

EXERCISE 6.1

1. Sketch the graphs of the following functions:

(i) $f(x) = 2 \sin x, 0 \leq x \leq \pi$

(ii) $g(x) = 3 \sin(x - \pi/4), 0 \leq x \leq 5\pi/4$

(iii) $h(x) = 2 \sin 3x, 0 \leq x \leq 2\pi/3$

(iv) $\phi(x) = 2 \sin(2x - \pi/3), 0 \leq x \leq 7\pi/3$

(v) $\Psi(x) = 4 \sin 3(x - \pi/4), 0 \leq x \leq 2\pi$

(vi) $\theta(x) = \sin(x/2 - \pi/4), 0 \leq x \leq 4\pi$

(vii) $u(x) = \sin^2 x, 0 \leq x \leq 2\pi$ $v(x) = |\sin x|, 0 \leq x \leq 2\pi$

(viii) $f(x) = 2 \sin \pi x, 0 \leq x \leq 2$

Solution:

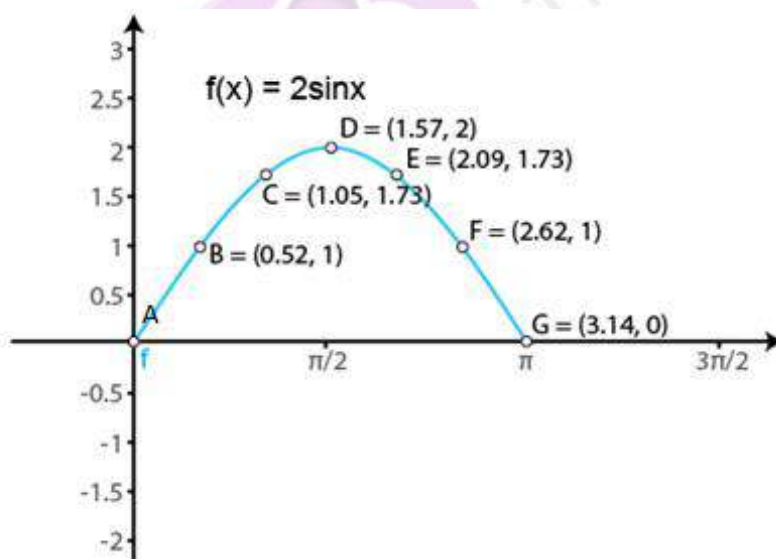
(i) $f(x) = 2 \sin x, 0 \leq x \leq \pi$

We know that $g(x) = \sin x$ is a periodic function with period π .

So, $f(x) = 2 \sin x$ is a periodic function with period π . So, we will draw the graph of $f(x) = 2 \sin x$ in the interval $[0, \pi]$. The values of $f(x) = 2 \sin x$ at various points in $[0, \pi]$ are listed in the following table:

x	0(A)	$\pi/6$ (B)	$\pi/3$ (C)	$\pi/2$ (D)	$2\pi/3$ (E)	$5\pi/6$ (F)	π (G)
$f(x) = 2 \sin x$	0	1	$\sqrt{3} = 1.73$	2	$\sqrt{3} = 1.73$	1	0

The required curve is:



(ii) $g(x) = 3 \sin(x - \pi/4), 0 \leq x \leq 5\pi/4$

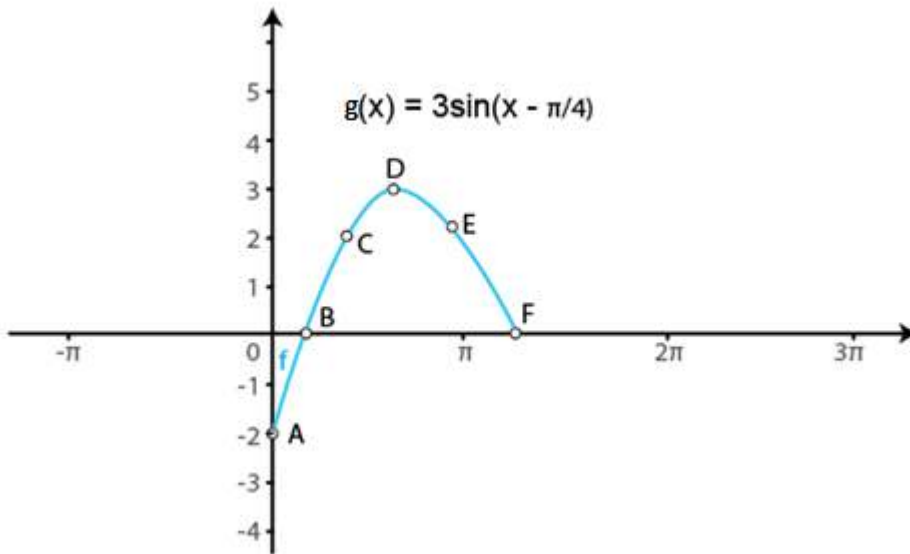
We know that if $f(x)$ is a periodic function with period T , then $f(ax + b)$ is periodic with

period $T/|a|$.

So, $g(x) = 3 \sin(x - \pi/4)$ is a periodic function with period π . So, we will draw the graph of $g(x) = 3 \sin(x - \pi/4)$ in the interval $[0, 5\pi/4]$. The values of $g(x) = 3 \sin(x - \pi/4)$ at various points in $[0, 5\pi/4]$ are listed in the following table:

x	0(A)	$\pi/4$ (B)	$\pi/2$ (C)	$3\pi/4$ (D)	π (E)	$5\pi/4$ (F)
$g(x) = 3 \sin(x - \pi/4)$	$-3/\sqrt{2} = -2.1$	0	$3/\sqrt{2} = 2.12$	3	$3/\sqrt{2} = 2.12$	0

The required curve is:



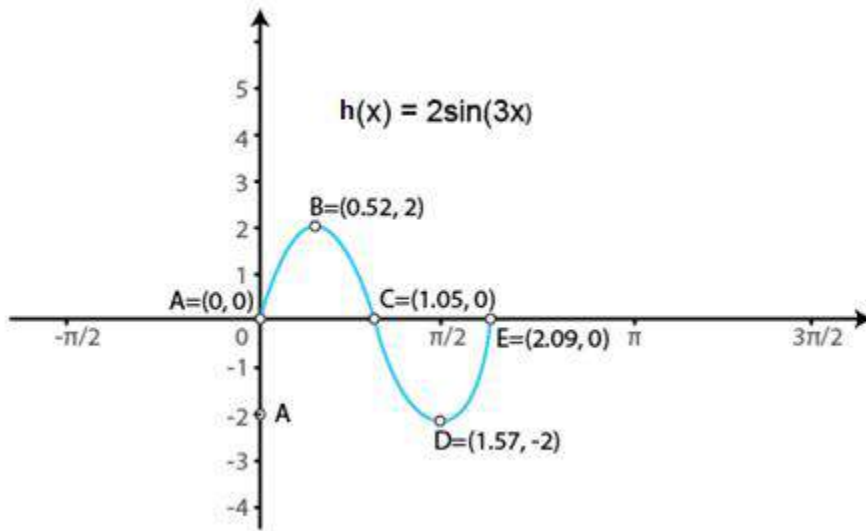
(iii) $h(x) = 2 \sin 3x, 0 \leq x \leq 2\pi/3$

We know that $g(x) = \sin x$ is a periodic function with period 2π .

So, $h(x) = 2 \sin 3x$ is a periodic function with period $2\pi/3$. So, we will draw the graph of $h(x) = 2 \sin 3x$ in the interval $[0, 2\pi/3]$. The values of $h(x) = 2 \sin 3x$ at various points in $[0, 2\pi/3]$ are listed in the following table:

x	0 (A)	$\pi/6$ (B)	$\pi/3$ (C)	$\pi/2$ (D)	$2\pi/3$ (E)
$h(x) = 2 \sin 3x$	0	2	0	-2	0

The required curve is:



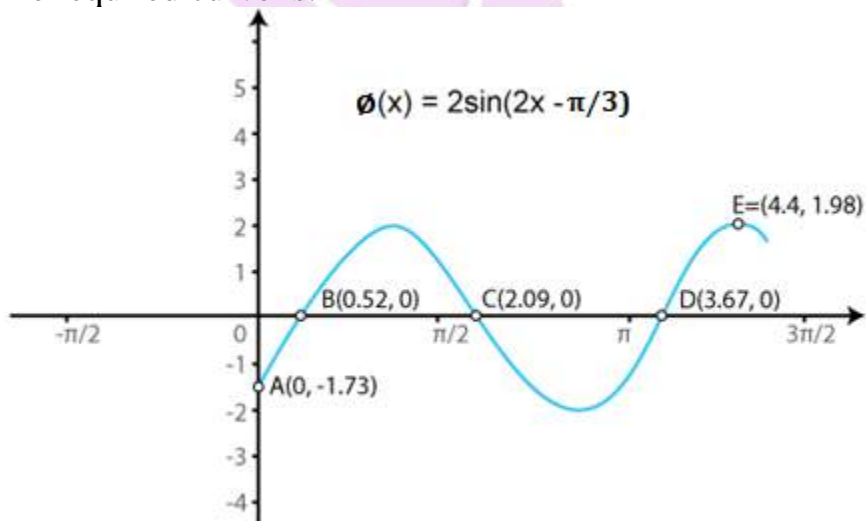
(iv) $\phi(x) = 2 \sin(2x - \pi/3), 0 \leq x \leq 7\pi/3$

We know that if $f(x)$ is a periodic function with period T , then $f(ax + b)$ is periodic with period $T/|a|$.

So, $\phi(x) = 2 \sin(2x - \pi/3)$ is a periodic function with period π . So, we will draw the graph of $\phi(x) = 2 \sin(2x - \pi/3)$, in the interval $[0, 7\pi/5]$. The values of $\phi(x) = 2 \sin(2x - \pi/3)$, at various points in $[0, 7\pi/5]$ are listed in the following table:

x	0 (A)	$\pi/6$ (B)	$2\pi/3$ (C)	$7\pi/6$ (D)	$7\pi/5$ (E)
$\phi(x) = 2 \sin(2x - \pi/3)$	$-\sqrt{3} = -1.73$	0	0	0	1.98

The required curve is:



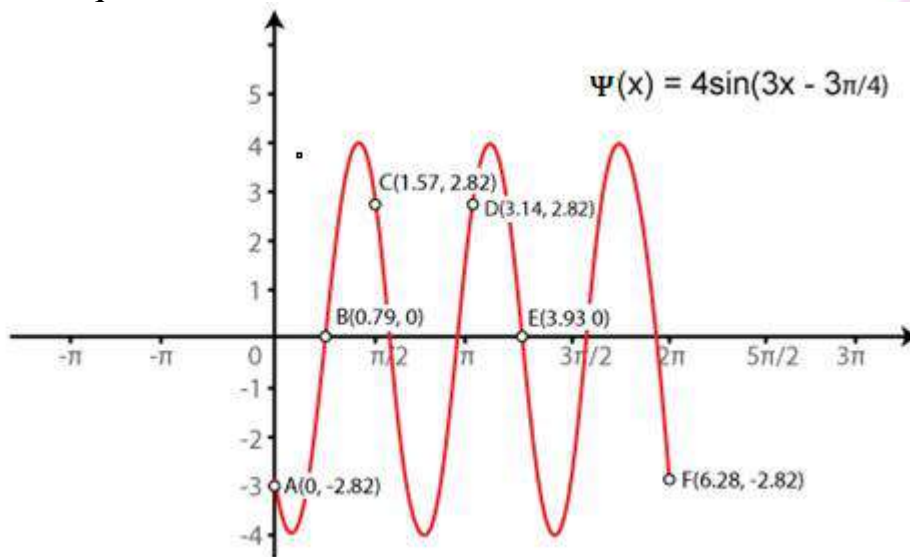
(v) $\Psi(x) = 4 \sin 3(x - \pi/4), 0 \leq x \leq 2\pi$

We know that if $f(x)$ is a periodic function with period T , then $f(ax + b)$ is periodic with period $T/|a|$.

So, $\Psi(x) = 4 \sin 3(x - \pi/4)$ is a periodic function with period 2π . So, we will draw the graph of $\Psi(x) = 4 \sin 3(x - \pi/4)$ in the interval $[0, 2\pi]$. The values of $\Psi(x) = 4 \sin 3(x - \pi/4)$ at various points in $[0, 2\pi]$ are listed in the following table:

x	0 (A)	$\pi/4$ (B)	$\pi/2$ (C)	π (D)	$5\pi/4$ (E)	2π (F)
$\Psi(x) = 4 \sin 3(x - \pi/4)$	$-2\sqrt{2} = -2.82$	0	$2\sqrt{2} = 2.82$	0	1.98	$-2\sqrt{2} = -2.82$

The required curve is:



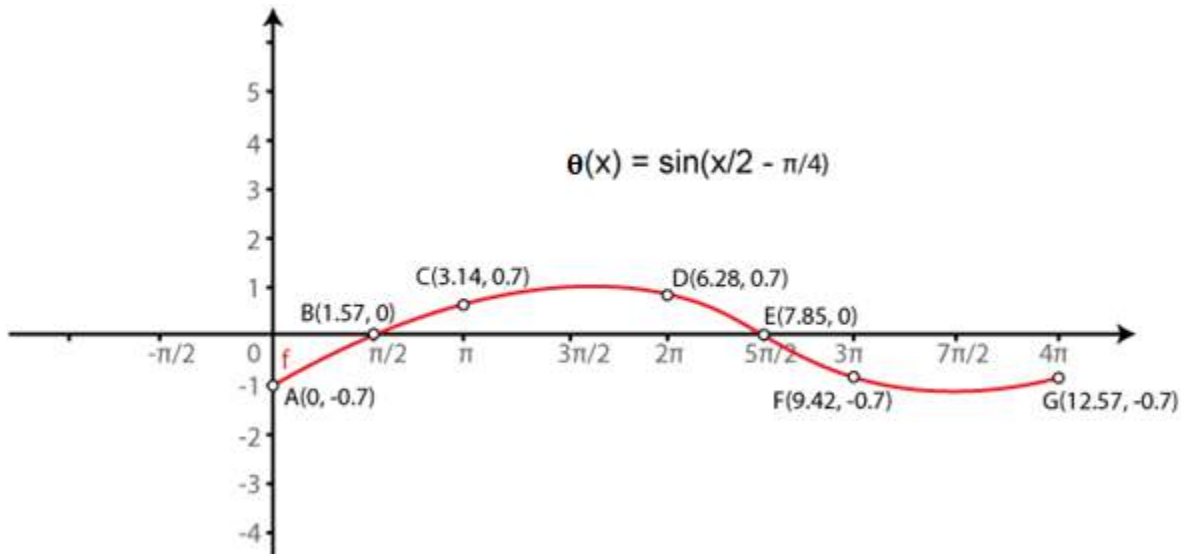
(vi) $\theta(x) = \sin(x/2 - \pi/4)$, $0 \leq x \leq 4\pi$

We know that if $f(x)$ is a periodic function with period T , then $f(ax + b)$ is periodic with period $T/|a|$.

So, $\theta(x) = \sin(x/2 - \pi/4)$ is a periodic function with period 4π . So, we will draw the graph of $\theta(x) = \sin(x/2 - \pi/4)$ in the interval $[0, 4\pi]$. The values of $\theta(x) = \sin(x/2 - \pi/4)$ at various points in $[0, 4\pi]$ are listed in the following table:

x	0 (A)	$\pi/2$ (B)	π (C)	2π (D)	$5\pi/2$ (E)	3π (F)	4π (G)
$\theta(x) = \sin(x/2 - \pi/4)$	$-1/\sqrt{2} = -0.7$	0	$1/\sqrt{2} = 0.7$	$1/\sqrt{2} = 0.7$	0	$-1/\sqrt{2} = -0.7$	$-1/\sqrt{2} = -0.7$

The required curve is:



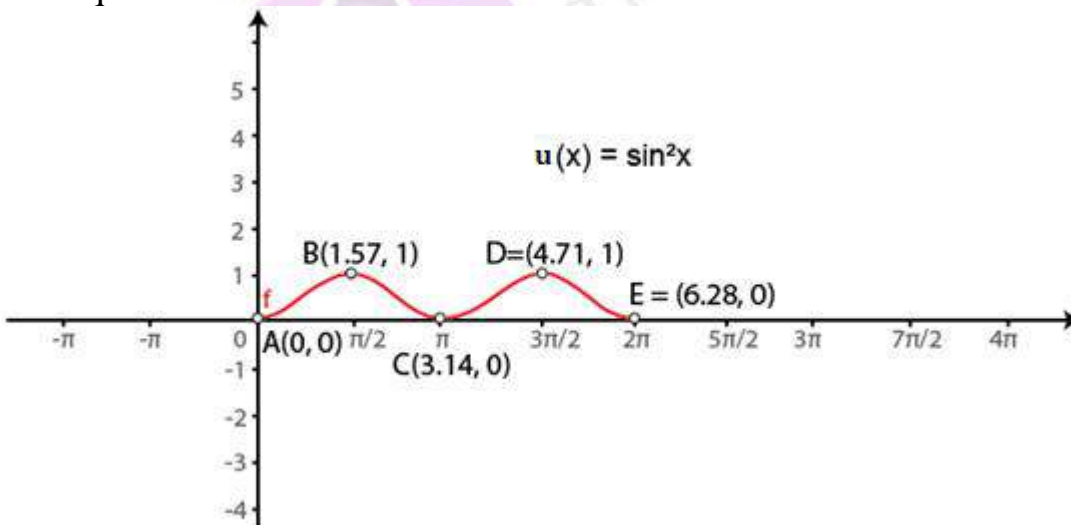
(vii) $u(x) = \sin^2 x$, $0 \leq x \leq 2\pi$ $v(x) = |\sin x|$, $0 \leq x \leq 2\pi$

We know that $g(x) = \sin x$ is a periodic function with period π .

So, $u(x) = \sin^2 x$ is a periodic function with period 2π . So, we will draw the graph of $u(x) = \sin^2 x$ in the interval $[0, 2\pi]$. The values of $u(x) = \sin^2 x$ at various points in $[0, 2\pi]$ are listed in the following table:

x	0 (A)	$\pi/2$ (B)	π (C)	$3\pi/2$ (D)	2π (E)
$u(x) = \sin^2 x$	0	1	0	1	0

The required curve is:



(viii) $f(x) = 2 \sin \pi x$, $0 \leq x \leq 2$

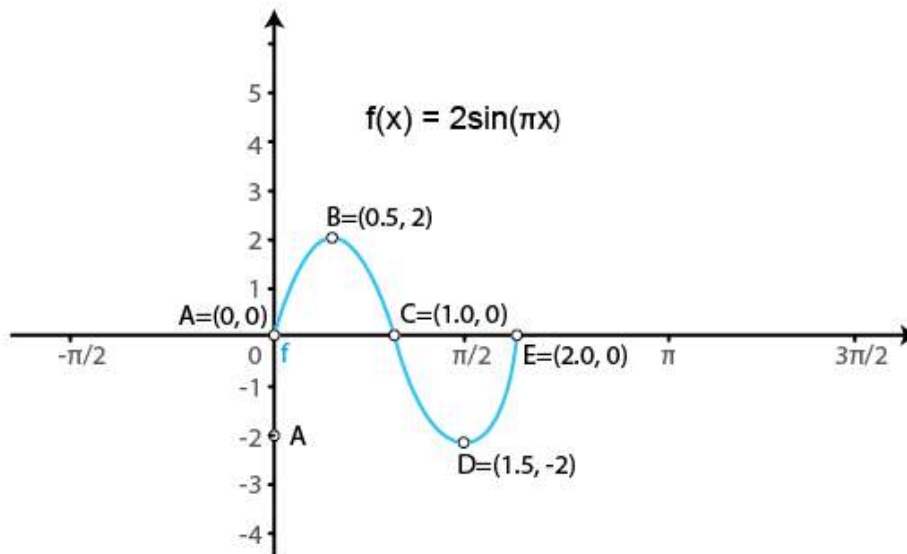
We know that $g(x) = \sin x$ is a periodic function with period 2π .

So, $f(x) = 2 \sin \pi x$ is a periodic function with period 2. So, we will draw the graph of f

$f(x) = 2 \sin \pi x$ in the interval $[0, 2]$. The values of $f(x) = 2 \sin \pi x$ at various points in $[0, 2]$ are listed in the following table:

x	0 (A)	1/2 (B)	1 (C)	3/2 (D)	2 (E)
$f(x) = 2 \sin \pi x$	0	2	0	-2	0

The required curve is:



2. Sketch the graphs of the following pairs of functions on the same axes:

(i) $f(x) = \sin x$, $g(x) = \sin(x + \pi/4)$

(ii) $f(x) = \sin x$, $g(x) = \sin 2x$

(iii) $f(x) = \sin 2x$, $g(x) = 2 \sin x$

(iv) $f(x) = \sin x/2$, $g(x) = \sin x$

Solution:

(i) $f(x) = \sin x$, $g(x) = \sin(x + \pi/4)$

We know that the functions $f(x) = \sin x$ and $g(x) = \sin(x + \pi/4)$ are periodic functions with periods 2π and $7\pi/4$.

The values of these functions are tabulated below:

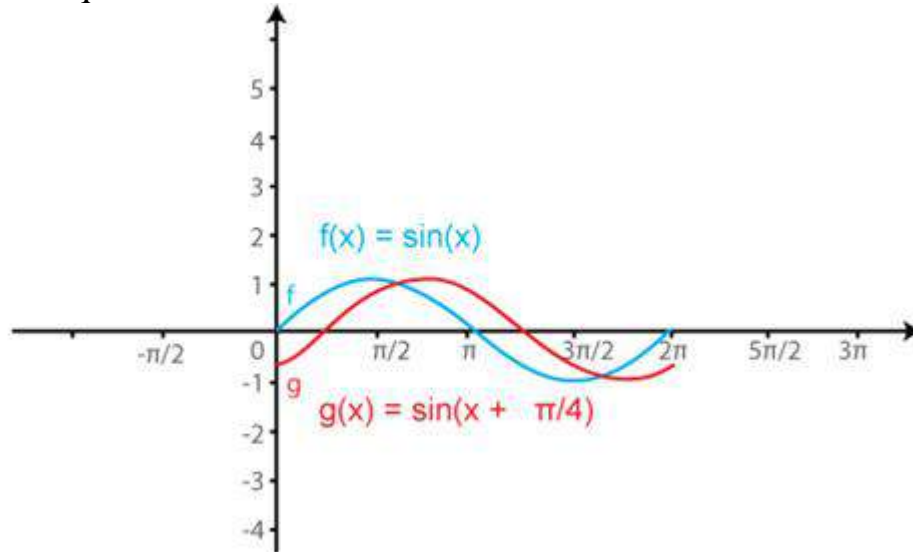
Values of $f(x) = \sin x$ in $[0, 2\pi]$

x	0	$\pi/2$	π	$3\pi/2$	2π
$f(x) = \sin x$	0	1	0	-1	0

Values of $g(x) = \sin(x + \pi/4)$ in $[0, 7\pi/4]$

x	0	$\pi/4$	$3\pi/4$	$5\pi/4$	$7\pi/4$
$g(x) = \sin(x + \pi/4)$	$1/\sqrt{2} = 0.7$	1	0	-1	0

The required curve is:



(ii) $f(x) = \sin x$, $g(x) = \sin 2x$

We know that the functions $f(x) = \sin x$ and $g(x) = \sin 2x$ are periodic functions with periods 2π and π .

The values of these functions are tabulated below:

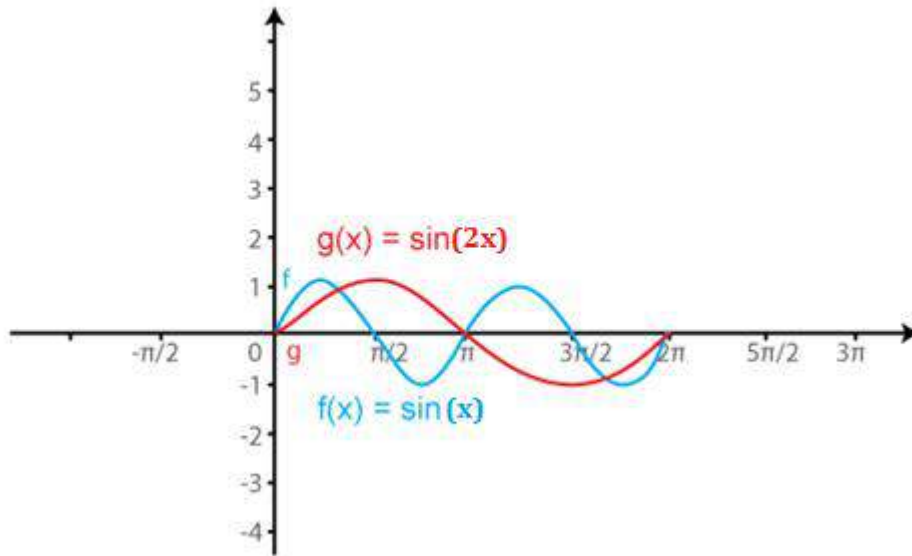
Values of $f(x) = \sin x$ in $[0, 2\pi]$

x	0	$\pi/2$	π	$3\pi/2$	2π
$f(x) = \sin x$	0	1	0	-1	0

Values of $g(x) = \sin(2x)$ in $[0, \pi]$

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$g(x) = \sin(2x)$	0	1	0	-1	0	1	0	-1	0

The required curve is:



(iii) $f(x) = \sin 2x$, $g(x) = 2 \sin x$

We know that the functions $f(x) = \sin 2x$ and $g(x) = 2 \sin x$ are periodic functions with periods π and π .

The values of these functions are tabulated below:

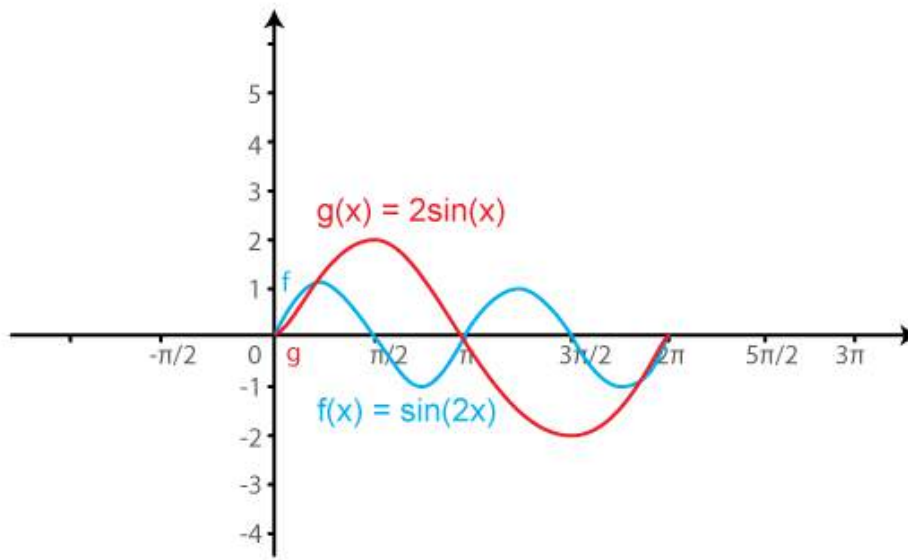
Values of $f(x) = \sin(2x)$ in $[0, \pi]$

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	π	$5\pi/4$	$3\pi/2$	$7\pi/4$	2π
$f(x) = \sin(2x)$	0	1	0	-1	0	1	0	-1	0

Values of $g(x) = 2 \sin x$ in $[0, \pi]$

x	0	$\pi/2$	π	$3\pi/2$	2π
$g(x) = 2 \sin x$	0	1	0	-1	0

The required curve is:



(iv) $f(x) = \sin x/2$, $g(x) = \sin x$

We know that the functions $f(x) = \sin x/2$ and $g(x) = \sin x$ are periodic functions with periods π and 2π .

The values of these functions are tabulated below:

Values of $f(x) = \sin x/2$ in $[0, \pi]$

x	0	π	2π	3π	4π
$f(x) = \sin x/2$	0	1	0	-1	0

Values of $g(x) = \sin(x)$ in $[0, 2\pi]$

x	0	$\pi/2$	π	$3\pi/2$	2π	$5\pi/2$	3π	$7\pi/2$	4π
$g(x) = \sin(x)$	0	1	0	-1	0	1	0	-1	0

The required curve is:

