EXERCISE 5.1 PAGE NO: 5.18

Prove the following identities:

1. $\sec^4 x - \sec^2 x = \tan^4 x + \tan^2 x$

Solution:

Let us consider LHS: $\sec^4 x - \sec^2 x$

$(\sec^2 x)^2 - \sec^2 x$

By using the formula, $\sec^2 \theta = 1 + \tan^2 \theta$.

$(1 + \tan^2 x)^2 - (1 + \tan^2 x)$

$1 + 2\tan^2 x + \tan^4 x - 1 - \tan^2 x$

$\tan^4 x + \tan^2 x$

= RHS

∴ LHS = RHS

Hence proved.

2. $\sin^6 x + \cos^6 x = 1 - 3 \sin^2 x \cos^2 x$

Solution:

Let us consider LHS: $\sin^6 x + \cos^6 x$

$(\sin^2 x)^3 + (\cos^2 x)^3$

By using the formula, $a^3 + b^3 = (a + b) (a^2 + b^2 - ab)$

$(\sin^2 x + \cos^2 x) [(\sin^2 x)^2 + (\cos^2 x)^2 - \sin^2 x \cos^2 x]$

By using the formula, $\sin^2 x + \cos^2 x = 1$ and $a^2 + b^2 = (a + b)^2 - 2ab$

$1 \times [(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x - \sin^2 x \cos^2 x$

$1^2 - 3\sin^2 x \cos^2 x$

$1 - 3\sin^2 x \cos^2 x$

= RHS

∴ LHS = RHS

Hence proved.

3. (cosec x – sin x) (sec x – cos x) (tan x + cot x) = 1

Solution:

Let us consider LHS: (cosec x – sin x) (sec x – cos x) (tan x + cot x)

By using the formulas

$\text{cosec } \theta = 1/\sin \theta$;

$\sec \theta = 1/\cos \theta$;

$\tan \theta = \sin \theta / \cos \theta$;

$\cot \theta = \cos \theta / \sin \theta$

Now,

$$\left(\frac{1}{\sin x}-\sin x\right)\left(\frac{1}{\cos x}-\cos x\right)\left(\frac{\sin x}{\cos x}+\frac{\cos x}{\sin x}\right)$$

$$\frac{1-\sin^2 x}{\sin x}\times\frac{1-\cos^2 x}{\cos x}\times\frac{\sin^2 x+\cos^2 x}{\sin x\cos x}$$

By using the formula,

$\text{Sin}^2 x + \cos^2 x = 1$;

$$\frac{\cos^2 x}{\sin x}\times\frac{\sin^2 x}{\cos x}\times\frac{1}{\sin x\cos x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

4. cosec x (sec x – 1) – cot x (1 – cos x) = tan x – sin x

Solution:

Let us consider LHS: cosec x (sec x – 1) – cot x (1 – cos x)

By using the formulas

cosec θ = 1/sin θ;

sec θ = 1/cos θ;

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

$$\frac{1}{\sin x}\left(\frac{1}{\cos x}-1\right)-\frac{\cos x}{\sin x}(1-\cos x)$$

$$\frac{1}{\sin x}\left(\frac{1-\cos x}{\cos x}\right)-\frac{\cos x}{\sin x}(1-\cos x)$$

$$\left(\frac{1-\cos x}{\sin x}\right)\left(\frac{1}{\cos x}-\cos x\right)$$

$$\left(\frac{1-\cos x}{\sin x}\right)\left(\frac{1-\cos^2 x}{\cos x}\right)$$

By using the formula, $1 – \cos^2 x = \sin^2 x$;

$$\left(\frac{1-\cos x}{\sin x}\right)\left(\frac{\sin^2 x}{\cos x}\right)$$

$$(1-\cos x)\left(\frac{\sin x}{\cos x}\right)$$

$$\frac{\sin x}{\cos x}-\sin x$$

$$\tan x-\sin x$$

= RHS

∴ LHS = RHS

Hence Proved.

5.

$$\frac{1-\sin x\cos x}{\cos x(\sec x-\operatorname{cosec} x)}\cdot\frac{\sin^2 x-\cos^2 x}{\sin^3 x+\cos^3 x}=\sin x$$

Solution:

Let us consider the LHS:

$$\frac{1-\sin x\cos x}{\cos x(\sec x-\operatorname{cosec} x)}\cdot\frac{\sin^2 x-\cos^2 x}{\sin^3 x+\cos^3 x}$$

By using the formula,

cosec θ = 1/sin θ;

sec θ = 1/cos θ;

Now,

$$\frac{1-\sin x\cos x}{\cos x\left(\frac{1}{\cos x}-\frac{1}{\sin x}\right)}\times\frac{(\sin x)^2-(\cos x)^2}{(\sin x)^3+(\cos x)^3}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 – ab)$

$$\frac{1-\sin x\cos x}{\cos x\left(\frac{\sin x-\cos x}{\cos x\sin x}\right)}\times\frac{(\sin x+\cos x)(\sin x-\cos x)}{(\sin x+\cos x)\,[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

$$\frac{\sin x(1-\sin x\cos x)}{\sin x-\cos x}\times\frac{(\sin x+\cos x)(\sin x-\cos x)}{(\sin x+\cos x)\,[(\sin x)^2+(\cos x)^2-\sin x\cos x]}$$

$$\frac{\sin x(1-\sin x\cos x)}{1}\times\frac{1}{[(\sin x)^2+(\cos x)^2-\sin x\ \cos x]}$$

By using the formula, $\sin^2 x + \cos^2 x = 1$.

$$\sin x(1-\sin x\cos x)\times\frac{1}{(1-\sin x\cos x)}$$

sin x
= RHS
∴ LHS = RHS
Hence Proved.

6.
$$\frac{\tan x}{1-\cot x}+\frac{\cot x}{1-tanx}=(\sec x\ cosec\ x+1)$$

Solution:
Let us consider the LHS:

$$\frac{\tan x}{1-\cot x}+\frac{\cot x}{1-tanx}$$

By using the formula,
tan θ = sin θ / cos θ;
cot θ = cos θ / sin θ
Now,

$$\frac{\frac{\sin x}{\cos x}}{1-\frac{\cos x}{\sin x}}+\frac{\frac{\cos x}{\sin x}}{1-\frac{\sin x}{\cos x}}$$

$$\frac{\frac{\sin x}{\cos x}}{\frac{\sin x-\cos x}{\sin x}}+\frac{\frac{\cos x}{\sin x}}{\frac{\cos x-\sin x}{\cos x}}$$

$$\frac{\sin^2 x}{\cos x(\sin x-\cos x)}-\frac{\cos^2 x}{\sin x(\sin x-\cos x)}$$

$$\frac{\sin^3 x-\cos^3 x}{\sin x\cos x(\sin x-\cos x)}$$

By using the formula, $a^3 - b^3 = (a - b)(a^2 + b^2 + ab)$

$$\frac{(\sin x - \cos x)\,[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x \cos x(\sin x - \cos x)}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\frac{[1 + \sin x \cos x]}{\sin x \cos x}$$

$$\frac{1}{\sin x \cos x} + \frac{\sin x \cos x}{\sin x \cos x}$$

$$\frac{1}{\sin x} \times \frac{1}{\cos x} + 1$$

By using the formula,
cosec $\theta = 1/\sin\theta$,
sec $\theta = 1/\cos\theta$;
cosec $x \times$ sec $x + 1$
sec x cosec $x + 1$
=RHS
∴ LHS = RHS
Hence Proved.

7.

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = 2$$

Solution:
Let us consider LHS:

$$\frac{\sin^3 x + \cos^3 x}{\sin x + \cos x} + \frac{\sin^3 x - \cos^3 x}{\sin x - \cos x}$$

By using the formula $a^3 \pm b^3 = (a \pm b)(a^2 + b^2 \mp ab)$

$$\frac{(\sin x + \cos x)\,[(\sin x)^2 + (\cos x)^2 - \sin x \cos x]}{\sin x + \cos x} + \frac{(\sin x - \cos x)\,[(\sin x)^2 + (\cos x)^2 + \sin x \cos x]}{\sin x - \cos x}$$

We know, $\sin^2x + \cos^2x = 1$.
1 - sinx cosx + 1 + sinx cosx
2
= RHS
∴ LHS = RHS
Hence Proved.

8. $(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2 = 1$
Solution:
Let us consider LHS:
$(\sec x \sec y + \tan x \tan y)^2 - (\sec x \tan y + \tan x \sec y)^2$
Expanding the above equation we get,
$[(\sec x \sec y)^2 + (\tan x \tan y)^2 + 2 (\sec x \sec y) (\tan x \tan y)] - [(\sec x \tan y)^2 + (\tan x \sec y)^2 + 2 (\sec x \tan y) (\tan x \sec y)]$
$[\sec^2 x \sec^2 y + \tan^2 x \tan^2 y + 2 (\sec x \sec y) (\tan x \tan y)] - [\sec^2 x \tan^2 y + \tan^2 x \sec^2 y + 2 (\sec^2 x \tan^2 y) (\tan x \sec y)]$
$\sec^2 x \sec^2 y - \sec^2 x \tan^2 y + \tan^2 x \tan^2 y - \tan^2 x \sec^2 y$
$\sec^2 x (\sec^2 y - \tan^2 y) + \tan^2 x (\tan^2 y - \sec^2 y)$
$\sec^2 x (\sec^2 y - \tan^2 y) - \tan^2 x (\sec^2 y - \tan^2 y)$
We know, $\sec^2 x - \tan^2 x = 1$.
$\sec^2 x \times 1 - \tan^2 x \times 1$
$\sec^2 x - \tan^2 x$
1 = RHS
∴ LHS = RHS
Hence proved.

9.

$$\frac{\cos x}{1-\sin x} = \frac{1+\cos x+\sin x}{1+\cos x-\sin x}$$

Solution:
Let us Consider RHS:

$$\frac{1+\cos x+\sin x}{1+\cos x-\sin x}$$

$$\frac{(1+\cos x)+(\sin x)}{(1+\cos x)-(\sin x)}$$

$$\frac{(1+\cos x)+(\sin x)}{(1+\cos x)-(\sin x)} \times \frac{(1+\cos x)+(\sin x)}{(1+\cos x)+(\sin x)}$$

$$\frac{[(1+\cos x)+(\sin x)]^2}{(1+\cos x)^2-(\sin x)^2}$$

$$\frac{[(1+\cos x)+(\sin x)]^2}{(1+\cos x)^2-(\sin x)^2}$$

$$\frac{(1+\cos x)^2+(\sin x)^2+2(1+\cos x)(\sin x)}{(1+\cos^2 x+2\cos x)-(\sin^2 x)}$$

$$\frac{(1+\cos x)^2+(\sin x)^2+2(1+\cos x)(\sin x)}{(1+\cos^2 x+2\cos x)-(\sin^2 x)}$$

$$\frac{(1+\cos x)^2+(\sin x)^2+2(1+\cos x)(\sin x)}{(1+\cos^2 x+2\cos x)-(\sin^2 x)}$$

$$\frac{1+\cos^2 x+2\cos x+\sin^2 x+2\sin x+2\sin x\cos x}{1+\cos^2 x+2\cos x-\sin^2 x}$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\frac{1+1+2\cos x+2\sin x+2\sin x\cos x}{(1-\sin^2 x)+\cos^2 x+2\cos x}$$

We know, $1 - \cos^2 x = \sin^2 x$.

$$\frac{2+2\cos x+2\sin x+2\sin x\cos x}{\cos^2 x+\cos^2 x+2\cos x}$$

$$\frac{2+2\cos x+2\sin x+2\sin x\cos x}{2\cos^2 x+2\cos x}$$

$$\frac{2+2\cos x+2\sin x+2\sin x\cos x}{\cos^2 x+\cos^2 x+2\cos x}$$

$$\frac{1+\cos x+\sin x+\sin x\cos x}{\cos x(\cos x+1)}$$

$$\frac{1(1+\cos x)+\sin x(\cos x+1)}{\cos x(\cos x+1)}$$

$$\frac{(1+\sin x)(\cos x+1)}{\cos x(\cos x+1)}$$

$$\frac{1+\sin x}{\cos x} \times \frac{\cos x}{\cos x}$$

$$\frac{(1+\sin x)\cos x}{\cos^2 x}$$

We know, $1 - \sin^2 x = \cos^2 x$.

$$\frac{(1+\sin x)\cos x}{1-\sin^2 x}$$

$$\frac{(1+\sin x)\cos x}{(1-\sin x)(1+\sin x)}$$

$$\frac{\cos x}{1-\sin x}$$

= LHS

∴ LHS = RHS

Hence Proved.

10.

$$\frac{\tan^3 x}{1+\tan^2 x} + \frac{\cot^3 x}{1+\cot^2 x} = \frac{1-2\sin^2 x\cos^2 x}{\sin x\cos x}$$

Solution:

Let us consider LHS:

$$\frac{\tan^3 x}{1+\tan^2 x} + \frac{\cot^3 x}{1+\cot^2 x}$$

By using the formulas,

$1 + \tan^2 x = \sec^2 x$ and $1 + \cot^2 x = \mathrm{cosec}^2 x$

$$\frac{\tan^3 x}{\sec^2 x} + \frac{\cot^3 x}{\mathrm{cosec}^2 x}$$

$$\frac{\frac{\sin^3 x}{\cos^3 x}}{\frac{1}{\cos^2 x}} + \frac{\frac{\cos^3 x}{\sin^3 x}}{\frac{1}{\sin^2 x}}$$

$$\frac{\sin^3 x}{\cos x} + \frac{\cos^3 x}{\sin x}$$

$$\frac{\sin^4 x + \cos^4 x}{\cos x \sin x}$$

$$\frac{(\sin^2 x)^2 + (\cos^2 x)^2}{\cos x \sin x}$$

We know, $a^2 + b^2 = (a + b)^2 - 2ab$

$$\frac{(\sin^2 x + \cos^2 x)^2 - 2\sin^2 x \cos^2 x}{\sin x \cos x}$$

$$\frac{1^2 - 2\sin^2 x \cos^2 x}{\sin x \cos x}$$

$$\frac{1 - 2\sin^2 x \cos^2 x}{\sin x \cos x}$$

= RHS

∴ LHS = RHS

Hence Proved.

11.

$$1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x} = \sin x \cos x$$

Solution:

Let us consider LHS:

$$1 - \frac{\sin^2 x}{1 + \cot x} - \frac{\cos^2 x}{1 + \tan x}$$

By using the formula,

tan θ = sin θ / cos θ;

cot θ = cos θ / sin θ

Now,

$$1 - \frac{\sin^2 x}{1 + \frac{\cos x}{\sin x}} - \frac{\cos^2 x}{1 + \frac{\sin x}{\cos x}}$$

$$1 - \frac{\sin^3 x}{\sin x + \cos x} - \frac{\cos^3 x}{\sin x + \cos x}$$

$$\frac{\sin x + \cos x - (\sin^3 x + \cos^3 x)}{\sin x + \cos x}$$

By using the formula, $a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$

$$\frac{\sin x + \cos x - ((\sin x + \cos x)(\sin x)^2 + (\cos x)^2 - \sin x \cos x))}{\sin x + \cos x}$$

$$\frac{(\sin x + \cos x)(1 - \sin^2 x - \cos^2 x + \sin x \cos x)}{\sin x + \cos x}$$

$1 - (\sin^2 x + \cos^2 x) + \sin x \cos x$

We know, $\sin^2 x + \cos^2 x = 1$.

$1 - 1 + \sin x \cos x$

Sin x cos x

= RHS

∴ LHS = RHS

Hence proved.

12.

$$\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{cosec^2 x - \sin^2 x}\right)\sin^2 x \cos^2 x = \frac{1 - \sin^2 x \cos^2 x}{2 + \sin^2 x \cos^2 x}$$

Solution:

Let us consider LHS:

$$\left(\frac{1}{\sec^2 x - \cos^2 x} + \frac{1}{cosec^2 x - \sin^2 x}\right)\sin^2 x \cos^2 x$$

By using the formula,

cosec θ = 1/sin θ,

sec θ = 1/cos θ;

$$\left(\frac{1}{\frac{1}{\cos^2 x} - \cos^2 x} + \frac{1}{\frac{1}{\sin^2 x} - \sin^2 x}\right)\sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x}{1 - \cos^4 x} + \frac{\sin^2 x}{1 - \sin^4 x}\right)\sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x(1 - \sin^4 x) + \sin^2 x(1 - \cos^4 x)}{(1 - \cos^4 x)(1 - \sin^4 x)}\right)\sin^2 x \cos^2 x$$

$$\left(\frac{\cos^2 x - \cos^2 x \sin^4 x + \sin^2 x - \sin^2 x \cos^4 x}{(1+\sin^2 x)(1-\sin^2 x)(1+\cos^2 x)(1-\cos^2 x)}\right)\sin^2 x \cos^2 x$$

We know, $\sin^2 x + \cos^2 x = 1$.

$$\left(\frac{1 - \cos^2 x \sin^4 x - \sin^2 x \cos^4 x}{(1+\sin^2 x)\cos^2 x(1+\cos^2 x)\sin^2 x}\right)\sin^2 x \cos^2 x$$

$$\left(\frac{1 - \cos^2 x \sin^2 x(\sin^2 x + \cos^2 x)}{(1+\sin^2 x)(1+\cos^2 x)}\right)$$

$$\frac{1-\sin^2 x \cos^2 x}{2+\sin^2 x \cos^2 x}$$

= RHS

∴ LHS = RHS

Hence proved.

13. $(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2 = \sec^2\alpha \sec^2\beta$

Solution:

Let us consider LHS: $(1 + \tan\alpha \tan\beta)^2 + (\tan\alpha - \tan\beta)^2$

$1 + \tan^2\alpha \tan^2\beta + 2\tan\alpha \tan\beta + \tan^2\alpha + \tan^2\beta - 2\tan\alpha \tan\beta$

$1 + \tan^2\alpha \tan^2\beta + \tan^2\alpha + \tan^2\beta$

$\tan^2\alpha(\tan^2\beta + 1) + 1(1 + \tan^2\beta)$

$(1 + \tan^2\beta)(1 + \tan^2\alpha)$

We know, $1 + \tan^2\theta = \sec^2\theta$

So,

$\sec^2\alpha \sec^2\beta$

= RHS

∴ LHS = RHS

Hence proved.

EXERCISE 5.2 PAGE NO: 5.25

1. Find the values of the other five trigonometric functions in each of the following:
(i) cot x = 12/5, x in quadrant III
(ii) cos x = -1/2, x in quadrant II
(iii) tan x = 3/4, x in quadrant III
(iv) sin x = 3/5, x in quadrant I
Solution:
(i) cot x = 12/5, x in quadrant III
In third quadrant, tan x and cot x are positive. sin x, cos x, sec x, cosec x are negative.
By using the formulas,
tan x = 1/cot x
= 1/(12/5)
= 5/12

cosec x = $-\sqrt{(1 + \cot^2 x)}$
= $-\sqrt{(1 + (12/5)^2)}$
= $-\sqrt{(25+144)/25}$
= $-\sqrt{(169/25)}$
= -13/5

sin x = 1/cosec x
= 1/(-13/5)
= -5/13

cos x = $-\sqrt{(1 - \sin^2 x)}$
= $-\sqrt{(1 - (-5/13)^2)}$
= $-\sqrt{(169-25)/169}$
= $-\sqrt{(144/169)}$
= -12/13

sec x = 1/cos x
= 1/(-12/13)
= -13/12

∴ sin x = -5/13, cos x = -12/13, tan x = 5/12, cosec x = -13/5, sec x = -13/12

(ii) cos x = -1/2, x in quadrant II
In second quadrant, sin x and cosec x are positive. tan x, cot x, cos x, sec x are negative.

By using the formulas,
sin x = √(1 – $\cos^2$ x)
= √(1 – $(-1/2)^2$)
= √(4-1)/4
= √(3/4)
= √3/2

tan x = sin x/cos x
= (√3/2)/(-1/2)
= -√3

cot x = 1/tan x
= 1/-√3
= -1/√3

cosec x = 1/sin x
= 1/(√3/2)
= 2/√3

sec x = 1/cos x
= 1/(-1/2)
= -2

∴ sin x = √3/2, tan x = -√3, cosec x = 2/√3, cot x = -1/√3 sec x = -2

(iii) tan x = 3/4, x in quadrant III
In third quadrant, tan x and cot x are positive. sin x, cos x, sec x, cosec x are negative.
By using the formulas,
sin x = √(1 – $\cos^2$ x)
= - √(1-$(-4/5)^2$)
= - √(25-16)/25
= - √(9/25)
= - 3/5

cos x = 1/sec x
= 1/(-5/4)
= -4/5

cot x = 1/tan x

$= 1/(3/4)$
$= 4/3$

cosec x = 1/sin x
$= 1/(-3/5)$
$= -5/3$

sec x = $-\sqrt{(1 + \tan^2 x)}$
$= -\sqrt{(1+(3/4)^2)}$
$= -\sqrt{(16+9)/16}$
$= -\sqrt{(25/16)}$
$= -5/4$

∴ sin x = -3/5, cos x = -4/5, cosec x = -5/3, sec x = -5/4, cot x = 4/3

(iv) sin x = 3/5, x in quadrant I
In first quadrant, all trigonometric ratios are positive.
So, by using the formulas,
tan x = sin x/cos x
$= (3/5)/(4/5)$
$= 3/4$

cosec x = 1/sin x
$= 1/(3/5)$
$= 5/3$

cos x = $\sqrt{(1-\sin^2 x)}$
$= \sqrt{(1 - (-3/5)^2)}$
$= \sqrt{(25-9)/25}$
$= \sqrt{(16/25)}$
$= 4/5$

sec x = 1/cos x
$= 1/(4/5)$
$= 5/4$

cot x = 1/tan x
$= 1/(3/4)$
$= 4/3$

∴ cos x = 4/5, tan x = 3/4, cosec x = 5/3, sec x = 5/4, cot x = 4/3

2. If sin x = 12/13 and lies in the second quadrant, find the value of sec x + tan x.
Solution:
Given:
Sin x = 12/13 and x lies in the second quadrant.
We know, in second quadrant, sin x and cosec x are positive and all other ratios are negative.
By using the formulas,
Cos x = $\sqrt{(1-\sin^2 x)}$
= $-\sqrt{(1-(12/13)^2)}$
= $-\sqrt{(1-(144/169))}$
= $-\sqrt{(169-144)/169}$
= $-\sqrt{(25/169)}$
= - 5/13

We know,
tan x = sin x/cos x
sec x = 1/cos x
Now,
tan x = (12/13)/(-5/13)
= -12/5

sec x = 1/(-5/13)
= -13/5

Sec x + tan x = -13/5 + (-12/5)
= (-13-12)/5
= -25/5
= -5
∴ Sec x + tan x = -5

3. If sin x = 3/5, tan y = 1/2 and π/2 < x< π< y< 3π/2 find the value of 8 tan x -√5 sec y.
Solution:
Given:
sin x = 3/5, tan y = 1/2 and π/2 < x< π< y< 3π/2
We know that, x is in second quadrant and y is in third quadrant.
In second quadrant, cos x and tan x are negative.

In third quadrant, sec y is negative.
By using the formula,
$\cos x = -\sqrt{(1-\sin^2 x)}$
$\tan x = \sin x/\cos x$
Now,
$\cos x = -\sqrt{(1-\sin^2 x)}$
$= -\sqrt{(1 - (3/5)^2)}$
$= -\sqrt{(1 - 9/25)}$
$= -\sqrt{((25-9)/25)}$
$= -\sqrt{(16/25)}$
$= -4/5$

$\tan x = \sin x/\cos x$
$= (3/5)/(-4/5)$
$= 3/5 \times -5/4$
$= -3/4$

We know that $\sec y = -\sqrt{(1+\tan^2 y)}$
$= -\sqrt{(1 + (1/2)^2)}$
$= -\sqrt{(1 + 1/4)}$
$= -\sqrt{((4+1)/4)}$
$= -\sqrt{(5/4)}$
$= -\sqrt{5}/2$

Now, $8 \tan x - \sqrt{5} \sec y = 8(-3/4) - \sqrt{5}(-\sqrt{5}/2)$
$= -6 + 5/2$
$= (-12+5)/2$
$= -7/2$
$\therefore 8 \tan x - \sqrt{5} \sec y = -7/2$

4. If sin x + cos x = 0 and x lies in the fourth quadrant, find sin x and cos x.
Solution:
Given:
Sin x + cos x = 0 and x lies in fourth quadrant.
Sin x = -cos x
Sin x/cos x = -1
So, tan x = -1 (since, tan x = sin x/cos x)
We know that, in fourth quadrant, cos x and sec x are positive and all other ratios are negative.

By using the formulas,
Sec x = $\sqrt{(1 + \tan^2 x)}$
Cos x = 1/sec x
Sin x = - $\sqrt{(1- \cos^2 x)}$
Now,
Sec x = $\sqrt{(1 + \tan^2 x)}$
= $\sqrt{(1 + (-1)^2)}$
= $\sqrt{2}$

Cos x = 1/sec x
= $1/\sqrt{2}$

Sin x = - $\sqrt{(1 - \cos^2 x)}$
= - $\sqrt{(1 - (1/\sqrt{2})^2)}$
= - $\sqrt{(1 - (1/2))}$
= - $\sqrt{((2-1)/2)}$
= - $\sqrt{(1/2)}$
= $-1/\sqrt{2}$
∴ sin x = $-1/\sqrt{2}$ and cos x = $1/\sqrt{2}$

5. If cos x = -3/5 and π<x<3π/2 find the values of other five trigonometric functions and hence evaluate $\frac{cosec\ x + \cot x}{\sec x - \tan x}$
Solution:
Given:
cos x= -3/5 and π <x < 3π/2
We know that in the third quadrant, tan x and cot x are positive and all other rations are negative.
By using the formulas,
Sin x = - $\sqrt{(1-\cos^2 x)}$
Tan x = sin x/cos x
Cot x = 1/tan x
Sec x = 1/cos x
Cosec x = 1/sin x
Now,
Sin x = - $\sqrt{(1-\cos^2 x)}$
= - $\sqrt{(1-(-3/5)^2)}$
= - $\sqrt{(1-9/25)}$
= - $\sqrt{((25-9)/25)}$

$= -\sqrt{(16/25)}$
$= -4/5$

Tan x = sin x/cos x
$= (-4/5)/(-3/5)$
$= -4/5 \times -5/3$
$= 4/3$

Cot x = 1/tan x
$= 1/(4/3)$
$= 3/4$

Sec x = 1/cos x
$= 1/(-3/5)$
$= -5/3$

Cosec x = 1/sin x
$= 1/(-4/5)$
$= -5/4$

$\therefore \frac{cosec\ x + cot\ x}{sec\ x - tan\ x} = [(-5/4) + (3/4)] / [(-5/3) – (4/3)]$
$= [(-5+3)/4] / [(-5-4)/3]$
$= [-2/4] / [-9/3]$
$= [-1/2] / [-3]$
$= 1/6$

EXERCISE 5.3 PAGE NO: 5.39

1. Find the values of the following trigonometric ratios:
(i) sin 5π/3
(ii) sin 17π
(iii) tan 11π/6
(iv) cos (-25π/4)
(v) tan 7π/4
(vi) sin 17π/6
(vii) cos 19π/6
(viii) sin (-11π/6)
(ix) cosec (-20π/3)
(x) tan (-13π/4)
(xi) cos 19π/4
(xii) sin 41π/4
(xiii) cos 39π/4
(xiv) sin 151π/6
Solution:
(i) sin 5π/3
$5\pi/3 = (5\pi/3 \times 180)^o$
$= 300^o$
$= (90\times3 + 30)^o$
Since, 300^o lies in IV quadrant in which sine function is negative.
$\sin 5\pi/3 = \sin (300)^o$
$= \sin (90\times3 + 30)^o$
$= - \cos 30^o$
$= - \sqrt{3}/2$

(ii) sin 17π
$\text{Sin } 17\pi = \sin 3060^o$
$= \sin (90\times34 + 0)^o$
Since, 3060^o lies in the negative direction of x-axis i.e., on boundary line of II and III quadrants.
$\text{Sin } 17\pi = \sin (90\times34 + 0)^o$
$= - \sin 0^o$
$= 0$

(iii) tan 11π/6
$\tan 11\pi/6 = (11/6 \times 180)^o$

$= 330^o$

Since, 330^o lies in the IV quadrant in which tangent function is negative.

$\tan 11\pi/6 = \tan (300)^o$
$= \tan (90\times3 + 60)^o$
$= - \cot 60^o$
$= - 1/\sqrt{3}$

(iv) $\cos (-25\pi/4)$

$\cos (-25\pi/4) = \cos (-1125)^o$
$= \cos (1125)^o$

Since, 1125^o lies in the I quadrant in which cosine function is positive.

$\cos (1125)^o = \cos (90\times12 + 45)^o$
$= \cos 45^o$
$= 1/\sqrt{2}$

(v) $\tan 7\pi/4$

$\tan 7\pi/4 = \tan 315^o$
$= \tan (90\times3 + 45)^o$

Since, 315^o lies in the IV quadrant in which tangent function is negative.

$\tan 315^o = \tan (90\times3 + 45)^o$
$= - \cot 45^o$
$= -1$

(vi) $\sin 17\pi/6$

$\sin 17\pi/6 = \sin 510^o$
$= \sin (90\times5 + 60)^o$

Since, 510^o lies in the II quadrant in which sine function is positive.

$\sin 510^o = \sin (90\times5 + 60)^o$
$= \cos 60^o$
$= 1/2$

(vii) $\cos 19\pi/6$

$\cos 19\pi/6 = \cos 570^o$
$= \cos (90\times6 + 30)^o$

Since, 570^o lies in III quadrant in which cosine function is negative.

$\cos 570^o = \cos (90\times6 + 30)^o$
$= - \cos 30^o$
$= - \sqrt{3}/2$

(viii) sin $(-11\pi/6)$

sin $(-11\pi/6)$ = sin (-330^o)

= - sin $(90\times3 + 60)^o$

Since, 330^o lies in the IV quadrant in which the sine function is negative.

sin (-330^o) = - sin $(90\times3 + 60)^o$

= - (-cos 60^o)

= - (-1/2)

= 1/2

(ix) cosec $(-20\pi/3)$

cosec $(-20\pi/3)$ = cosec $(-1200)^o$

= - cosec $(1200)^o$

= - cosec $(90\times13 + 30)^o$

Since, 1200^o lies in the II quadrant in which cosec function is positive.

cosec $(-1200)^o$ = - cosec $(90\times13 + 30)^o$

= - sec 30^o

= $-2/\sqrt{3}$

(x) tan $(-13\pi/4)$

tan $(-13\pi/4)$ = tan $(-585)^o$

= - tan $(90\times6 + 45)^o$

Since, 585^o lies in the III quadrant in which the tangent function is positive.

tan $(-585)^o$ = - tan $(90\times6 + 45)^o$

= - tan 45^o

= -1

(xi) cos $19\pi/4$

cos $19\pi/4$ = cos 855^o

= cos $(90\times9 + 45)^o$

Since, 855^o lies in the II quadrant in which the cosine function is negative.

cos 855^o = cos $(90\times9 + 45)^o$

= - sin 45^o

= $- 1/\sqrt{2}$

(xii) sin $41\pi/4$

sin $41\pi/4$ = sin 1845^o

= sin $(90\times20 + 45)^o$

Since, 1845^o lies in the I quadrant in which the sine function is positive.

sin 1845^o = sin $(90\times20 + 45)^o$

= sin 45°
= 1/√2

(xiii) cos 39π/4
cos 39π/4 = cos 1755°
= cos (90×19 + 45)°
Since, 1755° lies in the IV quadrant in which the cosine function is positive.
cos 1755° = cos (90×19 + 45)°
= sin 45°
= 1/√2

(xiv) sin 151π/6
sin 151π/6 = sin 4530°
= sin (90×50 + 30)°
Since, 4530° lies in the III quadrant in which the sine function is negative.
sin 4530° = sin (90×50 + 30)°
= - sin 30°
= -1/2

2. prove that:
(i) tan 225° cot 405° + tan 765° cot 675° = 0
(ii) sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6 = 1/2
(iii) cos 24° + cos 55° + cos 125° + cos 204° + cos 300° = 1/2
(iv) tan (-125°) cot (-405°) – tan (-765°) cot (675°) = 0
(v) cos 570° sin 510° + sin (-330°) cos (-390°) = 0
(vi) tan 11π/3 – 2 sin 4π/6 – 3/4 cosec^2 π/4 + 4 cos^2 17π/6 = (3 - 4√3)/2
(vii) 3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4 = 1
Solution:
(i) tan 225° cot 405° + tan 765° cot 675° = 0
Let us consider LHS:
tan 225° cot 405° + tan 765° cot 675°
tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)
We know that when n is odd, cot → tan.
tan 45° cot 45° + tan 45° [-tan 45°]
tan 45° cot 45° - tan 45° tan 45°
1 × 1 – 1 × 1
1 – 1
0 = RHS
∴ LHS = RHS

Hence proved.

(ii) sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6 = 1/2
Let us consider LHS:
sin 8π/3 cos 23π/6 + cos 13π/3 sin 35π/6
sin 480° cos 690° + cos 780° sin 1050°
sin (90° × 5 + 30°) cos (90° × 7 + 60°) + cos (90° × 8 + 60°) sin (90° × 11 + 60°)
We know that when n is odd, sin → cos and cos → sin.
cos 30° sin 60° + cos 60° [-cos 60°]
√3/2 × √3/2 – 1/2 × 1/2
3/4 - 1/4
2/4
1/2
= RHS
∴ LHS = RHS
Hence proved.

(iii) cos 24° + cos 55° + cos 125° + cos 204° + cos 300° = 1/2
Let us consider LHS:
cos 24° + cos 55° + cos 125° + cos 204° + cos 300°
cos 24° + cos (90° × 1 – 35°) + cos (90° × 1 + 35°) + cos (90° × 2 + 24°) + cos (90° × 3 + 30°)
We know that when n is odd, cos → sin.
cos 24° + sin 35° - sin 35° - cos 24° + sin 30°
0 + 0 + 1/2
1/2
= RHS
∴ LHS = RHS
Hence proved.

(iv) tan (-125°) cot (-405°) – tan (-765°) cot (675°) = 0
Let us consider LHS:
tan (-125°) cot (-405°) – tan (-765°) cot (675°)
We know that tan (-x) = -tan (x) and cot (-x) = -cot (x).
[-tan (225°)] [-cot (405°)] – [-tan (765°)] cot (675°)
tan (225°) cot (405°) + tan (765°) cot (675°)
tan (90° × 2 + 45°) cot (90° × 4 + 45°) + tan (90° × 8 + 45°) cot (90° × 7 + 45°)
tan 45° cot 45° + tan 45° [-tan 45°]
1 × 1 + 1 × (-1)

1 – 1
0
= RHS
∴ LHS = RHS
Hence proved.

(v) $\cos 570^o \sin 510^o + \sin(-330^o) \cos(-390^o) = 0$
Let us consider LHS:
$\cos 570^o \sin 510^o + \sin(-330^o) \cos(-390^o)$
We know that $\sin(-x) = -\sin(x)$ and $\cos(-x) = +\cos(x)$.
$\cos 570^o \sin 510^o + [-\sin(330^o)] \cos(390^o)$
$\cos 570^o \sin 510^o - \sin(330^o) \cos(390^o)$
$\cos(90° \times 6 + 30°) \sin(90° \times 5 + 60°) – \sin(90° \times 3 + 60°) \cos(90° \times 4 + 30°)$
We know that cos is negative at $90° + \theta$ i.e. in Q_2 and when n is odd, sin → cos and cos → sin.
$-\cos 30° \cos 60° - [-\cos 60°] \cos 30°$
$-\cos 30° \cos 60° + \cos 60° \cos 30°$
0
= RHS
∴ LHS = RHS
Hence proved.

(vi) $\tan 11\pi/3 – 2 \sin 4\pi/6 – 3/4 \text{ cosec}^2 \pi/4 + 4 \cos^2 17\pi/6 = (3 - 4\sqrt{3})/2$
Let us consider LHS:
$\tan 11\pi/3 – 2 \sin 4\pi/6 – 3/4 \text{ cosec}^2 \pi/4 + 4 \cos^2 17\pi/6$
$\tan (11 \times 180^o)/3 – 2 \sin (4 \times 180^o)/6 – 3/4 \text{ cosec}^2 180^o/4 + 4 \cos^2 (17 \times 180^o)/6$
$\tan 660^o – 2 \sin 120^o - 3/4 (\text{cosec } 45^o)^2 + 4 (\cos 510^o)^2$
$\tan(90° \times 7 + 30°) – 2 \sin(90° \times 1 + 30°) – 3/4 [\text{cosec } 45°]^2 + 4 [\cos(90° \times 5 + 60°)]^2$
We know that tan and cos is negative at $90° + \theta$ i.e. in Q_2 and when n is odd, tan → cot, sin → cos and cos → sin.
$[-\cot 30°] – 2 \cos 30° - 3/4 [\text{cosec } 45°]^2 + [-\sin 60°]^2$
$- \cot 30° - 2 \cos 30° - 3/4 [\text{cosec } 45°]^2 + [\sin 60°]^2$
$-\sqrt{3} - 2\sqrt{3}/2 – 3/4 (\sqrt{2})^2 + 4 (\sqrt{3}/2)^2$
$-\sqrt{3} - \sqrt{3} – 6/4 + 12/4$
$(3 - 4\sqrt{3})/2$
= RHS
∴ LHS = RHS
Hence proved.

(vii) 3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4 = 1
Let us consider LHS:
3 sin π/6 sec π/3 – 4 sin 5π/6 cot π/4
3 sin 180^o/6 sec 180^o/3 – 4 sin $5(180^o)$/6 cot 180^o/4
3 sin 30° sec 60° - 4 sin 150° cot 45°
3 sin 30° sec 60° - 4 sin (90° × 1 + 60°) cot 45°
We know that when n is odd, sin → cos.
3 sin 30° sec 60° - 4 cos 60° cot 45°
3 (1/2) (2) – 4 (1/2) (1)
3 – 2
1
= RHS
∴ LHS = RHS
Hence proved.

3. Prove that:

(i)
$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)} = 1$$

(ii)
$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)} = 2$$

(iii)
$$\frac{sin(\pi + x)\, cos(\frac{\pi}{2} + x)\, tan(\frac{3\pi}{2} - x)\, cot(2\pi - x)}{sin(2\pi - x)\, cos(2\pi + x)\, cosec(-x)\, sin(\frac{3\pi}{2} - x)} = 1$$

(iv)
$$\left\{1 + cot\,x - sec(\frac{\pi}{2} + x)\right\}\left\{1 + cot\,x + sec(\frac{\pi}{2} + x)\right\} = 2\,cot\,x$$

(v)
$$\frac{tan(\frac{\pi}{2} - x)\, sec(\pi - x)\, sin(-x)}{sin(\pi + x)\, cot(2\pi - x)\, cosec(\frac{\pi}{2} - x)} = 1$$

Solution:

(i)

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)} = 1$$

Let us consider LHS:

$$\frac{cos(2\pi + x)\, cosec(2\pi + x)\, tan(\pi/2 + x)}{sec(\pi/2 + x)\, cos\, x\, cot(\pi + x)}$$

$$\frac{cos\, x\, cosec\, x[-cot\, x]}{[-cosec\, x]\, cos\, x\, cot\, x}$$

$$\frac{-cos\, x cosec\, x\, cot\, x}{-cosec\, x\, cos\, x\, cot\, x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

(ii)

$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)} = 2$$

Let us consider LHS:

$$\frac{cosec\,(90^o + x) + cot(450^o + x)}{cosec(90^o - x) + tan(180^o - x)} + \frac{tan(180^o + x) + sec(180^o - x)}{tan(360^o + x) - sec(-x)}$$

$$\frac{cosec\,(90^o + x) + cot(90^o \times 5 + x)}{cosec(90^o - x) + tan(90^o \times 2 - x)} + \frac{tan(90^o \times 2 + x) + sec(90^o \times 2 - x)}{tan(90^o \times 4 + x) - sec(-x)}$$

We know that when n is odd, cosec → sec and also sec (-x) = sec x.

$$\frac{sec\, x + cot(90^o \times 5 + x)}{cosec(90^o - x) + tan(90^o \times 2 - x)} + \frac{tan(90^o \times 2 + x) + sec(90^o \times 2 - x)}{tan(90^o \times 4 + x) - sec(-x)}$$

$$\frac{sec\, x - tan\, x}{sec\, x - tan\, x} + \frac{tan\, x - sec\, x}{tan\, x - sec\, x}$$

1 + 1

2 = RHS

∴ LHS = RHS

Hence proved.

(iii)

$$\frac{sin(\pi + x)\ cos(\frac{\pi}{2} + x)\ tan(\frac{3\pi}{2} - x)\ cot(2\pi - x)}{sin(2\pi - x)\ cos(2\pi + x)\ cosec(-x)\ sin(\frac{3\pi}{2} - x)} = 1$$

Let us consider LHS:

$$\frac{sin(\pi + x)\ cos(\frac{\pi}{2} + x)\ tan(\frac{3\pi}{2} - x)\ cot(2\pi - x)}{sin(2\pi - x)\ cos(2\pi + x)\ cosec(-x)\ sin(\frac{3\pi}{2} - x)}$$

$$\frac{sin(180^o - x)\ cos(90^o + x)\ tan(270^o - x)\ cot(360^o - x)}{sin(360^o - x)\ cos(360^o + x)\ \ cosec(-x)\ sin(270^o - x)}$$

We know that cosec (-x) = - cosec x.

$$\frac{sin(90^o \times 2 - x)\ cos(90^o \times 1 + x)\ tan(90^o \times 3 - x)\ cot(90^o \times 4 - x)}{sin(90^o \times 4 - x)\ cos(90^o \times 4 + x)\ [-cosec(x)]\ sin(90^o \times 3 - x)}$$

We know that when n is odd, cos → sin, tan → cot and sin → cos.

$$\frac{(-sin\ x)(-sin\ x)\ cot\ x\ (-cot\ x)}{(-sin\ x)\ cos\ x\ (-cosec\ x)(-cos\ x)}$$

$$\frac{sin^2 x\ cot^2 x}{sin\ x\ cosec\ x\ cos\ x\ cos\ x}$$

$$\frac{sin^2 x\ \times \frac{cos^2 x}{sin^2 x}}{sin\ x\ \times \frac{1}{sin\ x} \times cos^2 x}$$

$$\frac{cos^2 x}{cos^2 x}$$

1 = RHS

∴ LHS = RHS

Hence proved.

(iv)

$$\left\{1 + cot\ x - sec(\frac{\pi}{2} + x)\right\}\left\{1 + cot\ x + sec(\frac{\pi}{2} + x)\right\} = 2\ cot\ x$$

Let us consider LHS:

$$\left\{1 + cot\ x - sec(\frac{\pi}{2} + x)\right\}\left\{1 + cot\ x + sec(\frac{\pi}{2} + x)\right\}$$

{1 + cot x – (-cosec x)} {1 + cot x + (-cosec x)}

{1 + cot x + cosec x} {1 + cot x – cosec x}

{(1 + cot x) + (cosec x)} {(1 + cot x) – (cosec x)}

By using the formula, $(a + b)(a - b) = a^2 - b^2$
$(1 + \cot x)^2 - (\text{cosec } x)^2$
$1 + \cot^2 x + 2 \cot x - \text{cosec}^2 x$
We know that $1 + \cot^2 x = \text{cosec}^2 x$
$\text{cosec}^2 x + 2 \cot x - \text{cosec}^2 x$
$2 \cot x$ = RHS
∴ LHS = RHS
Hence proved.

(v)

$$\frac{tan(\frac{\pi}{2}-x)\ sec(\pi-x)\ sin(-x)}{sin(\pi+x)\ cot(2\pi-x)\ cosec(\frac{\pi}{2}-x)} = 1$$

Let us consider LHS:

$$\frac{tan(\frac{\pi}{2}-x)\ sec(\pi-x)\ sin(-x)}{sin(\pi+x)\ cot(2\pi-x)\ cosec(\frac{\pi}{2}-x)}$$

$$\frac{tan(90^o-x)\ sec(180^o-x)\ sin(-x)}{sin(180^o+x)\ cot(360^o-x)cosec(90^o-x)}$$

We know that sin (-x) = -sin x.

$$\frac{tan(90^o\times 1-x)\ sec(90^o\times 2-x)\ [-sin(x)]}{sin(90^o\times 2+x)\ cot(90^o\times 4-x)cosec(90^o\times 1-x)}$$

We know that when n is odd, tan → cot and cosec → sec.

$$\frac{(cot\,x)(-sec\,x)(-sin\,x)}{(-sin\,x)(-cot\,x)(sec\,x)}$$

$$\frac{cot\,x\ sec\,x\ sin\,x}{sin\,x\ cot\,x\ sec\,x}$$

1 = RHS
∴ LHS = RHS
Hence proved.

4. Prove that: $\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9 = 2$
Solution:
Let us consider LHS:
$\sin^2 \pi/18 + \sin^2 \pi/9 + \sin^2 7\pi/18 + \sin^2 4\pi/9$

$\sin^2 \pi/18 + \sin^2 2\pi/18 + \sin^2 7\pi/18 + \sin^2 8\pi/18$
$\sin^2 \pi/18 + \sin^2 2\pi/18 + \sin^2 (\pi/2 - 2\pi/18) + \sin^2 (\pi/2 – \pi/18)$
We know that when n is odd, sin → cos.
$\sin^2 \pi/18 + \sin^2 2\pi/18 + \cos^2 2\pi/18 + \cos^2 2\pi/18$
when rearranged,
$\sin^2 \pi/18 + \cos^2 2\pi/18 + \sin^2 \pi/18 + \cos^2 2\pi/18$
We know that $\sin^2 + \cos^2x = 1$.
So,
1 + 1
2 = RHS
∴ LHS = RHS
Hence proved.