

**EXERCISE 6.2**

**1. Sketch the graphs of the following trigonometric functions:**

- (i)  $f(x) = \cos(x - \pi/4)$
- (ii)  $g(x) = \cos(x + \pi/4)$
- (iii)  $h(x) = \cos^2 2x$
- (iv)  $\phi(x) = 2 \cos(x - \pi/6)$
- (v)  $\psi(x) = \cos(3x)$
- (vi)  $u(x) = \cos^2 x/2$
- (vii)  $f(x) = \cos \pi x$
- (viii)  $g(x) = \cos 2\pi x$

**Solution:**

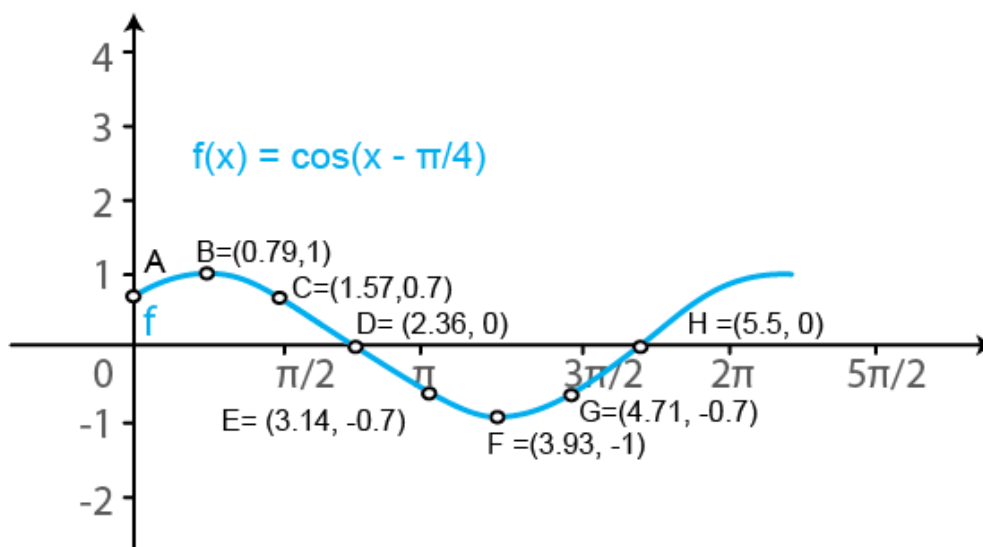
(i)  $f(x) = \cos(x - \pi/4)$

We know that  $g(x) = \cos x$  is a periodic function with period  $2\pi$ .

So,  $f(x) = \cos(x - \pi/4)$  is a periodic function with period  $\pi$ . So, we will draw the graph of  $f(x) = \cos(x - \pi/4)$  in the interval  $[0, \pi]$ . The values of  $f(x) = \cos(x - \pi/4)$  at various points in  $[0, \pi]$  are listed in the following table:

x	0 (A)	$\pi/4$ (B)	$\pi/2$ (C)	$3\pi/4$ (D)	$\pi$ (E)	$5\pi/4$ (F)	$3\pi/2$ (G)	$7\pi/4$ (H)
$f(x) = \cos(x - \pi/4)$	$1/\sqrt{2} = 0.7$	1	$1/\sqrt{2} = 0.7$	0	$-1/\sqrt{2} = -0.7$	-1	$-1/\sqrt{2} = -0.7$	0

The required curve is:



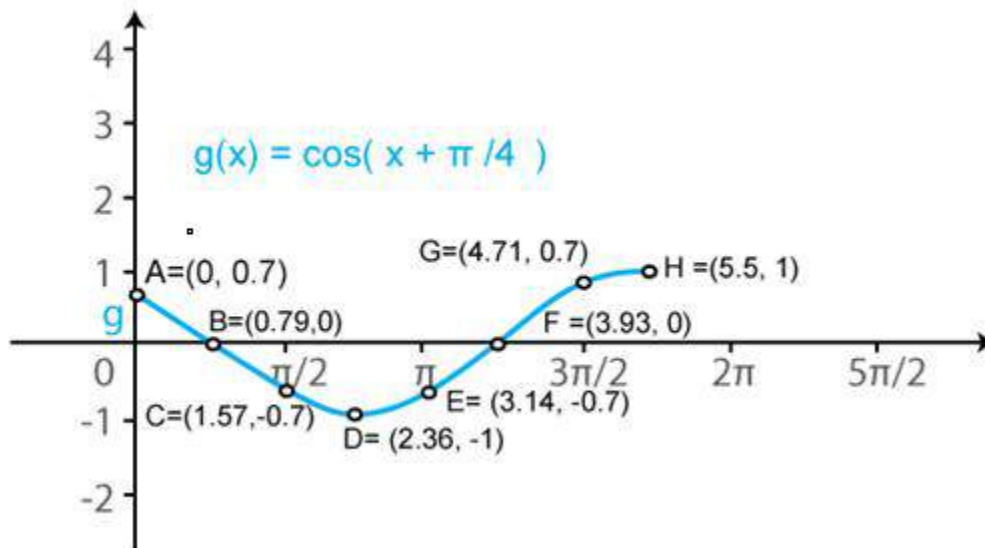
(ii)  $g(x) = \cos(x + \pi/4)$

We know that  $f(x) = \cos x$  is a periodic function with period  $2\pi$ .

So,  $g(x) = \cos(x + \pi/4)$  is a periodic function with period  $\pi$ . So, we will draw the graph of  $g(x) = \cos(x + \pi/4)$  in the interval  $[0, \pi]$ . The values of  $g(x) = \cos(x + \pi/4)$  at various points in  $[0, \pi]$  are listed in the following table:

x	0 (A)	$\pi/4$ (B)	$\pi/2$ (C)	$3\pi/4$ (D)	$\pi$ (E)	$5\pi/4$ (F)	$3\pi/2$ (G)	$7\pi/4$ (H)
$g(x) = \cos(x + \pi/4)$	$1/\sqrt{2} = 0.7$	0	$-1/\sqrt{2} = -0.7$	-1	$-1/\sqrt{2} = -0.7$	0	$1/\sqrt{2} = 0.7$	1

The required curve is:



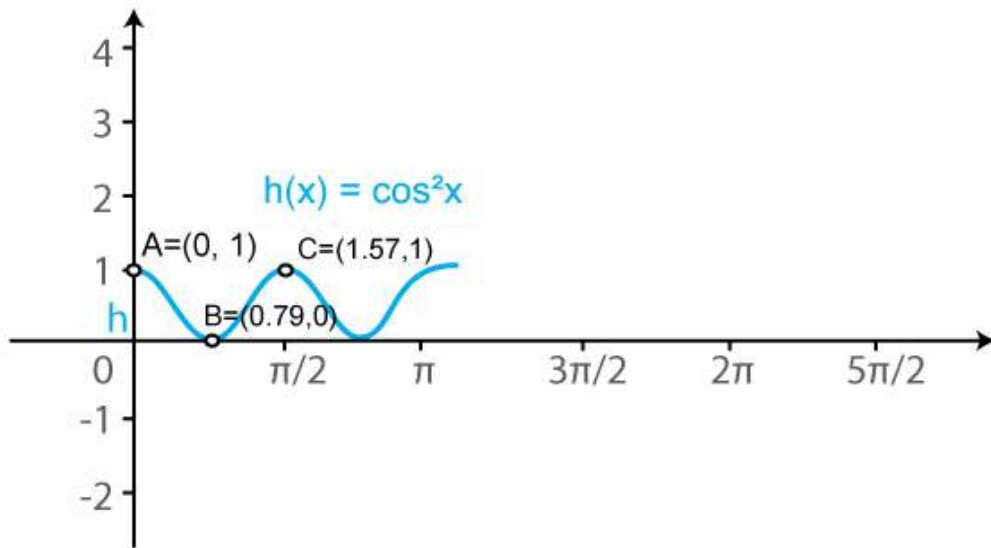
(iii)  $h(x) = \cos^2 2x$

We know that  $f(x) = \cos x$  is a periodic function with period  $2\pi$ .

So,  $h(x) = \cos^2 2x$  is a periodic function with period  $\pi$ . So, we will draw the graph of  $h(x) = \cos^2 2x$  in the interval  $[0, \pi]$ . The values of  $h(x) = \cos^2 2x$  at various points in  $[0, \pi]$  are listed in the following table:

x	0 (A)	$\pi/4$ (B)	$\pi/2$ (C)	$3\pi/4$ (D)	$\pi$ (E)	$5\pi/4$ (F)	$3\pi/2$ (G)
$h(x) = \cos^2 2x$	1	0	1	0	1	0	1

The required curve is:



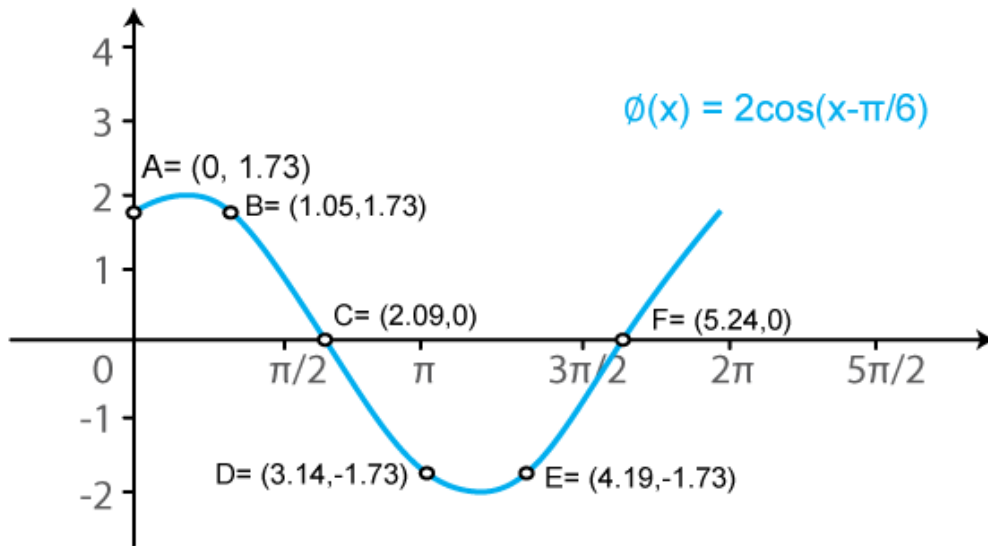
(iv)  $\phi(x) = 2 \cos(x - \pi/6)$

We know that  $f(x) = \cos x$  is a periodic function with period  $2\pi$ .

So,  $\phi(x) = 2 \cos(x - \pi/6)$  is a periodic function with period  $\pi$ . So, we will draw the graph of  $\phi(x) = 2 \cos(x - \pi/6)$  in the interval  $[0, \pi]$ . The values of  $\phi(x) = 2 \cos(x - \pi/6)$  at various points in  $[0, \pi]$  are listed in the following table:

x	0 (A)	$\pi/3$ (B)	$2\pi/3$ (C)	$\pi$ (D)	$4\pi/3$ (E)	$5\pi/3$ (F)
$\phi(x) = 2 \cos(x - \pi/6)$	$\sqrt{3} = 1.73$	$\sqrt{3} = 1.73$	0	$-\sqrt{3} = -1.73$	$-\sqrt{3} = -1.73$	0

The required curve is:



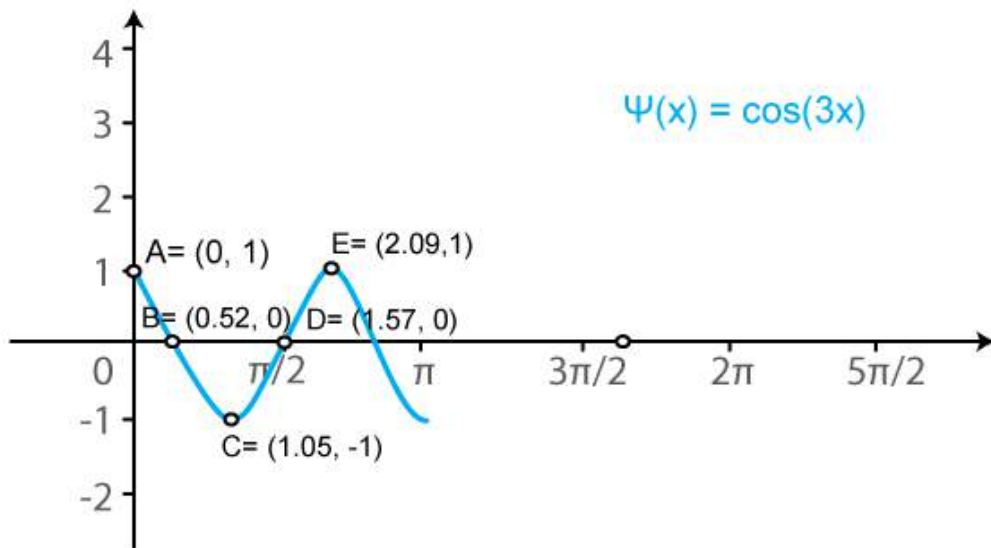
(v)  $\psi(x) = \cos(3x)$

We know that  $f(x) = \cos x$  is a periodic function with period  $2\pi$ .

So,  $\psi(x) = \cos(3x)$  is a periodic function with period  $2\pi/3$ . So, we will draw the graph of  $\psi(x) = \cos(3x)$  in the interval  $[0, 2\pi/3]$ . The values of  $\psi(x) = \cos(3x)$  at various points in  $[0, 2\pi/3]$  are listed in the following table:

x	0 (A)	$\pi/6$ (B)	$\pi/3$ (C)	$\pi/2$ (D)	$2\pi/3$ (E)	$5\pi/6$ (F)
$\psi(x) = \cos(3x)$	1	0	-1	0	1	0

The required curve is:



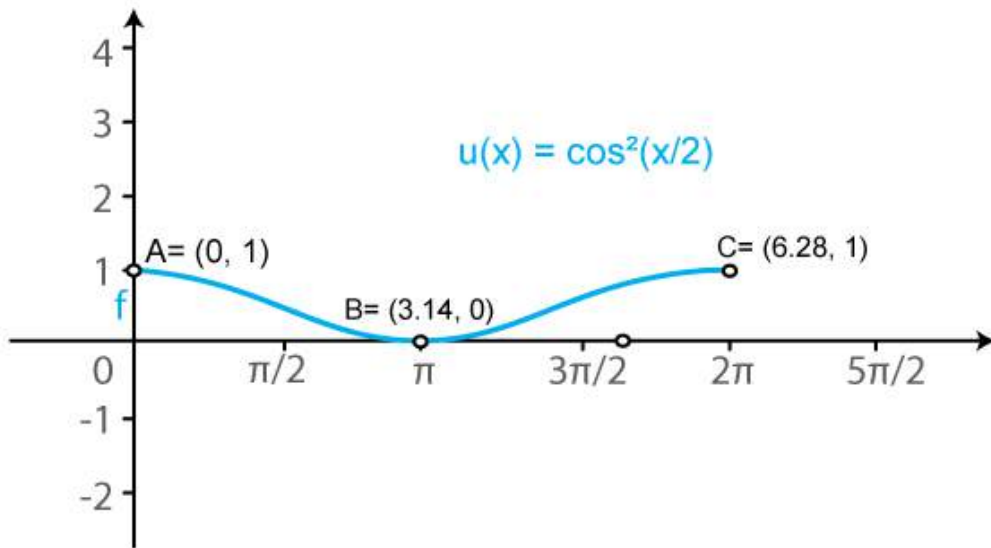
(vi)  $u(x) = \cos^2 x/2$

We know that  $f(x) = \cos x$  is a periodic function with period  $2\pi$ .

So,  $u(x) = \cos^2(x/2)$  is a periodic function with period  $\pi$ . So, we will draw the graph of  $u(x) = \cos^2(x/2)$  in the interval  $[0, \pi]$ . The values of  $u(x) = \cos^2(x/2)$  at various points in  $[0, \pi]$  are listed in the following table:

$x$	0 (A)	$\pi$ (B)	$2\pi$ (C)	$3\pi$ (D)
$u(x) = \cos^2 x/2$	1	0	1	0

The required curve is:



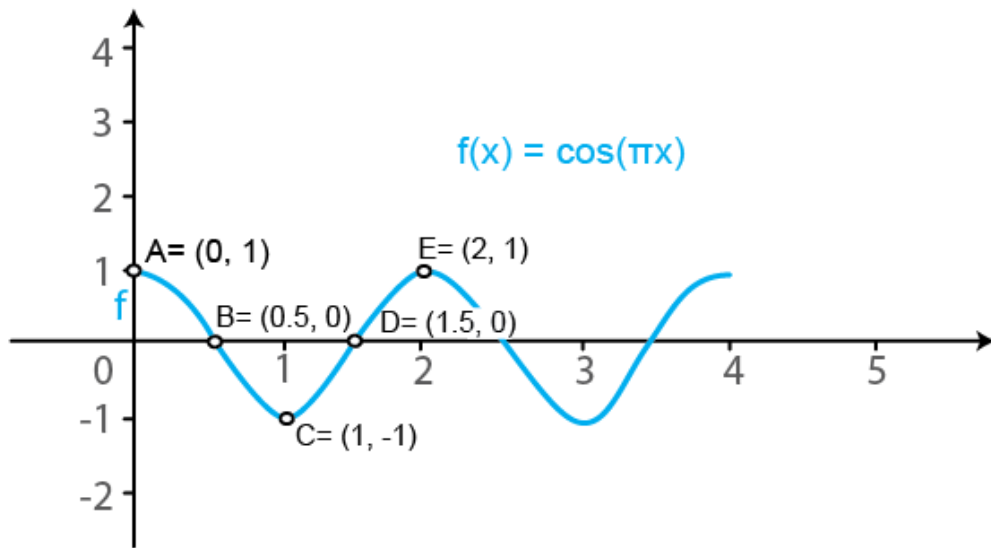
(vii)  $f(x) = \cos \pi x$

We know that  $g(x) = \cos x$  is a periodic function with period  $2\pi$ .

So,  $f(x) = \cos(\pi x)$  is a periodic function with period 2. So, we will draw the graph of  $f(x) = \cos(\pi x)$  in the interval  $[0, 2]$ . The values of  $f(x) = \cos(\pi x)$  at various points in  $[0, 2]$  are listed in the following table:

$x$	0 (A)	1/2 (B)	1 (C)	3/2 (D)	2 (E)	5/2 (F)
$f(x) = \cos \pi x$	1	0	-1	0	1	0

The required curve is:



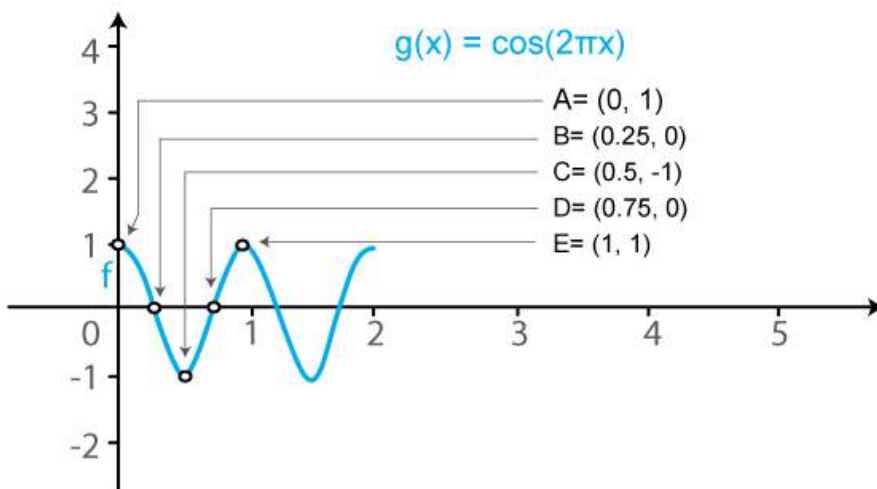
(viii)  $g(x) = \cos 2\pi x$

We know that  $f(x) = \cos x$  is a periodic function with period  $2\pi$ .

So,  $g(x) = \cos(2\pi x)$  is a periodic function with period 1. So, we will draw the graph of  $g(x) = \cos(2\pi x)$  in the interval  $[0, 1]$ . The values of  $g(x) = \cos(2\pi x)$  at various points in  $[0, 1]$  are listed in the following table:

x	0 (A)	1/4 (B)	1/2 (C)	3/4 (D)	1 (E)	5/4 (F)	3/2 (G)	7/4 (H)	2
$g(x) = \cos 2\pi x$	1	0	-1	0	1	0	-1	0	1

The required curve is:



2. Sketch the graphs of the following curves on the same scale and the same axes:

- (i)  $y = \cos x$  and  $y = \cos(x - \pi/4)$   
 (ii)  $y = \cos 2x$  and  $y = \cos(x - \pi/4)$   
 (iii)  $y = \cos x$  and  $y = \cos x/2$   
 (iv)  $y = \cos^2 x$  and  $y = \cos x$

**Solution:**

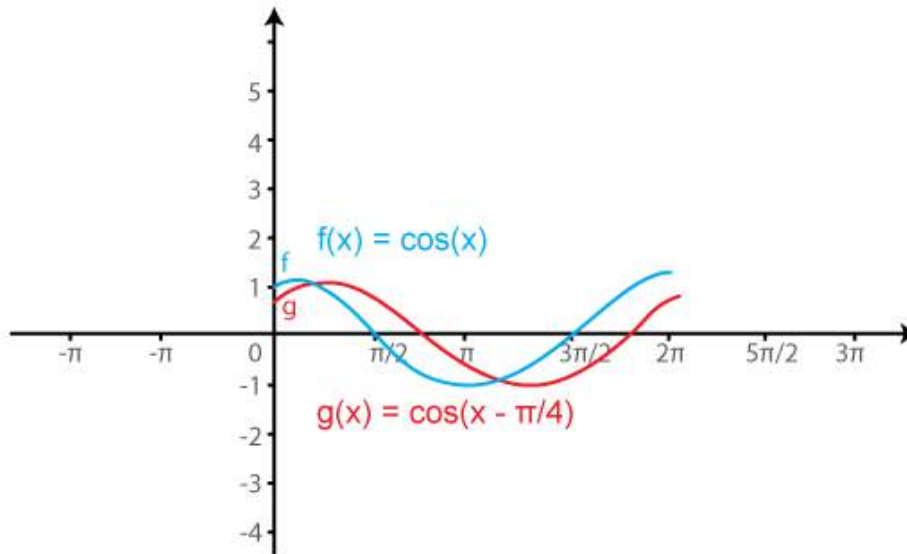
- (i)  $y = \cos x$  and  $y = \cos(x - \pi/4)$

We know that the functions  $y = \cos x$  and  $y = \cos(x - \pi/4)$  are periodic functions with periods  $\pi$  and  $\pi$ .

The values of these functions are tabulated below:

x	0	$\pi/4$	$\pi/2$	$3\pi/4$	$\pi$	$5\pi/4$	$3\pi/2$	$7\pi/4$
$y = \cos x$	1	$1/\sqrt{2} = 0.7$	0	$-1/\sqrt{2} = -0.7$	-1	$-1/\sqrt{2} = -0.7$	0	1
$y = \cos(x - \pi/4)$	$1/\sqrt{2} = 0.7$	1	$1/\sqrt{2} = 0.7$	0	$-1/\sqrt{2} = -0.7$	-1	$-1/\sqrt{2} = -0.7$	0

The required curve is:



- (ii)  $y = \cos 2x$  and  $y = \cos 2(x - \pi/4)$

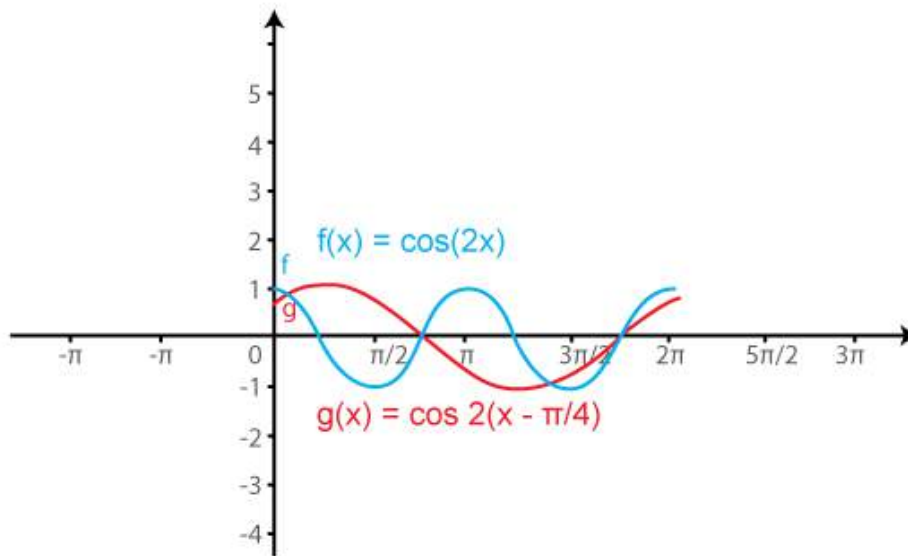
We know that the functions  $y = \cos 2x$  and  $y = \cos 2(x - \pi/4)$  are periodic functions with periods  $\pi$  and  $\pi$ .

The values of these functions are tabulated below:



x	0	$\pi/4$	$\pi/2$	$3\pi/4$	$\pi$	$5\pi/4$	$3\pi/2$	$7\pi/4$
$y = \cos x$	1	0	-1	0	1	0	-1	0
$y = \cos 2(x - \pi/4)$	0	1	0	-1	0	1	0	-1

The required curve is:



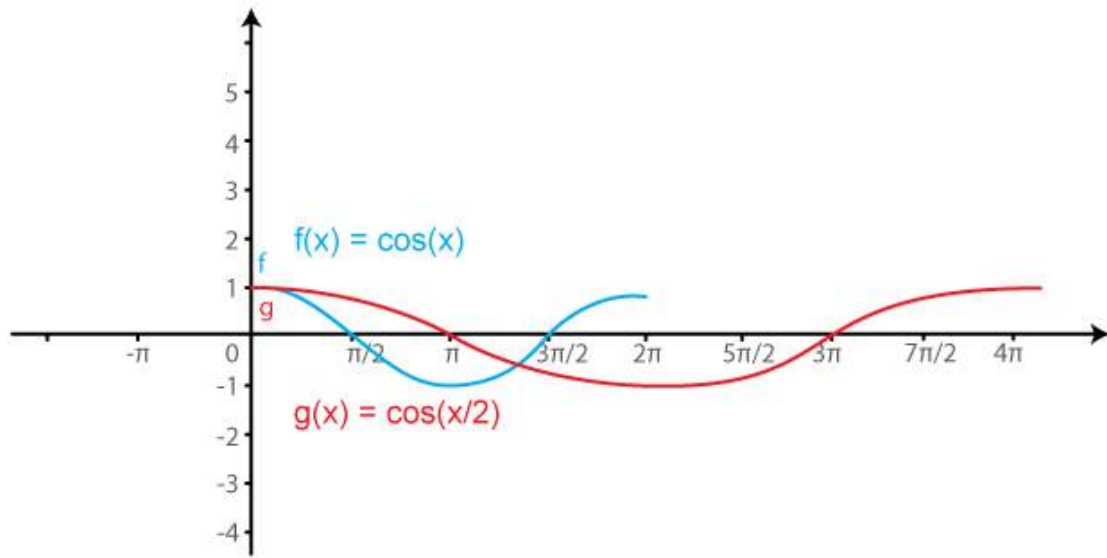
(iii)  $y = \cos x$  and  $y = \cos x/2$

We know that the functions  $y = \cos x$  and  $y = \cos (x/2)$  are periodic functions with periods  $\pi$  and  $\pi$ .

The values of these functions are tabulated below:

x	0	$\pi/2$	$\pi$	$3\pi/2$	$2\pi$
$y = \cos x$	1	0	-1	0	1
$y = \cos x/2$	1	$1/\sqrt{2} = 0.7$	0	$-1/\sqrt{2} = -0.7$	-1

The required curve is:



(iv)  $y = \cos^2 x$  and  $y = \cos x$

We know that the functions  $y = \cos^2 x$  and  $y = \cos x$  are periodic functions with period  $2\pi$ .

The values of these functions are tabulated below:

$x$	$0$	$\pi/2$	$\pi$	$3\pi/2$	$2\pi$
$y = \cos^2 x$	1	0	1	0	1
$y = \cos x$	1	0	-1	0	1

The required curve is:

