(y)$ .

**Solution:**

Given:



$$y = f(x) = (ax - b) / (bx - a) \Rightarrow f(y) = (ay - b) / (by - a)$$

Let us prove that  $x = f(y)$ .

We have,

$$y = (ax - b) / (bx - a)$$

By cross-multiplying,

$$y(bx - a) = ax - b$$

$$bxy - ay = ax - b$$

$$bxy - ax = ay - b$$

$$x(by - a) = ay - b$$

$$x = (ay - b) / (by - a) = f(y)$$

$$\therefore x = f(y)$$

Hence proved.

**4. If  $f(x) = 1 / (1 - x)$ , show that  $f[f\{f(x)\}] = x$ .**

**Solution:**

Given:

$$f(x) = 1 / (1 - x)$$

Let us prove that  $f[f\{f(x)\}] = x$ .

Firstly, let us solve for  $f\{f(x)\}$ .

$$\begin{aligned} f\{f(x)\} &= f\{1/(1-x)\} \\ &= 1 / 1 - (1/(1-x)) \\ &= 1 / [(1-x-1)/(1-x)] \\ &= 1 / (-x/(1-x)) \\ &= (1-x) / -x \\ &= (x-1) / x \end{aligned}$$

$$\therefore f\{f(x)\} = (x-1) / x$$

Now, we shall solve for  $f[f\{f(x)\}]$

$$\begin{aligned} f[f\{f(x)\}] &= f[(x-1)/x] \\ &= 1 / [1 - (x-1)/x] \\ &= 1 / [(x - (x-1))/x] \\ &= 1 / [(x - x + 1)/x] \\ &= 1 / (1/x) \end{aligned}$$

$$\therefore f[f\{f(x)\}] = x$$

Hence proved.

**5. If  $f(x) = (x + 1) / (x - 1)$ , show that  $f[f(x)] = x$ .**

**Solution:**

Given:



$$f(x) = (x + 1) / (x - 1)$$

Let us prove that  $f[f(x)] = x$ .

$$\begin{aligned} f[f(x)] &= f[(x+1)/(x-1)] \\ &= [(x+1)/(x-1) + 1] / [(x+1)/(x-1) - 1] \\ &= [[(x+1) + (x-1)]/(x-1)] / [[(x+1) - (x-1)]/(x-1)] \\ &= [(x+1) + (x-1)] / [(x+1) - (x-1)] \\ &= (x+1+x-1)/(x+1-x+1) \\ &= 2x/2 \\ &= x \end{aligned}$$

$$\therefore f[f(x)] = x$$

Hence proved.

**6. If**

$$f(x) = \begin{cases} x^2, & \text{when } x < 0 \\ x, & \text{when } 0 \leq x < 1 \\ \frac{1}{x}, & \text{when } x \geq 1 \end{cases}$$

**Find:**

(i)  $f(1/2)$

(ii)  $f(-2)$

(iii)  $f(1)$

(iv)  $f(\sqrt{3})$

(v)  $f(\sqrt{-3})$

**Solution:**

(i)  $f(1/2)$

When,  $0 \leq x \leq 1$ ,  $f(x) = x$

$$\therefore f(1/2) = 1/2$$

(ii)  $f(-2)$

When,  $x < 0$ ,  $f(x) = x^2$

$$\begin{aligned} f(-2) &= (-2)^2 \\ &= 4 \end{aligned}$$

$$\therefore f(-2) = 4$$

(iii)  $f(1)$

When,  $x \geq 1$ ,  $f(x) = 1/x$

$$f(1) = 1/1$$

$$\therefore f(1) = 1$$

(iv)  $f(\sqrt{3})$

We have  $\sqrt{3} = 1.732 > 1$

When,  $x \geq 1$ ,  $f(x) = 1/x$

$$\therefore f(\sqrt{3}) = 1/\sqrt{3}$$

(v)  $f(\sqrt{-3})$

We know  $\sqrt{-3}$  is not a real number and the function  $f(x)$  is defined only when  $x \in \mathbb{R}$ .

$\therefore f(\sqrt{-3})$  does not exist.



## EXERCISE 3.3

PAGE NO: 3.18

**1. Find the domain of each of the following real valued functions of real variable:**

**(i)**  $f(x) = 1/x$

**(ii)**  $f(x) = 1/(x-7)$

**(iii)**  $f(x) = (3x-2)/(x+1)$

**(iv)**  $f(x) = (2x+1)/(x^2-9)$

**(v)**  $f(x) = (x^2+2x+1)/(x^2-8x+12)$

**Solution:**

**(i)**  $f(x) = 1/x$

We know,  $f(x)$  is defined for all real values of  $x$ , except for the case when  $x = 0$ .

$$\therefore \text{Domain of } f = \mathbb{R} - \{0\}$$

**(ii)**  $f(x) = 1/(x-7)$

We know,  $f(x)$  is defined for all real values of  $x$ , except for the case when  $x - 7 = 0$  or  $x = 7$ .

$$\therefore \text{Domain of } f = \mathbb{R} - \{7\}$$

**(iii)**  $f(x) = (3x-2)/(x+1)$

We know,  $f(x)$  is defined for all real values of  $x$ , except for the case when  $x + 1 = 0$  or  $x = -1$ .

$$\therefore \text{Domain of } f = \mathbb{R} - \{-1\}$$

**(iv)**  $f(x) = (2x+1)/(x^2-9)$

We know,  $f(x)$  is defined for all real values of  $x$ , except for the case when  $x^2 - 9 = 0$ .

$$x^2 - 9 = 0$$

$$x^2 - 3^2 = 0$$

$$(x+3)(x-3) = 0$$

$$x+3 = 0 \text{ or } x-3 = 0$$

$$x = \pm 3$$

$$\therefore \text{Domain of } f = \mathbb{R} - \{-3, 3\}$$

**(v)**  $f(x) = (x^2+2x+1)/(x^2-8x+12)$

We know,  $f(x)$  is defined for all real values of  $x$ , except for the case when  $x^2 - 8x + 12 = 0$ .

$$x^2 - 8x + 12 = 0$$

$$x^2 - 2x - 6x + 12 = 0$$

$$x(x-2) - 6(x-2) = 0$$

$$(x-2)(x-6) = 0$$

$$x - 2 = 0 \text{ or } x - 6 = 0$$

$$x = 2 \text{ or } 6$$

$$\therefore \text{Domain of } f = \mathbb{R} - \{2, 6\}$$

**2. Find the domain of each of the following real valued functions of real variable:**

**(i)**  $f(x) = \sqrt{x-2}$

**(ii)**  $f(x) = 1/(\sqrt{x^2-1})$

**(iii)**  $f(x) = \sqrt{9-x^2}$

**(iv)**  $f(x) = \sqrt{x-2}/(3-x)$

**Solution:**

**(i)**  $f(x) = \sqrt{x-2}$

We know the square of a real number is never negative.

$f(x)$  takes real values only when  $x - 2 \geq 0$

$$x \geq 2$$

$$\therefore x \in [2, \infty)$$

$$\therefore \text{Domain } (f) = [2, \infty)$$

**(ii)**  $f(x) = 1/(\sqrt{x^2-1})$

We know the square of a real number is never negative.

$f(x)$  takes real values only when  $x^2 - 1 \geq 0$

$$x^2 - 1^2 \geq 0$$

$$(x + 1)(x - 1) \geq 0$$

$$x \leq -1 \text{ or } x \geq 1$$

$$\therefore x \in (-\infty, -1] \cup [1, \infty)$$

In addition,  $f(x)$  is also undefined when  $x^2 - 1 = 0$  because denominator will be zero and the result will be indeterminate.

$$x^2 - 1 = 0 \Rightarrow x = \pm 1$$

$$\text{So, } x \in (-\infty, -1] \cup [1, \infty) - \{-1, 1\}$$

$$x \in (-\infty, -1) \cup (1, \infty)$$

$$\therefore \text{Domain } (f) = (-\infty, -1) \cup (1, \infty)$$

**(iii)**  $f(x) = \sqrt{9-x^2}$

We know the square of a real number is never negative.

$f(x)$  takes real values only when  $9 - x^2 \geq 0$

$$9 \geq x^2$$

$$x^2 \leq 9$$

$$x^2 - 9 \leq 0$$

$$x^2 - 3^2 \leq 0$$

$$(x + 3)(x - 3) \leq 0$$

$$x \geq -3 \text{ and } x \leq 3$$

$$x \in [-3, 3]$$

$$\therefore \text{Domain (f)} = [-3, 3]$$

$$\text{(iv) } f(x) = \sqrt{(x-2)/(3-x)}$$

We know the square root of a real number is never negative.

$f(x)$  takes real values only when  $x - 2$  and  $3 - x$  are both positive and negative.

(a) Both  $x - 2$  and  $3 - x$  are positive

$$x - 2 \geq 0$$

$$x \geq 2$$

$$3 - x \geq 0$$

$$x \leq 3$$

Hence,  $x \geq 2$  and  $x \leq 3$

$$\therefore x \in [2, 3]$$

(b) Both  $x - 2$  and  $3 - x$  are negative

$$x - 2 \leq 0$$

$$x \leq 2$$

$$3 - x \leq 0$$

$$x \geq 3$$

Hence,  $x \leq 2$  and  $x \geq 3$

However, the intersection of these sets is null set. Thus, this case is not possible.

Hence,  $x \in [2, 3] - \{3\}$

$$x \in [2, 3]$$

$$\therefore \text{Domain (f)} = [2, 3]$$

**3. Find the domain and range of each of the following real valued functions:**

(i)  $f(x) = (ax+b)/(bx-a)$

(ii)  $f(x) = (ax-b)/(cx-d)$

(iii)  $f(x) = \sqrt{x-1}$

(iv)  $f(x) = \sqrt{x-3}$

(v)  $f(x) = (x-2)/(2-x)$

(vi)  $f(x) = |x-1|$

(vii)  $f(x) = -|x|$

(viii)  $f(x) = \sqrt{9-x^2}$

**Solution:**

(i)  $f(x) = (ax+b)/(bx-a)$

$f(x)$  is defined for all real values of  $x$ , except for the case when  $bx - a = 0$  or  $x = a/b$ .

$$\text{Domain } (f) = \mathbb{R} - (a/b)$$

$$\text{Let } f(x) = y$$

$$(ax+b)/(bx-a) = y$$

$$ax + b = y(bx - a)$$

$$ax + b = bxy - ay$$

$$ax - bxy = -ay - b$$

$$x(a - by) = -(ay + b)$$

$$\therefore x = -(ay+b)/(a-by)$$

$$\text{When } a - by = 0 \text{ or } y = a/b$$

Hence,  $f(x)$  cannot take the value  $a/b$ .

$$\therefore \text{Range } (f) = \mathbb{R} - (a/b)$$

$$\text{(ii) } f(x) = (ax-b)/(cx-d)$$

$f(x)$  is defined for all real values of  $x$ , except for the case when  $cx - d = 0$  or  $x = d/c$ .

$$\text{Domain } (f) = \mathbb{R} - (d/c)$$

$$\text{Let } f(x) = y$$

$$(ax-b)/(cx-d) = y$$

$$ax - b = y(cx - d)$$

$$ax - b = cxy - dy$$

$$ax - cxy = b - dy$$

$$x(a - cy) = b - dy$$

$$\therefore x = (b-dy)/(a-cy)$$

$$\text{When } a - cy = 0 \text{ or } y = a/c,$$

Hence,  $f(x)$  cannot take the value  $a/c$ .

$$\therefore \text{Range } (f) = \mathbb{R} - (a/c)$$

$$\text{(iii) } f(x) = \sqrt{x-1}$$

We know the square of a real number is never negative.

$f(x)$  takes real values only when  $x - 1 \geq 0$

$$x \geq 1$$

$$\therefore x \in [1, \infty)$$

$$\text{Thus, domain } (f) = [1, \infty)$$

When  $x \geq 1$ , we have  $x - 1 \geq 0$

$$\text{Hence, } \sqrt{x-1} \geq 0 \Rightarrow f(x) \geq 0$$

$$f(x) \in [0, \infty)$$

$$\therefore \text{Range } (f) = [0, \infty)$$

$$\text{(iv) } f(x) = \sqrt{x-3}$$

We know the square of a real number is never negative.

$f(x)$  takes real values only when  $x - 3 \geq 0$

$$x \geq 3$$

$$\therefore x \in [3, \infty)$$

$$\text{Domain}(f) = [3, \infty)$$

When  $x \geq 3$ , we have  $x - 3 \geq 0$

$$\text{Hence, } \sqrt{x-3} \geq 0 \Rightarrow f(x) \geq 0$$

$$f(x) \in [0, \infty)$$

$$\therefore \text{Range}(f) = [0, \infty)$$

$$(v) f(x) = (x-2)/(2-x)$$

$f(x)$  is defined for all real values of  $x$ , except for the case when  $2 - x = 0$  or  $x = 2$ .

$$\text{Domain}(f) = \mathbb{R} - \{2\}$$

$$\text{We have, } f(x) = (x-2)/(2-x)$$

$$f(x) = -(2-x)/(2-x) \\ = -1$$

$$\text{When } x \neq 2, f(x) = -1$$

$$\therefore \text{Range}(f) = \{-1\}$$

$$(vi) f(x) = |x-1|$$

$$\text{we know } |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Now we have,

$$|x-1| = \begin{cases} -(x-1), & x-1 < 0 \\ x-1, & x-1 \geq 0 \end{cases}$$

$$\therefore f(x) = |x-1| = \begin{cases} 1-x, & x < 1 \\ x-1, & x \geq 1 \end{cases}$$

Hence,  $f(x)$  is defined for all real numbers  $x$ .

$$\text{Domain}(f) = \mathbb{R}$$

When,  $x < 1$ , we have  $x - 1 < 0$  or  $1 - x > 0$ .

$$|x-1| > 0 \Rightarrow f(x) > 0$$

When,  $x \geq 1$ , we have  $x - 1 \geq 0$ .

$$|x-1| \geq 0 \Rightarrow f(x) \geq 0$$

$$\therefore f(x) \geq 0 \text{ or } f(x) \in [0, \infty)$$

$$\text{Range}(f) = [0, \infty)$$

$$(vii) f(x) = -|x|$$



$$\text{we know } |x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$$

Now we have,

$$-|x| = \begin{cases} -(-x), & x < 0 \\ -x, & x \geq 0 \end{cases}$$

$$\therefore f(x) = -|x| = \begin{cases} x, & x < 0 \\ -x, & x \geq 0 \end{cases}$$

Hence,  $f(x)$  is defined for all real numbers  $x$ .

Domain ( $f$ ) =  $\mathbb{R}$

When,  $x < 0$ , we have  $-|x| < 0$

$$f(x) < 0$$

When,  $x \geq 0$ , we have  $-x \leq 0$ .

$$-|x| \leq 0 \Rightarrow f(x) \leq 0$$

$$\therefore f(x) \leq 0 \text{ or } f(x) \in (-\infty, 0]$$

$$\text{Range } (f) = (-\infty, 0]$$

$$\text{(viii) } f(x) = \sqrt{9-x^2}$$

We know the square of a real number is never negative.

$f(x)$  takes real values only when  $9 - x^2 \geq 0$

$$9 \geq x^2$$

$$x^2 \leq 9$$

$$x^2 - 9 \leq 0$$

$$x^2 - 3^2 \leq 0$$

$$(x+3)(x-3) \leq 0$$

$$x \geq -3 \text{ and } x \leq 3$$

$$\therefore x \in [-3, 3]$$

$$\text{Domain } (f) = [-3, 3]$$

When,  $x \in [-3, 3]$ , we have  $0 \leq 9 - x^2 \leq 9$

$$0 \leq \sqrt{9-x^2} \leq 3 \Rightarrow 0 \leq f(x) \leq 3$$

$$\therefore f(x) \in [0, 3]$$

$$\text{Range } (f) = [0, 3]$$

## EXERCISE 3.4

PAGE NO: 3.38

**1. Find  $f + g$ ,  $f - g$ ,  $cf$  ( $c \in \mathbb{R}$ ,  $c \neq 0$ ),  $fg$ ,  $1/f$  and  $f/g$  in each of the following:**

**(i)  $f(x) = x^3 + 1$  and  $g(x) = x + 1$**

**(ii)  $f(x) = \sqrt{x-1}$  and  $g(x) = \sqrt{x+1}$**

**Solution:**

**(i)  $f(x) = x^3 + 1$  and  $g(x) = x + 1$**

We have  $f(x): \mathbb{R} \rightarrow \mathbb{R}$  and  $g(x): \mathbb{R} \rightarrow \mathbb{R}$

**(a)  $f + g$**

We know,  $(f + g)(x) = f(x) + g(x)$

$$\begin{aligned}(f + g)(x) &= x^3 + 1 + x + 1 \\ &= x^3 + x + 2\end{aligned}$$

So,  $(f + g)(x): \mathbb{R} \rightarrow \mathbb{R}$

$\therefore f + g: \mathbb{R} \rightarrow \mathbb{R}$  is given by  $(f + g)(x) = x^3 + x + 2$

**(b)  $f - g$**

We know,  $(f - g)(x) = f(x) - g(x)$

$$\begin{aligned}(f - g)(x) &= x^3 + 1 - (x + 1) \\ &= x^3 + 1 - x - 1 \\ &= x^3 - x\end{aligned}$$

So,  $(f - g)(x): \mathbb{R} \rightarrow \mathbb{R}$

$\therefore f - g: \mathbb{R} \rightarrow \mathbb{R}$  is given by  $(f - g)(x) = x^3 - x$

**(c)  $cf$  ( $c \in \mathbb{R}$ ,  $c \neq 0$ )**

We know,  $(cf)(x) = c \times f(x)$

$$\begin{aligned}(cf)(x) &= c(x^3 + 1) \\ &= cx^3 + c\end{aligned}$$

So,  $(cf)(x): \mathbb{R} \rightarrow \mathbb{R}$

$\therefore cf: \mathbb{R} \rightarrow \mathbb{R}$  is given by  $(cf)(x) = cx^3 + c$

**(d)  $fg$**

We know,  $(fg)(x) = f(x)g(x)$

$$\begin{aligned}(fg)(x) &= (x^3 + 1)(x + 1) \\ &= (x + 1)(x^2 - x + 1)(x + 1) \\ &= (x + 1)^2(x^2 - x + 1)\end{aligned}$$

So,  $(fg)(x): \mathbb{R} \rightarrow \mathbb{R}$

$\therefore fg: \mathbb{R} \rightarrow \mathbb{R}$  is given by  $(fg)(x) = (x + 1)^2(x^2 - x + 1)$

(e)  $1/f$

We know,  $(1/f)(x) = 1/f(x)$

$$1/f(x) = 1/(x^3 + 1)$$

Observe that  $1/f(x)$  is undefined when  $f(x) = 0$  or when  $x = -1$ .

So,  $1/f: \mathbb{R} - \{-1\} \rightarrow \mathbb{R}$  is given by  $1/f(x) = 1/(x^3 + 1)$

(f)  $f/g$

We know,  $(f/g)(x) = f(x)/g(x)$

$$(f/g)(x) = (x^3 + 1)/(x + 1)$$

Observe that  $(x^3 + 1)/(x + 1)$  is undefined when  $g(x) = 0$  or when  $x = -1$ .

Using  $x^3 + 1 = (x + 1)(x^2 - x + 1)$ , we have

$$\begin{aligned}(f/g)(x) &= [(x+1)(x^2 - x + 1)/(x+1)] \\ &= x^2 - x + 1\end{aligned}$$

$\therefore f/g: \mathbb{R} - \{-1\} \rightarrow \mathbb{R}$  is given by  $(f/g)(x) = x^2 - x + 1$

(ii)  $f(x) = \sqrt{x-1}$  and  $g(x) = \sqrt{x+1}$

We have  $f(x): [1, \infty) \rightarrow \mathbb{R}^+$  and  $g(x): [-1, \infty) \rightarrow \mathbb{R}^+$  as real square root is defined only for non-negative numbers.

(a)  $f + g$

We know,  $(f + g)(x) = f(x) + g(x)$

$$(f+g)(x) = \sqrt{x-1} + \sqrt{x+1}$$

Domain of  $(f + g) = \text{Domain of } f \cap \text{Domain of } g$

$$\text{Domain of } (f + g) = [1, \infty) \cap [-1, \infty)$$

$$\text{Domain of } (f + g) = [1, \infty)$$

$\therefore f + g: [1, \infty) \rightarrow \mathbb{R}$  is given by  $(f+g)(x) = \sqrt{x-1} + \sqrt{x+1}$

(b)  $f - g$

We know,  $(f - g)(x) = f(x) - g(x)$

$$(f-g)(x) = \sqrt{x-1} - \sqrt{x+1}$$

Domain of  $(f - g) = \text{Domain of } f \cap \text{Domain of } g$

$$\text{Domain of } (f - g) = [1, \infty) \cap [-1, \infty)$$

$$\text{Domain of } (f - g) = [1, \infty)$$

$\therefore f - g: [1, \infty) \rightarrow \mathbb{R}$  is given by  $(f-g)(x) = \sqrt{x-1} - \sqrt{x+1}$

(c)  $cf$  ( $c \in \mathbb{R}, c \neq 0$ )

We know,  $(cf)(x) = c \times f(x)$

$$(cf)(x) = c\sqrt{x-1}$$

Domain of  $(cf) = \text{Domain of } f$

Domain of  $(cf) = [1, \infty)$

$\therefore cf: [1, \infty) \rightarrow \mathbb{R}$  is given by  $(cf)(x) = c\sqrt{x-1}$

(d)  $fg$

We know,  $(fg)(x) = f(x)g(x)$

$$\begin{aligned}(fg)(x) &= \sqrt{x-1} \sqrt{x+1} \\ &= \sqrt{x^2 - 1}\end{aligned}$$

Domain of  $(fg) = \text{Domain of } f \cap \text{Domain of } g$

$$\text{Domain of } (fg) = [1, \infty) \cap [-1, \infty)$$

$$\text{Domain of } (fg) = [1, \infty)$$

$\therefore fg: [1, \infty) \rightarrow \mathbb{R}$  is given by  $(fg)(x) = \sqrt{x^2 - 1}$

(e)  $1/f$

We know,  $(1/f)(x) = 1/f(x)$

$$(1/f)(x) = 1/\sqrt{x-1}$$

Domain of  $(1/f) = \text{Domain of } f$

$$\text{Domain of } (1/f) = [1, \infty)$$

Observe that  $1/\sqrt{x-1}$  is also undefined when  $x - 1 = 0$  or  $x = 1$ .

$\therefore 1/f: (1, \infty) \rightarrow \mathbb{R}$  is given by  $(1/f)(x) = 1/\sqrt{x-1}$

(f)  $f/g$

We know,  $(f/g)(x) = f(x)/g(x)$

$$(f/g)(x) = \sqrt{x-1}/\sqrt{x+1}$$

$$(f/g)(x) = \sqrt{[(x-1)/(x+1)]}$$

Domain of  $(f/g) = \text{Domain of } f \cap \text{Domain of } g$

$$\text{Domain of } (f/g) = [1, \infty) \cap [-1, \infty)$$

$$\text{Domain of } (f/g) = [1, \infty)$$

$\therefore f/g: [1, \infty) \rightarrow \mathbb{R}$  is given by  $(f/g)(x) = \sqrt{[(x-1)/(x+1)]}$

**2. Let  $f(x) = 2x + 5$  and  $g(x) = x^2 + x$ . Describe**

**(i)  $f + g$**

**(ii)  $f - g$**

**(iii)  $fg$**

**(iv)  $f/g$**

**Find the domain in each case.**

**Solution:**

Given:

$$f(x) = 2x + 5 \text{ and } g(x) = x^2 + x$$

Both  $f(x)$  and  $g(x)$  are defined for all  $x \in \mathbb{R}$ .

So, domain of  $f$  = domain of  $g$  =  $\mathbb{R}$

(i)  $f + g$

We know,  $(f + g)(x) = f(x) + g(x)$

$$\begin{aligned}(f + g)(x) &= 2x + 5 + x^2 + x \\ &= x^2 + 3x + 5\end{aligned}$$

$(f + g)(x)$  is defined for all real numbers  $x$ .

$\therefore$  The domain of  $(f + g)$  is  $\mathbb{R}$

(ii)  $f - g$

We know,  $(f - g)(x) = f(x) - g(x)$

$$\begin{aligned}(f - g)(x) &= 2x + 5 - (x^2 + x) \\ &= 2x + 5 - x^2 - x \\ &= 5 + x - x^2\end{aligned}$$

$(f - g)(x)$  is defined for all real numbers  $x$ .

$\therefore$  The domain of  $(f - g)$  is  $\mathbb{R}$

(iii)  $fg$

We know,  $(fg)(x) = f(x)g(x)$

$$\begin{aligned}(fg)(x) &= (2x + 5)(x^2 + x) \\ &= 2x(x^2 + x) + 5(x^2 + x) \\ &= 2x^3 + 2x^2 + 5x^2 + 5x \\ &= 2x^3 + 7x^2 + 5x\end{aligned}$$

$(fg)(x)$  is defined for all real numbers  $x$ .

$\therefore$  The domain of  $fg$  is  $\mathbb{R}$

(iv)  $f/g$

We know,  $(f/g)(x) = f(x)/g(x)$

$$(f/g)(x) = (2x+5)/(x^2+x)$$

$(f/g)(x)$  is defined for all real values of  $x$ , except for the case when  $x^2 + x = 0$ .

$$x^2 + x = 0$$

$$x(x + 1) = 0$$

$$x = 0 \text{ or } x + 1 = 0$$

$$x = 0 \text{ or } -1$$

When  $x = 0$  or  $-1$ ,  $(f/g)(x)$  will be undefined as the division result will be indeterminate.

$\therefore$  The domain of  $f/g$  =  $\mathbb{R} - \{-1, 0\}$

**3. If  $f(x)$  be defined on  $[-2, 2]$  and is given by  $f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x - 1, & 0 < x \leq 2 \end{cases}$  and  $g(x) = f(|x|) + |f(x)|$ . Find  $g(x)$ .**

**Solution:**

Given:

$$f(x) = \begin{cases} -1, & -2 \leq x \leq 0 \\ x-1, & 0 < x \leq 2 \end{cases} \text{ and}$$

$$g(x) = f(|x|) + |f(x)|$$

Now we have,

$$f(|x|) = \begin{cases} -1, & -2 \leq |x| \leq 0 \\ |x|-1, & 0 < |x| \leq 2 \end{cases}$$

However,  $|x| \geq 0 \Rightarrow f(|x|) = |x| - 1$  when  $0 < |x| \leq 2$

We also have,

$$\begin{aligned} |f(x)| &= \begin{cases} |-1|, & -2 \leq x \leq 0 \\ |x-1|, & 0 < x \leq 2 \end{cases} \\ &= \begin{cases} 1, & -2 \leq x \leq 0 \\ |x-1|, & 0 < x \leq 2 \end{cases} \end{aligned}$$

We also know,

$$\begin{aligned} |x-1| &= \begin{cases} -(x-1), & x-1 < 0 \\ x-1, & x-1 \geq 0 \end{cases} \\ &= \begin{cases} -(x-1), & x < 1 \\ x-1, & x \geq 1 \end{cases} \end{aligned}$$

Here, we shall only the range between  $[0, 2]$ .

$$|x-1| = \begin{cases} -(x-1), & 0 < x < 1 \\ x-1, & 1 \leq x \leq 2 \end{cases}$$

Substituting this value of  $|x-1|$  in  $|f(x)|$ , we get

$$\begin{aligned} |f(x)| &= \begin{cases} 1, & -2 \leq x \leq 0 \\ -(x-1), & 0 < x < 1 \\ x-1, & 1 \leq x \leq 2 \end{cases} \\ &= \begin{cases} 1, & -2 \leq x \leq 0 \\ 1-x, & 0 < x < 1 \\ x-1, & 1 \leq x \leq 2 \end{cases} \end{aligned}$$

Now, we need to find  $g(x)$

$$\begin{aligned} g(x) &= f(|x|) + |f(x)| \\ &= |x| - 1 \text{ when } 0 < |x| \leq 2 + \begin{cases} 1, & -2 \leq x \leq 0 \\ 1-x, & 0 < x < 1 \\ x-1, & 1 \leq x \leq 2 \end{cases} \end{aligned}$$

$$\begin{aligned}
 g(x) &= \begin{cases} -x-1, -2 \leq x \leq 0 \\ x-1, 0 < x < 1 \\ x-1, 1 \leq x \leq 2 \end{cases} + \begin{cases} 1, -2 \leq x \leq 0 \\ 1-x, 0 < x < 1 \\ x-1, 1 \leq x \leq 2 \end{cases} \\
 &= \begin{cases} -x-1+1, -2 \leq x \leq 0 \\ x-1+1-x, 0 < x < 1 \\ x-1+x-1, 1 \leq x \leq 2 \end{cases} \\
 &= \begin{cases} -x, -2 \leq x \leq 0 \\ 0, 0 < x < 1 \\ 2(x-1), 1 \leq x \leq 2 \end{cases}
 \end{aligned}$$

$$\begin{aligned}
 \therefore g(x) &= f(|x|) + |f(x)| \\
 &= \begin{cases} -x, -2 \leq x \leq 0 \\ 0, 0 < x < 1 \\ 2(x-1), 1 \leq x \leq 2 \end{cases}
 \end{aligned}$$

4. Let  $f, g$  be two real functions defined by  $f(x) = \sqrt{x+1}$  and  $g(x) = \sqrt{9-x^2}$ . Then, describe each of the following functions.

- (i)  $f + g$
- (ii)  $g - f$
- (iii)  $fg$
- (iv)  $f/g$
- (v)  $g/f$
- (vi)  $2f - \sqrt{5}g$
- (vii)  $f^2 + 7f$
- (viii)  $5/g$

**Solution:**

Given:

$$f(x) = \sqrt{x+1} \text{ and } g(x) = \sqrt{9-x^2}$$

We know the square of a real number is never negative.

So,  $f(x)$  takes real values only when  $x+1 \geq 0$

$$x \geq -1, x \in [-1, \infty)$$

Domain of  $f = [-1, \infty)$

Similarly,  $g(x)$  takes real values only when  $9 - x^2 \geq 0$

$$9 \geq x^2$$

$$x^2 \leq 9$$

$$x^2 - 9 \leq 0$$

$$x^2 - 3^2 \leq 0$$

$$(x+3)(x-3) \leq 0$$



$$x \geq -3 \text{ and } x \leq 3$$

$$\therefore x \in [-3, 3]$$

$$\text{Domain of } g = [-3, 3]$$

(i)  $f + g$

$$\text{We know, } (f + g)(x) = f(x) + g(x)$$

$$(f + g)(x) = \sqrt{x+1} + \sqrt{9-x^2}$$

$$\text{Domain of } f + g = \text{Domain of } f \cap \text{Domain of } g$$

$$= [-1, \infty) \cap [-3, 3]$$

$$= [-1, 3]$$

$$\therefore f + g: [-1, 3] \rightarrow \mathbb{R} \text{ is given by } (f + g)(x) = f(x) + g(x) = \sqrt{x+1} + \sqrt{9-x^2}$$

(ii)  $g - f$

$$\text{We know, } (g - f)(x) = g(x) - f(x)$$

$$(g - f)(x) = \sqrt{9-x^2} - \sqrt{x+1}$$

$$\text{Domain of } g - f = \text{Domain of } g \cap \text{Domain of } f$$

$$= [-3, 3] \cap [-1, \infty)$$

$$= [-1, 3]$$

$$\therefore g - f: [-1, 3] \rightarrow \mathbb{R} \text{ is given by } (g - f)(x) = g(x) - f(x) = \sqrt{9-x^2} - \sqrt{x+1}$$

(iii)  $fg$

$$\text{We know, } (fg)(x) = f(x)g(x)$$

$$(fg)(x) = \sqrt{x+1} \sqrt{9-x^2}$$

$$= \sqrt{[(x+1)(9-x^2)]}$$

$$= \sqrt{[x(9-x^2) + (9-x^2)]}$$

$$= \sqrt{(9x-x^3+9-x^2)}$$

$$= \sqrt{(9+9x-x^2-x^3)}$$

$$\text{Domain of } fg = \text{Domain of } f \cap \text{Domain of } g$$

$$= [-1, \infty) \cap [-3, 3]$$

$$= [-1, 3]$$

$$\therefore fg: [-1, 3] \rightarrow \mathbb{R} \text{ is given by } (fg)(x) = f(x)g(x) = \sqrt{x+1} \sqrt{9-x^2} = \sqrt{(9+9x-x^2-x^3)}$$

(iv)  $f/g$

$$\text{We know, } (f/g)(x) = f(x)/g(x)$$

$$(f/g)(x) = \sqrt{x+1} / \sqrt{9-x^2}$$

$$= \sqrt{[(x+1) / (9-x^2)]}$$

$$\text{Domain of } f/g = \text{Domain of } f \cap \text{Domain of } g$$

$$= [-1, \infty) \cap [-3, 3]$$

$$= [-1, 3]$$

However,  $(f/g)(x)$  is defined for all real values of  $x \in [-1, 3]$ , except for the case when  $9 - x^2 = 0$  or  $x = \pm 3$

When  $x = \pm 3$ ,  $(f/g)(x)$  will be undefined as the division result will be indeterminate.

Domain of  $f/g = [-1, 3] - \{-3, 3\}$

Domain of  $f/g = [-1, 3)$

$\therefore f/g: [-1, 3) \rightarrow \mathbb{R}$  is given by  $(f/g)(x) = f(x)/g(x) = \sqrt{x+1} / \sqrt{9-x^2}$

(v)  $g/f$

We know,  $(g/f)(x) = g(x)/f(x)$

$$(g/f)(x) = \sqrt{9-x^2} / \sqrt{x+1}$$

$$= \sqrt{(9-x^2) / (x+1)}$$

Domain of  $g/f = \text{Domain of } f \cap \text{Domain of } g$

$$= [-1, \infty) \cap [-3, 3]$$

$$= [-1, 3]$$

However,  $(g/f)(x)$  is defined for all real values of  $x \in [-1, 3]$ , except for the case when  $x + 1 = 0$  or  $x = -1$

When  $x = -1$ ,  $(g/f)(x)$  will be undefined as the division result will be indeterminate.

Domain of  $g/f = [-1, 3] - \{-1\}$

Domain of  $g/f = (-1, 3]$

$\therefore g/f: (-1, 3] \rightarrow \mathbb{R}$  is given by  $(g/f)(x) = g(x)/f(x) = \sqrt{9-x^2} / \sqrt{x+1}$

(vi)  $2f - \sqrt{5}g$

We know,  $(2f - \sqrt{5}g)(x) = 2f(x) - \sqrt{5}g(x)$

$$(2f - \sqrt{5}g)(x) = 2f(x) - \sqrt{5}g(x)$$

$$= 2\sqrt{x+1} - \sqrt{5}\sqrt{9-x^2}$$

$$= 2\sqrt{x+1} - \sqrt{45-5x^2}$$

Domain of  $2f - \sqrt{5}g = \text{Domain of } f \cap \text{Domain of } g$

$$= [-1, \infty) \cap [-3, 3]$$

$$= [-1, 3]$$

$\therefore 2f - \sqrt{5}g: [-1, 3] \rightarrow \mathbb{R}$  is given by  $(2f - \sqrt{5}g)(x) = 2f(x) - \sqrt{5}g(x) = 2\sqrt{x+1} - \sqrt{45-5x^2}$

(vii)  $f^2 + 7f$

We know,  $(f^2 + 7f)(x) = f^2(x) + (7f)(x)$

$$(f^2 + 7f)(x) = f(x)f(x) + 7f(x)$$

$$= \sqrt{x+1}\sqrt{x+1} + 7\sqrt{x+1}$$

$$= x + 1 + 7\sqrt{x+1}$$

Domain of  $f^2 + 7f$  is same as domain of  $f$ .

Domain of  $f^2 + 7f = [-1, \infty)$

$\therefore f^2 + 7f: [-1, \infty) \rightarrow \mathbb{R}$  is given by  $(f^2 + 7f)(x) = f(x)f(x) + 7f(x) = x + 1 + 7\sqrt{x+1}$

(viii)  $5/g$

We know,  $(5/g)(x) = 5/g(x)$

$$(5/g)(x) = 5/\sqrt{9-x^2}$$

Domain of  $5/g$  = Domain of  $g = [-3, 3]$

However,  $(5/g)(x)$  is defined for all real values of  $x \in [-3, 3]$ , except for the case when  $9 - x^2 = 0$  or  $x = \pm 3$

When  $x = \pm 3$ ,  $(5/g)(x)$  will be undefined as the division result will be indeterminate.

Domain of  $5/g = [-3, 3] - \{-3, 3\}$

$$= (-3, 3)$$

$\therefore 5/g: (-3, 3) \rightarrow \mathbb{R}$  is given by  $(5/g)(x) = 5/g(x) = 5/\sqrt{9-x^2}$

**5. If  $f(x) = \log_e(1-x)$  and  $g(x) = [x]$ , then determine each of the following functions:**

(i)  $f + g$

(ii)  $fg$

(iii)  $f/g$

(iv)  $g/f$

**Also, find  $(f + g)(-1)$ ,  $(fg)(0)$ ,  $(f/g)(1/2)$  and  $(g/f)(1/2)$ .**

**Solution:**

Given:

$$f(x) = \log_e(1-x) \text{ and } g(x) = [x]$$

We know,  $f(x)$  takes real values only when  $1 - x > 0$

$$1 > x$$

$$x < 1, \therefore x \in (-\infty, 1)$$

$$\text{Domain of } f = (-\infty, 1)$$

Similarly,  $g(x)$  is defined for all real numbers  $x$ .

$$\text{Domain of } g = [x], x \in \mathbb{R}$$

$$= \mathbb{R}$$

(i)  $f + g$

We know,  $(f + g)(x) = f(x) + g(x)$

$$(f + g)(x) = \log_e(1-x) + [x]$$

Domain of  $f + g$  = Domain of  $f \cap$  Domain of  $g$

$$\text{Domain of } f + g = (-\infty, 1) \cap \mathbb{R}$$

$$= (-\infty, 1)$$

$\therefore f + g: (-\infty, 1) \rightarrow \mathbb{R}$  is given by  $(f + g)(x) = \log_e(1-x) + [x]$

(ii)  $fg$

We know,  $(fg)(x) = f(x)g(x)$

$$\begin{aligned}(fg)(x) &= \log_e(1-x) \times [x] \\ &= [x] \log_e(1-x)\end{aligned}$$

$$\begin{aligned}\text{Domain of } fg &= \text{Domain of } f \cap \text{Domain of } g \\ &= (-\infty, 1) \cap \mathbb{R} \\ &= (-\infty, 1)\end{aligned}$$

$\therefore fg: (-\infty, 1) \rightarrow \mathbb{R}$  is given by  $(fg)(x) = [x] \log_e(1-x)$

**(iii)  $f/g$**

We know,  $(f/g)(x) = f(x)/g(x)$

$$(f/g)(x) = \log_e(1-x) / [x]$$

$$\begin{aligned}\text{Domain of } f/g &= \text{Domain of } f \cap \text{Domain of } g \\ &= (-\infty, 1) \cap \mathbb{R} \\ &= (-\infty, 1)\end{aligned}$$

However,  $(f/g)(x)$  is defined for all real values of  $x \in (-\infty, 1)$ , except for the case when  $[x] = 0$ .

We have,  $[x] = 0$  when  $0 \leq x < 1$  or  $x \in [0, 1)$

When  $0 \leq x < 1$ ,  $(f/g)(x)$  will be undefined as the division result will be indeterminate.

$$\begin{aligned}\text{Domain of } f/g &= (-\infty, 1) - [0, 1) \\ &= (-\infty, 0)\end{aligned}$$

$\therefore f/g: (-\infty, 0) \rightarrow \mathbb{R}$  is given by  $(f/g)(x) = \log_e(1-x) / [x]$

**(iv)  $g/f$**

We know,  $(g/f)(x) = g(x)/f(x)$

$$(g/f)(x) = [x] / \log_e(1-x)$$

However,  $(g/f)(x)$  is defined for all real values of  $x \in (-\infty, 1)$ , except for the case when  $\log_e(1-x) = 0$ .

$$\log_e(1-x) = 0 \Rightarrow 1-x = 1 \text{ or } x = 0$$

When  $x = 0$ ,  $(g/f)(x)$  will be undefined as the division result will be indeterminate.

$$\begin{aligned}\text{Domain of } g/f &= (-\infty, 1) - \{0\} \\ &= (-\infty, 0) \cup (0, 1)\end{aligned}$$

$\therefore g/f: (-\infty, 0) \cup (0, 1) \rightarrow \mathbb{R}$  is given by  $(g/f)(x) = [x] / \log_e(1-x)$

**(a) We need to find  $(f+g)(-1)$ .**

We have,  $(f+g)(x) = \log_e(1-x) + [x]$ ,  $x \in (-\infty, 1)$

Substituting  $x = -1$  in the above equation, we get

$$\begin{aligned}(f+g)(-1) &= \log_e(1-(-1)) + [-1] \\ &= \log_e(1+1) + (-1) \\ &= \log_e 2 - 1\end{aligned}$$

$$\therefore (f + g)(-1) = \log_e 2 - 1$$

(b) We need to find  $(fg)(0)$ .

We have,  $(fg)(x) = [x] \log_e(1 - x)$ ,  $x \in (-\infty, 1)$

Substituting  $x = 0$  in the above equation, we get

$$\begin{aligned}(fg)(0) &= [0] \log_e(1 - 0) \\ &= 0 \times \log_e 1\end{aligned}$$

$$\therefore (fg)(0) = 0$$

(c) We need to find  $(f/g)(1/2)$

We have,  $(f/g)(x) = \log_e(1 - x) / [x]$ ,  $x \in (-\infty, 0) \cup (0, \infty)$

However,  $1/2$  is not in the domain of  $f/g$ .

$\therefore (f/g)(1/2)$  does not exist.

(d) We need to find  $(g/f)(1/2)$

We have,  $(g/f)(x) = [x] / \log_e(1 - x)$ ,  $x \in (-\infty, 0) \cup (0, \infty)$

Substituting  $x = 1/2$  in the above equation, we get

$$\begin{aligned}(g/f)(1/2) &= [x] / \log_e(1 - x) \\ &= (1/2) / \log_e(1 - 1/2) \\ &= 0.5 / \log_e(1/2) \\ &= 0 / \log_e(1/2) \\ &= 0\end{aligned}$$

$$\therefore (g/f)(1/2) = 0$$