

EXERCISE 5.1

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Prove the following identities:

1. $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

Solution:

Let us consider LHS: $\sec^4 x - \sec^2 x$

$$(\sec^2 x)^2 - \sec^2 x$$

By using the formula, $\sec^2 \theta = 1 + \tan^2 \theta$.

$$(1 + \tan^2 x)^2 - (1 + \tan^2 x)$$

$$1 + 2\tan^2 x + \tan^4 x - 1 - \tan^2 x$$

$$\tan^4 x + \tan^2 x$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

2. $\sin^6 x + \cos^6 x = 1 - 3 \sin^2 x \cos^2 x$

Solution:

Let us consider LHS: $\sin^6 x + \cos^6 x$

$$(\sin^2 x)^3 + (\cos^2 x)^3$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$(\sin^2 x + \cos^2 x) [(\sin^2 x)^2 + (\cos^2 x)^2 - \sin^2 x \cos^2 x]$$

By using the formula, $\sin^2 x + \cos^2 x = 1$ and $a^2 + b^2 = (a + b)^2 - 2ab$

$$1 \times [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x]$$

$$1^2 - 3\sin^2 x \cos^2 x$$

$$1 - 3\sin^2 x \cos^2 x$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

3. $(\operatorname{cosec} x - \sin x)(\sec x - \cos x)(\tan x + \cot x) = 1$

Solution:

Let us consider LHS: $(\operatorname{cosec} x - \sin x)(\sec x - \cos x)(\tan x + \cot x)$

By using the formulas

$$\operatorname{cosec} \theta = 1/\sin \theta;$$

$$\sec \theta = 1/\cos \theta;$$

$$\tan \theta = \sin \theta / \cos \theta;$$

$$\cot \theta = \cos \theta / \sin \theta$$

Now,

$$\left(\frac{1}{\sin x} - \sin x\right)\left(\frac{1}{\cos x} - \cos x\right)\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$\frac{1 - \sin^2 x}{\sin x} \times \frac{1 - \cos^2 x}{\cos x} \times \frac{\sin^2 x + \cos^2 x}{\sin x \cos x}$$

By using the formula,

$$\sin^2 x + \cos^2 x = 1;$$

$$\frac{\cos^2 x}{\sin x} \times \frac{\sin^2 x}{\cos x} \times \frac{1}{\sin x \cos x}$$

$$1 = \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

$$4. \operatorname{cosec} x (\sec x - 1) - \cot x (1 - \cos x) = \tan x - \sin x$$

Solution:

Let us consider LHS: $\operatorname{cosec} x (\sec x - 1) - \cot x (1 - \cos x)$

By using the formulas

$$\operatorname{cosec} \theta = 1/\sin \theta;$$

$$\sec \theta = 1/\cos \theta;$$

$$\tan \theta = \sin \theta / \cos \theta;$$

$$\cot \theta = \cos \theta / \sin \theta$$

Now,

$$\frac{1}{\sin x} \left(\frac{1}{\cos x} - 1 \right) - \frac{\cos x}{\sin x} (1 - \cos x)$$

$$\frac{1}{\sin x} \left(\frac{1 - \cos x}{\cos x} \right) - \frac{\cos x}{\sin x} (1 - \cos x)$$

$$\left(\frac{1 - \cos x}{\sin x} \right) \left(\frac{1}{\cos x} - \cos x \right)$$

$$\left(\frac{1 - \cos x}{\sin x} \right) \left(\frac{1 - \cos^2 x}{\cos x} \right)$$

By using the formula, $1 - \cos^2 x = \sin^2 x$;

$$\left(\frac{1 - \cos x}{\sin x}\right) \left(\frac{\sin^2 x}{\cos x}\right)$$

$$(1 - \cos x) \left(\frac{\sin x}{\cos x}\right)$$

$$\frac{\sin x}{\cos x} - \sin x$$

$$\tan x - \sin x$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved.

5.

$$\frac{1 - \sin x \cos x}{\cos x (\sec x - \operatorname{cosec} x)} \cdot \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x} = \sin x$$

Solution:

Let us consider the LHS:

$$\frac{1 - \sin x \cos x}{\cos x (\sec x - \operatorname{cosec} x)} \cdot \frac{\sin^2 x - \cos^2 x}{\sin^3 x + \cos^3 x}$$

By using the formula,

$$\operatorname{cosec} \theta = 1/\sin \theta;$$

$$\sec \theta = 1/\cos \theta;$$

Now,

$$\frac{1 - \sin x \cos x}{\cos x \left(\frac{1}{\cos x} - \frac{1}{\sin x}\right)} \times \frac{(\sin x)^2 - (\cos x)^2}{(\sin x)^3 + (\cos x)^3}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{1 - \sin x \cos x}{\cos x \left(\frac{\sin x - \cos x}{\cos x \sin x}\right)} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

$$\frac{\sin x (1 - \sin x \cos x)}{\sin x - \cos x} \times \frac{(\sin x + \cos x)(\sin x - \cos x)}{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

$$\frac{\sin x (1 - \sin x \cos x)}{1} \times \frac{1}{[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}$$

By using the formula, $\sin^2 x + \cos^2 x = 1$.

$$\sin x (1 - \sin x \cos x) \times \frac{1}{(1 - \sin x \cos x)}$$

$\sin x$

= RHS

$\therefore$ LHS = RHS

Hence Proved.

6.

$$\frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x} = (\sec x \operatorname{cosec} x + 1)$$

Solution:

Let us consider the LHS:

$$\frac{\tan x}{1 - \cot x} + \frac{\cot x}{1 - \tan x}$$

By using the formula,

$\tan \theta = \sin \theta / \cos \theta$;

$\cot \theta = \cos \theta / \sin \theta$

Now,

$$\frac{\frac{\sin x}{\cos x}}{1 - \frac{\cos x}{\sin x}} + \frac{\frac{\cos x}{\sin x}}{1 - \frac{\sin x}{\cos x}}$$

$$\frac{\frac{\sin x}{\cos x}}{\frac{\sin x - \cos x}{\sin x}} + \frac{\frac{\cos x}{\sin x}}{\frac{\cos x - \sin x}{\cos x}}$$

$$\frac{\sin^2 x}{\cos x (\sin x - \cos x)} - \frac{\cos^2 x}{\sin x (\sin x - \cos x)}$$

$$\frac{\sin^3 x - \cos^3 x}{\sin x \cos x (\sin x - \cos x)}$$

By using the formula, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$\frac{(\sin x - \cos x)[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x \cos x (\sin x - \cos x)}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\frac{[1 + \sin x \cos x]}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} + \frac{\sin x \cos x}{\sin x \cos x}$$

$$\frac{1}{\sin x} \times \frac{1}{\cos x} + 1$$

By using the formula,

$$\operatorname{cosec} \theta = 1/\sin \theta,$$

$$\sec \theta = 1/\cos \theta;$$

$$\operatorname{cosec} x \times \sec x + 1$$

$$\sec x \operatorname{cosec} x + 1$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence Proved.

7.

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 2$$

Solution:

Let us consider LHS:

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$$

By using the formula $a^3 \pm b^3 = (a \pm b)(a^2 + b^2 \mp ab)$

$$\frac{(\sin x + \cos x)[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}{\sin x + \cos x} + \frac{(\sin x - \cos x)[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x - \cos x}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$1 - \sin x \cos x + 1 + \sin x \cos x$$

2

= RHS

$\therefore$ LHS = RHS

Hence Proved.

8. $(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2 = 1$

Solution:

Let us consider LHS:

$$(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2$$

Expanding the above equation we get,

$$[(\sec x \sec y)^2 + (\tan x \tan y)^2 + 2 (\sec x \sec y) (\tan x \tan y)] - [(\sec x \tan y)^2 + (\tan x \sec y)^2 + 2 (\sec x \tan y) (\tan x \sec y)]$$

$$[\sec^2 x \sec^2 y + \tan^2 x \tan^2 y + 2 (\sec x \sec y) (\tan x \tan y)] - [\sec^2 x \tan^2 y + \tan^2 x \sec^2 y + 2 (\sec^2 x \tan^2 y) (\tan x \sec y)]$$

$$\sec^2 x \sec^2 y - \sec^2 x \tan^2 y + \tan^2 x \tan^2 y - \tan^2 x \sec^2 y$$

$$\sec^2 x (\sec^2 y - \tan^2 y) + \tan^2 x (\tan^2 y - \sec^2 y)$$

$$\sec^2 x (\sec^2 y - \tan^2 y) - \tan^2 x (\sec^2 y - \tan^2 y)$$

We know, $\sec^2 x - \tan^2 x = 1$.

$$\sec^2 x \times 1 - \tan^2 x \times 1$$

$$\sec^2 x - \tan^2 x$$

1 = RHS

$\therefore$ LHS = RHS

Hence proved.

9.

$$\frac{\cos x}{1 - \sin x} = \frac{1 + \cos x + \sin x}{1 + \cos x - \sin x}$$

Solution:

Let us Consider RHS:

$$\frac{1 + \cos x + \sin x}{1 + \cos x - \sin x}$$

$$\frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) - (\sin x)}$$

$$\frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) - (\sin x)} \times \frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) + (\sin x)}$$

$$\frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) - (\sin x)} \times \frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) + (\sin x)}$$

$$\frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) - (\sin x)} \times \frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) + (\sin x)}$$

$$\frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) - (\sin x)} \times \frac{(1 + \cos x) + (\sin x)}{(1 + \cos x) + (\sin x)}$$

$$\frac{[(1 + \cos x) + (\sin x)]^2}{(1 + \cos x)^2 - (\sin x)^2}$$

$$\frac{[(1 + \cos x) + (\sin x)]^2}{(1 + \cos x)^2 - (\sin x)^2}$$

$$\frac{(1 + \cos x)^2 + (\sin x)^2 + 2(1 + \cos x)(\sin x)}{(1 + \cos^2 x + 2 \cos x) - (\sin^2 x)}$$

$$\frac{(1 + \cos x)^2 + (\sin x)^2 + 2(1 + \cos x)(\sin x)}{(1 + \cos^2 x + 2 \cos x) - (\sin^2 x)}$$

$$\frac{(1 + \cos x)^2 + (\sin x)^2 + 2(1 + \cos x)(\sin x)}{(1 + \cos^2 x + 2 \cos x) - (\sin^2 x)}$$

$$\frac{1 + \cos^2 x + 2 \cos x + \sin^2 x + 2 \sin x + 2 \sin x \cos x}{1 + \cos^2 x + 2 \cos x - \sin^2 x}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\frac{1 + 1 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{(1 - \sin^2 x) + \cos^2 x + 2 \cos x}$$

We know, $1 - \cos^2 x = \sin^2 x$.

$$\frac{2 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{\cos^2 x + \cos^2 x + 2 \cos x}$$

$$\frac{2 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{2 \cos^2 x + 2 \cos x}$$

$$\frac{2 + 2 \cos x + 2 \sin x + 2 \sin x \cos x}{\cos^2 x + \cos^2 x + 2 \cos x}$$

$$\frac{1 + \cos x + \sin x + \sin x \cos x}{\cos x (\cos x + 1)}$$

$$\frac{1(1 + \cos x) + \sin x (\cos x + 1)}{\cos x (\cos x + 1)}$$

$$\frac{(1 + \sin x)(\cos x + 1)}{\cos x (\cos x + 1)}$$

$$\frac{1 + \sin x}{\cos x} \times \frac{\cos x}{\cos x}$$

$$\frac{(1 + \sin x) \cos x}{\cos^2 x}$$

We know, $1 - \sin^2 x = \cos^2 x$.

$$\frac{(1 + \sin x) \cos x}{1 - \sin^2 x}$$

$$\frac{(1 + \sin x) \cos x}{(1 - \sin x)(1 + \sin x)}$$

$$\frac{\cos x}{1 - \sin x}$$

= LHS

∴ LHS = RHS

Hence Proved.

10.

$$\frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x} = \frac{1 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

Solution:

Let us consider LHS:

$$\frac{\tan^3 x}{1 + \tan^2 x} + \frac{\cot^3 x}{1 + \cot^2 x}$$

By using the formulas,

$1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \operatorname{cosec}^2 x$

$$\frac{\tan^3 x}{\sec^2 x} + \frac{\cot^3 x}{\operatorname{cosec}^2 x}$$

$$\frac{\sin^3 x}{\cos^3 x} + \frac{\cos^3 x}{\sin^3 x}$$

$$\frac{1}{\cos^2 x} + \frac{1}{\sin^2 x}$$

$$\frac{\sin^3 x}{\cos x} + \frac{\cos^3 x}{\sin x}$$

$$\frac{\sin^4 x + \cos^4 x}{\cos x \sin x}$$

$$\frac{(\sin^2 x)^2 + (\cos^2 x)^2}{\cos x \sin x}$$

We know, $a^2 + b^2 = (a + b)^2 - 2ab$

$$\frac{(\sin^2 x + \cos^2 x)^2 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

$$\frac{1^2 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

$$\frac{1 - 2 \sin^2 x \cos^2 x}{\sin x \cos x}$$

= RHS

∴ LHS = RHS

Hence Proved.

11.

$$1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x} = \sin x \cos x$$

Solution:

Let us consider LHS:

$$1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x}$$

By using the formula,

$$\tan \theta = \sin \theta / \cos \theta;$$

$$\cot \theta = \cos \theta / \sin \theta$$

Now,

$$1 - \frac{\sin^2 x}{1 + \frac{\cos x}{\sin x}} - \frac{\cos^2 x}{1 + \frac{\sin x}{\cos x}}$$

$$1 - \frac{\sin^3 x}{\sin x + \cos x} - \frac{\cos^3 x}{\sin x + \cos x}$$

$$\frac{\sin x + \cos x - (\sin^3 x + \cos^3 x)}{\sin x + \cos x}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{\sin x + \cos x - ((\sin x + \cos x)(\sin x)^2 + (\cos x)^2 - \sin x \cos x))}{\sin x + \cos x}$$

$$\frac{(\sin x + \cos x)(1 - \sin^2 x - \cos^2 x + \sin x \cos x)}{\sin x + \cos x}$$

$$1 - (\sin^2 x + \cos^2 x) + \sin x \cos x$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$1 - 1 + \sin x \cos x$$

$$\sin x \cos x$$

$$= \text{RHS}$$

$$\therefore \text{LHS} = \text{RHS}$$

Hence proved.

12.

$$\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{\operatorname{cosec}^2 x - \sin^2 x} \right) \sin^2 x \cos^2 x = \frac{1 - \sin^2 x \cos^2 x}{2 + \sin^2 x \cos^2 x}$$

Solution:

Let us consider LHS:

$$\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{\operatorname{cosec}^2 x - \sin^2 x} \right) \sin^2 x \cos^2 x$$

By using the formula,

$$\operatorname{cosec} \theta = 1/\sin \theta,$$

$$\sec \theta = 1/\cos \theta;$$

$$\left(\frac{1}{\frac{1}{\cos^2 x} - \cos^2 x} + \frac{1}{\frac{1}{\sin^2 x} - \sin^2 x} \right) \sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x}{1 - \cos^4 x} + \frac{\sin^2 x}{1 - \sin^4 x} \right) \sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x(1 - \sin^4 x) + \sin^2 x(1 - \cos^4 x)}{(1 - \cos^4 x)(1 - \sin^4 x)} \right) \sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x - \cos^2 x \sin^4 x + \sin^2 x - \sin^2 x \cos^4 x}{(1 + \sin^2 x)(1 - \sin^2 x)(1 + \cos^2 x)(1 - \cos^2 x)} \right) \sin^2 x \cos^2 x$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\left(\frac{1 - \cos^2 x \sin^4 x - \sin^2 x \cos^4 x}{(1 + \sin^2 x) \cos^2 x (1 + \cos^2 x) \sin^2 x} \right) \sin^2 x \cos^2 x$$

$$\left(\frac{1 - \cos^2 x \sin^2 x (\sin^2 x + \cos^2 x)}{(1 + \sin^2 x)(1 + \cos^2 x)} \right)$$

$$\frac{1 - \sin^2 x \cos^2 x}{2 + \sin^2 x \cos^2 x}$$

= RHS

∴ LHS = RHS

Hence proved.

$$13. (1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2 = \sec^2 \alpha \sec^2 \beta$$

Solution:

Let us consider LHS: $(1 + \tan \alpha \tan \beta)^2 + (\tan \alpha - \tan \beta)^2$

$$1 + \tan^2 \alpha \tan^2 \beta + 2 \tan \alpha \tan \beta + \tan^2 \alpha + \tan^2 \beta - 2 \tan \alpha \tan \beta$$

$$1 + \tan^2 \alpha \tan^2 \beta + \tan^2 \alpha + \tan^2 \beta$$

$$\tan^2 \alpha (\tan^2 \beta + 1) + 1 (1 + \tan^2 \beta)$$

$$(1 + \tan^2 \beta) (1 + \tan^2 \alpha)$$

We know, $1 + \tan^2 \theta = \sec^2 \theta$

So,

$$\sec^2 \alpha \sec^2 \beta$$

= RHS

∴ LHS = RHS

Hence proved.