EXERCISE 5.2 PAGE NO: 5.25

1. Find the values of the other five trigonometric functions in each of the following:
(i) cot x = 12/5, x in quadrant III
(ii) cos x = -1/2, x in quadrant II
(iii) tan x = 3/4, x in quadrant III
(iv) sin x = 3/5, x in quadrant I
Solution:
(i) cot x = 12/5, x in quadrant III
In third quadrant, tan x and cot x are positive. sin x, cos x, sec x, cosec x are negative.
By using the formulas,
tan x = 1/cot x
= 1/(12/5)
= 5/12

cosec x = $-\sqrt{(1 + \cot^2 x)}$
= $-\sqrt{(1 + (12/5)^2)}$
= $-\sqrt{(25+144)/25}$
= $-\sqrt{(169/25)}$
= -13/5

sin x = 1/cosec x
= 1/(-13/5)
= -5/13

cos x = $-\sqrt{(1 - \sin^2 x)}$
= $-\sqrt{(1 – (-5/13)^2)}$
= $-\sqrt{(169-25)/169}$
= $-\sqrt{(144/169)}$
= -12/13

sec x = 1/cos x
= 1/(-12/13)
= -13/12

∴ sin x = -5/13, cos x = -12/13, tan x = 5/12, cosec x = -13/5, sec x = -13/12

(ii) cos x = -1/2, x in quadrant II
In second quadrant, sin x and cosec x are positive. tan x, cot x, cos x, sec x are negative.

By using the formulas,
$\sin x = \sqrt{(1 - \cos^2 x)}$
$= \sqrt{(1 - (-1/2)^2)}$
$= \sqrt{(4-1)/4}$
$= \sqrt{(3/4)}$
$= \sqrt{3}/2$

$\tan x = \sin x/\cos x$
$= (\sqrt{3}/2)/(-1/2)$
$= -\sqrt{3}$

$\cot x = 1/\tan x$
$= 1/-\sqrt{3}$
$= -1/\sqrt{3}$

$\text{cosec } x = 1/\sin x$
$= 1/(\sqrt{3}/2)$
$= 2/\sqrt{3}$

$\sec x = 1/\cos x$
$= 1/(-1/2)$
$= -2$

$\therefore \sin x = \sqrt{3}/2, \tan x = -\sqrt{3}, \text{cosec } x = 2/\sqrt{3}, \cot x = -1/\sqrt{3} \sec x = -2$

(iii) $\tan x = 3/4$, x in quadrant III
In third quadrant, tan x and cot x are positive. sin x, cos x, sec x, cosec x are negative.
By using the formulas,
$\sin x = \sqrt{(1 - \cos^2 x)}$
$= - \sqrt{(1-(-4/5)^2)}$
$= - \sqrt{(25-16)/25}$
$= - \sqrt{(9/25)}$
$= - 3/5$

$\cos x = 1/\sec x$
$= 1/(-5/4)$
$= -4/5$

$\cot x = 1/\tan x$

= 1/(3/4)
= 4/3

cosec x = 1/sin x
= 1/(-3/5)
= -5/3

sec x = -√(1 + tan^2 x)
= - √(1+(3/4)^2)
= - √(16+9)/16
= - √ (25/16)
= -5/4

∴ sin x = -3/5, cos x = -4/5, cosec x = -5/3, sec x = -5/4, cot x = 4/3

(iv) sin x = 3/5, x in quadrant I
In first quadrant, all trigonometric ratios are positive.
So, by using the formulas,
tan x = sin x/cos x
= (3/5)/(4/5)
= 3/4

cosec x = 1/sin x
= 1/(3/5)
= 5/3

cos x = √(1-sin^2 x)
= √(1 – (-3/5)^2)
= √(25-9)/25
= √(16/25)
= 4/5

sec x = 1/cos x
= 1/(4/5)
= 5/4

cot x = 1/tan x
= 1/(3/4)
= 4/3

∴ cos x = 4/5, tan x = 3/4, cosec x = 5/3, sec x = 5/4, cot x = 4/3

2. If sin x = 12/13 and lies in the second quadrant, find the value of sec x + tan x.
Solution:
Given:
Sin x = 12/13 and x lies in the second quadrant.
We know, in second quadrant, sin x and cosec x are positive and all other ratios are negative.
By using the formulas,
Cos x = $\sqrt{(1-\sin^2 x)}$
= - $\sqrt{(1-(12/13)^2)}$
= - $\sqrt{(1- (144/169))}$
= - $\sqrt{(169-144)/169}$
= -$\sqrt{(25/169)}$
= - 5/13

We know,
tan x = sin x/cos x
sec x = 1/cos x
Now,
tan x = (12/13)/(-5/13)
= -12/5

sec x = 1/(-5/13)
= -13/5

Sec x + tan x = -13/5 + (-12/5)
= (-13-12)/5
= -25/5
= -5
∴ Sec x + tan x = -5

3. If sin x = 3/5, tan y = 1/2 and π/2 < x< π< y< 3π/2 find the value of 8 tan x -√5 sec y.
Solution:
Given:
sin x = 3/5, tan y = 1/2 and π/2 < x< π< y< 3π/2
We know that, x is in second quadrant and y is in third quadrant.
In second quadrant, cos x and tan x are negative.

In third quadrant, sec y is negative.
By using the formula,
$\cos x = -\sqrt{(1-\sin^2 x)}$
$\tan x = \sin x/\cos x$
Now,
$\cos x = -\sqrt{(1-\sin^2 x)}$
$= -\sqrt{(1 – (3/5)^2)}$
$= -\sqrt{(1 – 9/25)}$
$= -\sqrt{((25-9)/25)}$
$= -\sqrt{(16/25)}$
$= - 4/5$

$\tan x = \sin x/\cos x$
$= (3/5)/(-4/5)$
$= 3/5 \times -5/4$
$= -3/4$

We know that $\sec y = -\sqrt{(1+\tan^2 y)}$
$= -\sqrt{(1 + (1/2)^2)}$
$= -\sqrt{(1 + 1/4)}$
$= -\sqrt{((4+1)/4)}$
$= -\sqrt{(5/4)}$
$= -\sqrt{5}/2$

Now, $8 \tan x - \sqrt{5} \sec y = 8(-3/4) - \sqrt{5}(-\sqrt{5}/2)$
$= -6 + 5/2$
$= (-12+5)/2$
$= -7/2$
$\therefore 8 \tan x - \sqrt{5} \sec y = -7/2$

4. If sin x + cos x = 0 and x lies in the fourth quadrant, find sin x and cos x.
Solution:
Given:
Sin x + cos x = 0 and x lies in fourth quadrant.
Sin x = -cos x
Sin x/cos x = -1
So, tan x = -1 (since, tan x = sin x/cos x)
We know that, in fourth quadrant, cos x and sec x are positive and all other ratios are negative.

By using the formulas,
Sec x = $\sqrt{(1 + \tan^2 x)}$
Cos x = 1/sec x
Sin x = - $\sqrt{(1- \cos^2 x)}$
Now,
Sec x = $\sqrt{(1 + \tan^2 x)}$
= $\sqrt{(1 + (-1)^2)}$
= $\sqrt{2}$

Cos x = 1/sec x
= $1/\sqrt{2}$

Sin x = - $\sqrt{(1 – \cos^2 x)}$
= - $\sqrt{(1 – (1/\sqrt{2})^2)}$
= - $\sqrt{(1 – (1/2))}$
= - $\sqrt{((2-1)/2)}$
= - $\sqrt{(1/2)}$
= $-1/\sqrt{2}$
∴ sin x = $-1/\sqrt{2}$ and cos x = $1/\sqrt{2}$

5. If cos x = -3/5 and π<x<3π/2 find the values of other five trigonometric functions and hence evaluate $\frac{cosec\ x + cot\ x}{sec\ x - tan\ x}$

Solution:
Given:
cos x= -3/5 and π <x < 3π/2
We know that in the third quadrant, tan x and cot x are positive and all other rations are negative.
By using the formulas,
Sin x = - $\sqrt{(1-\cos^2 x)}$
Tan x = sin x/cos x
Cot x = 1/tan x
Sec x = 1/cos x
Cosec x = 1/sin x
Now,
Sin x = - $\sqrt{(1-\cos^2 x)}$
= - $\sqrt{(1-(-3/5)^2)}$
= - $\sqrt{(1-9/25)}$
= - $\sqrt{((25-9)/25)}$

= - √(16/25)
= - 4/5

Tan x = sin x/cos x
= (-4/5)/(-3/5)
= -4/5 × -5/3
= 4/3

Cot x = 1/tan x
= 1/(4/3)
= 3/4

Sec x = 1/cos x
= 1/(-3/5)
= -5/3

Cosec x = 1/sin x
= 1/(-4/5)
= -5/4

$\therefore \frac{cosec\ x + cot\ x}{sec\ x - tan\ x}$ = [(-5/4) + (3/4)] / [(-5/3) – (4/3)]
= [(-5+3)/4] / [(-5-4)/3]
= [-2/4] / [-9/3]
= [-1/2] / [-3]
= 1/6