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1. If sin A = 4/5 and cos B = 5/13, where 0 <A, B < π/2, find the values of the following:
(i) sin (A + B)
(ii) cos (A + B)
(iii) sin (A – B)
(iv) cos (A – B)
Solution:
Given:
sin A = 4/5 and cos B = 5/13
We know that cos A = $\sqrt{(1 - \sin^2 A)}$ and sin B = $\sqrt{(1 - \cos^2 B)}$, where 0 <A, B < π/2
So let us find the value of sin A and cos B
cos A = $\sqrt{(1 - \sin^2 A)}$
= $\sqrt{(1 - (4/5)^2)}$
= √(1 – (16/25))
= √((25 – 16)/25)
= √(9/25)
= 3/5

sin B = $\sqrt{(1 - \cos^2 B)}$
= $\sqrt{(1 - (5/13)^2)}$
= √(1 – (25/169))
= √(169 – 25)/169)
= √(144/169)
= 12/13

(i) sin (A + B)
We know that sin (A +B) = sin A cos B + cos A sin B
So,
sin (A +B) = sin A cos B + cos A sin B
= 4/5 × 5/13 + 3/5 × 12/13
= 20/65 + 36/65
= (20+36)/65
= 56/65

(ii) cos (A + B)
We know that cos (A +B) = cos A cos B – sin A sin B
So,

cos (A + B) = cos A cos B – sin A sin B
= 3/5 × 5/13 – 4/5 × 12/13
= 15/65 – 48/65
= -33/65

(iii) sin (A – B)
We know that sin (A - B) = sin A cos B – cos A sin B
So,
sin (A - B) = sin A cos B – cos A sin B
= 4/5 × 5/13 – 3/5 × 12/13
= 20/65 – 36/65
= -16/65

(iv) cos (A – B)
We know that cos (A -B) = cos A cos B + sin A sin B
So,
cos (A -B) = cos A cos B + sin A sin B
= 3/5 × 5/13 + 4/5 × 12/13
= 15/65 + 48/65
= 63/65

2. (a) If Sin A = 12/13 and sin B = 4/5, where $\pi/2 < A < \pi$ and $0 < B < \pi/2$, find the following:
(i) sin (A + B) (ii) cos (A + B)
(b) If sin A = 3/5, cos B = –12/13, where A and B, both lie in second quadrant, find the value of sin (A +B).
Solution:
(a) Given:
Sin A = 12/13 and sin B = 4/5, where $\pi/2 < A < \pi$ and $0 < B < \pi/2$
We know that cos A = $-\sqrt{(1 - \sin^2 A)}$ and cos B = $\sqrt{(1 - \sin^2 B)}$
So let us find the value of cos A and cos B
cos A = $-\sqrt{(1 - \sin^2 A)}$
= $-\sqrt{(1 - (12/13)^2)}$
= $-\sqrt{(1-144/169)}$
= $-\sqrt{((169-144)/169)}$
= $-\sqrt{(25/169)}$
= - 5/13

cos B = $\sqrt{(1 - \sin^2 B)}$
= $\sqrt{(1 - (4/5)^2)}$
= $\sqrt{(1-16/25)}$
= $\sqrt{((25-16)/25)}$
= $\sqrt{(9/25)}$
= 3/5

(i) sin (A +B)
We know that sin (A + B) = sin A cos B + cos A sin B
So,
sin (A +B) = sin A cos B + cos A sin B
= 12/13 × 3/5 + (-5/13) × 4/5
= 36/65 – 20/65
= 16/65

(ii) cos (A + B)
We know that cos (A +B) = cos A cos B – sin A sin B
So,
cos (A +B) = cos A cos B – sin A sin B
= -5/13 × 3/5 – 12/13 × 4/5
= -15/65 – 48/65
= - 63/65

(b) Given:
sin A = 3/5, cos B = –12/13, where A and B, both lie in second quadrant.
We know that cos A = - $\sqrt{(1 - \sin^2 A)}$ and sin B = $\sqrt{(1 - \cos^2 B)}$
So let us find the value of cos A and sin B
cos A = - $\sqrt{(1 - \sin^2 A)}$
= - $\sqrt{(1 - (3/5)^2)}$
= - $\sqrt{(1- 9/25)}$
= - $\sqrt{((25-9)/25)}$
= - $\sqrt{(16/25)}$
= - 4/5

sin B = $\sqrt{(1 - \cos^2 B)}$
= $\sqrt{(1 - (-12/13)^2)}$
= $\sqrt{(1 - 144/169)}$
= $\sqrt{((169-144)/169)}$
= $\sqrt{(25/169)}$
= 5/13

We need to find sin (A + B)
Since, sin (A + B) = sin A cos B + cos A sin B
= 3/5 × (-12/13) + (-4/5) × 5/13
= -36/65 – 20/65
= -56/65

3. If cos A = – 24/25 and cos B = 3/5, where π <A < 3π/2 and 3π/2 <B < 2π, find the following:
(i) sin (A + B) (ii) cos (A + B)
Solution:
Given:
cos A = – 24/25 and cos B = 3/5, where π <A < 3π/2 and 3π/2 <B < 2π
We know that A is in third quadrant, B is in fourth quadrant. So sine function is negative.
By using the formulas,
sin A = - $\sqrt{(1 - \cos^2 A)}$ and sin B = -$\sqrt{(1 - \cos^2 B)}$
So let us find the value of sin A and sin B
sin A = - $\sqrt{(1 - \cos^2 A)}$
= - $\sqrt{(1-(-24/25)^2)}$
= - √(1-576/625)
= - √((625-576)/625)
= - √(49/625)
= -7/25

sin B = -$\sqrt{(1 - \cos^2 B)}$
= - $\sqrt{(1-(3/5)^2)}$
= - √(1-9/25)
= - √((25-9)/25)
= - √(16/25)
= - 4/5

(i) sin (A + B)
We know that sin (A + B) = sin A cos B + cos A sin B
So,
sin (A + B) = sin A cos B + cos A sin B
= -7/25 × 3/5 + (-24/25) × (-4/5)
= -21/125 + 96/125
= 75/125
= 3/5

(ii) cos (A + B)

We know that cos (A + B) = cos A cos B – sin A sin B

So,

cos (A + B) = cos A cos B – sin A sin B

= (-24/25) × 3/5 – (-7/25) × (-4/5)

= -72/125 – 28/125

= -100/125

= - 4/5

4. If tan A = 3/4, cos B = 9/41, where π<A < 3π/2 and 0 <B < π/2, find tan (A + B).

Solution:

Given:

tan A = 3/4 and cos B = 9/41, where π <A < 3π/2 and 0 <B < π/2

We know that, A is in third quadrant, B is in first quadrant.

So, tan function And sine function are positive.

By using the formula,

sin B = $\sqrt{(1 - \cos^2 B)}$

Let us find the value of sin B.

sin B = $\sqrt{(1 - \cos^2 B)}$

= $\sqrt{(1- (9/41)^2)}$

= √(1- 81/1681)

= √((1681-81)/1681)

= √(1600/1681)

= 40/41

We know, tan B = sin B/cos B

= (40/41) / (9/41)

= 40/9

So, tan (A + B) = (tan A + tan B) / (1 – tan A tan B)

= (3/4 + 40/9) / (1 – 3/4 × 40/9)

= (187/36) / (1- 120/36)

= (187/36) / ((36-120)/36)

= (187/36) / (-84/36)

= -187/84

5. If sin A = 1/2, cos B = 12/13, where π/2<A < π and 3π/2 <B < 2π, find tan(A - B).

Solution:

Given:

sin A = 1/2, cos B = 12/13, where π/2<A < π and 3π/2 <B < 2π

We know that, A is in second quadrant, B is in fourth quadrant.
In the second quadrant, sine function is positive, cosine and tan functions are negative.
In the fourth quadrant, sine and tan functions are negative, cosine function is positive.
By using the formulas,
$\cos A = -\sqrt{(1 - \sin^2 A)}$ and $\sin B = -\sqrt{(1 - \cos^2 B)}$
So let us find the value of cos A and sin B
$\cos A = -\sqrt{(1 - \sin^2 A)}$
$= -\sqrt{(1 - (1/2)^2)}$
$= -\sqrt{(1 - 1/4)}$
$= -\sqrt{((4-1)/4)}$
$= -\sqrt{(3/4)}$
$= -\sqrt{3}/2$

$\sin B = -\sqrt{(1 - \cos^2 B)}$
$= -\sqrt{(1-(12/13)^2)}$
$= -\sqrt{(1 - 144/169)}$
$= -\sqrt{((169-144)/169)}$
$= -\sqrt{(25/169)}$
$= -5/13$

We know, tan A = sin A / cos A and tan B = sin B / cos B
$\tan A = (1/2)/(-\sqrt{3}/2) = -1/\sqrt{3}$ and
$\tan B = (-5/13)/(12/13) = -5/12$

So, $\tan(A - B) = (\tan A - \tan B) / (1 + \tan A \tan B)$
$= ((-1/\sqrt{3}) - (-5/12)) / (1 + (-1/\sqrt{3}) \times (-5/12))$
$= ((-12+5\sqrt{3})/12\sqrt{3}) / (1 + 5/12\sqrt{3})$
$= ((-12+5\sqrt{3})/12\sqrt{3}) / ((12\sqrt{3} + 5)/12\sqrt{3})$
$= (5\sqrt{3} - 12) / (5 + 12\sqrt{3})$

6. If $\sin A = 1/2$, $\cos B = \sqrt{3}/2$, where $\pi/2 < A < \pi$ and $0 < B < \pi/2$, find the following:
(i) tan (A + B) (ii) tan (A - B)
Solution:
Given:
$\sin A = 1/2$ and $\cos B = \sqrt{3}/2$, where $\pi/2 < A < \pi$ and $0 < B < \pi/2$
We know that, A is in second quadrant, B is in first quadrant.
In the second quadrant, sine function is positive. cosine and tan functions are negative.
In first quadrant, all functions are positive.
By using the formulas,

cos A = $-\sqrt{(1 - \sin^2 A)}$ and sin B = $\sqrt{(1 - \cos^2 B)}$
So let us find the value of cos A and sin B

$\cos A = -\sqrt{(1 - \sin^2 A)}$
$= -\sqrt{(1 - (1/2)^2)}$
$= -\sqrt{(1 - 1/4)}$
$= -\sqrt{((4-1)/4)}$
$= -\sqrt{(3/4)}$
$= -\sqrt{3}/2$

$\sin B = \sqrt{(1 - \cos^2 B)}$
$= \sqrt{(1 - (\sqrt{3}/2)^2)}$
$= \sqrt{(1 - 3/4)}$
$= \sqrt{((4-3)/4)}$
$= \sqrt{(1/4)}$
$= 1/2$

We know, tan A = sin A / cos A and tan B = sin B / cos B
tan A = $(1/2)/(-\sqrt{3}/2) = -1/\sqrt{3}$ and
tan B = $(1/2)/(\sqrt{3}/2) = 1/\sqrt{3}$

(i) $\tan(A + B) = (\tan A + \tan B) / (1 - \tan A \tan B)$
$= (-1/\sqrt{3} + 1/\sqrt{3}) / (1 - (-1/\sqrt{3}) \times 1/\sqrt{3})$
$= 0 / (1 + 1/3)$
$= 0$

(ii) $\tan(A - B) = (\tan A - \tan B) / (1 + \tan A \tan B)$
$= ((-1/\sqrt{3}) - (1/\sqrt{3})) / (1 + (-1/\sqrt{3}) \times (1/\sqrt{3}))$
$= ((-2/\sqrt{3}) / (1 - 1/3)$
$= ((-2/\sqrt{3}) / (3-1)/3)$
$= ((-2/\sqrt{3}) / 2/3)$
$= -\sqrt{3}$

7. Evaluate the following:
(i) $\sin 78^0 \cos 18^0 - \cos 78^0 \sin 18^0$
(ii) $\cos 47^0 \cos 13^0 - \sin 47^0 \sin 13^0$
(iii) $\sin 36^0 \cos 9^0 + \cos 36^0 \sin 9^0$
(iv) $\cos 80^0 \cos 20^0 + \sin 80^0 \sin 20^0$
Solution:
(i) $\sin 78^0 \cos 18^0 - \cos 78^0 \sin 18^0$

We know that sin (A - B) = sin A cos B – cos A sin B

$\sin 78^0 \cos 18^0 - \cos 78^0 \sin 18^0 = \sin(78 - 18)°$

$= \sin 60°$

$= \sqrt{3}/2$

(ii) $\cos 47^0 \cos 13^0 - \sin 47^0 \sin 13^0$

We know that cos A cos B – sin A sin B = cos (A + B)

$\cos 47^0 \cos 13^0 - \sin 47^0 \sin 13^0 = \cos (47 + 13)°$

$= \cos 60°$

$= 1/2$

(iii) $\sin 36^0 \cos 9^0 + \cos 36^0 \sin 9^0$

We know that sin (A +B) = sin A cos B + cos A sin B

$\sin 36^0 \cos 9^0 + \cos 36^0 \sin 9^0 = \sin (36 + 9)°$

$= \sin 45°$

$= 1/\sqrt{2}$

(iv) $\cos 80^0 \cos 20^0 + \sin 80^0 \sin 20^0$

We know that cos A cos B + sin A sin B = cos (A - B)

$\cos 80^0 \cos 20^0 + \sin 80^0 \sin 20^0 = \cos (80 - 20)°$

$= \cos 60°$

$= ½$

8. If cos A = –12/13 and cot B = 24/7, where A lies in the second quadrant and B in the third quadrant, find the values of the following:

(i) sin (A + B) (ii) cos (A + B) (iii) tan (A + B)

Solution:

Given:

cos A = -12/13 and cot B = 24/7

We know that, A lies in second quadrant, B in the third quadrant.

In the second quadrant sine function is positive.

In the third quadrant, both sine and cosine functions are negative.

By using the formulas,

$\sin A = \sqrt{(1 - \cos^2 A)}$, $\sin B = - 1/\sqrt{(1 + \cot^2 B)}$ and $\cos B = -\sqrt{(1 - \sin^2 B)}$,

So let us find the value of sin A and sin B

$\sin A = \sqrt{(1 - \cos^2 A)}$

$= \sqrt{(1 - (-12/13)^2)}$

$= \sqrt{(1 - 144/169)}$

$= \sqrt{((169-144)/169)}$

$= \sqrt{(25/169)}$
$= 5/13$

$\sin B = -1/\sqrt{(1 + \cot^2 B)}$
$= -1/\sqrt{(1 + (24/7)^2)}$
$= -1/\sqrt{(1 + 576/49)}$
$= -1/\sqrt{((49+576)/49)}$
$= -1/\sqrt{(625/49)}$
$= -1/(25/7)$
$= -7/25$

$\cos B = -\sqrt{(1 - \sin^2 B)}$
$= -\sqrt{(1-(-7/25)^2)}$
$= -\sqrt{(1-(49/625))}$
$= -\sqrt{((625-49)/625)}$
$= -\sqrt{(576/625)}$
$= -24/25$

So, now let us find

(i) sin (A + B)

We know that sin (A + B) = sin A cos B + cos A sin B

So,

sin (A + B) = sin A cos B + cos A sin B
$= 5/13 \times (-24/25) + (-12/13) \times (-7/25)$
$= -120/325 + 84/325$
$= -36/325$

(ii) cos (A + B)

We know that cos (A + B) = cos A cos B – sin A sin B

So,

cos (A + B) = cos A cos B – sin A sin B
$= -12/13 \times (-24/25) – (5/13) \times (-7/25)$
$= 288/325 + 35/325$
$= 323/325$

(iii) tan (A + B)

We know that tan (A + B) = sin (A+B) / cos (A+B)
$= (-36/325) / (323/325)$
$= -36/323$

9. Prove that: cos 7π/12 + cos π/12 = sin 5π/12 – sin π/12
Solution:
We know that, 7π/12 = 105°, π/12 = 15°; 5π/12 = 75°
Let us consider LHS: cos 105° + cos 15°
cos (90° + 15°) + sin (90° - 75°)
-sin 15° + sin 75°
sin 75° - sin 15°
= RHS
∴ LHS = RHS
Hence proved.

10. Prove that: (tan A + tan B) / (tan A – tan B) = sin (A + B) / sin (A - B)
Solution:
Let us consider LHS: (tan A + tan B) / (tan A – tan B)

$$\frac{\tan A+\tan B}{\tan A-\tan B}=\frac{\frac{\sin A}{\cos A}+\frac{\sin B}{\cos B}}{\frac{\sin A}{\cos A}-\frac{\sin B}{\cos B}}$$

$$=\frac{\frac{\sin A\cos B+\cos A\sin B}{\cos A\cos B}}{\frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}}$$

We know that sin (A ±B) = sin A cos B ± cos A sin B

$$=\frac{\sin(A+B)}{\sin(A-B)}$$

= RHS
∴ LHS = RHS
Hence proved.

11. Prove that:
(i) (cos 11° + sin 11°) / (cos 11° – sin 11°) = tan 56°
(ii) (cos 9° + sin 9°) / (cos 9° – sin 9°) = tan 54°
(iii) (cos 8° – sin 8°) / (cos 8° + sin 8°) = tan 37°
Solution:
(i) (cos 11° + sin 11°) / (cos 11° – sin 11°) = tan 56°
Let us consider LHS:
(cos 11° + sin 11°) / (cos 11° – sin 11°)
Now let us divide the numerator and denominator by cos 11° we get,
(cos 11° + sin 11°) / (cos 11° – sin 11°) = (1 + tan 11°) / (1 – tan 11°)
= (1 + tan 11°) / (1- 1×tan 11°)

$= (\tan 45^o + \tan 11^o) / (1 - \tan 45^o \times \tan 11^o)$

We know that $\tan (A+B) = (\tan A + \tan B) / (1 - \tan A \tan B)$

So,

$(\tan 45^o + \tan 11^o) / (1 - \tan 45^o \times \tan 11^o) = \tan (45^o + 11^o)$

$= \tan 56^o$

= RHS

∴ LHS = RHS

Hence proved.

(ii) $(\cos 9^o + \sin 9^o) / (\cos 9^o - \sin 9^o) = \tan 54^o$

Let us consider LHS:

$(\cos 9^o + \sin 9^o) / (\cos 9^o - \sin 9^o)$

Now let us divide the numerator and denominator by $\cos 9^o$ we get,

$(\cos 9^o + \sin 9^o) / (\cos 9^o - \sin 9^o) = (1 + \tan 9^o) / (1 - \tan 9^o)$

$= (1 + \tan 9^o) / (1 - 1 \times \tan 9^o)$

$= (\tan 45^o + \tan 9^o) / (1 - \tan 45^o \times \tan 9^o)$

We know that $\tan (A+B) = (\tan A + \tan B) / (1 - \tan A \tan B)$

So,

$(\tan 45^o + \tan 9^o) / (1 - \tan 45^o \times \tan 9^o) = \tan (45^o + 9^o)$

$= \tan 54^o$

= RHS

∴ LHS = RHS

Hence proved.

(iii) $(\cos 8^o - \sin 8^o) / (\cos 8^o + \sin 8^o) = \tan 37^o$

Let us consider LHS:

$(\cos 8^o - \sin 8^o) / (\cos 8^o + \sin 8^o)$

Now let us divide the numerator and denominator by $\cos 8^o$ we get,

$(\cos 8^o - \sin 8^o) / (\cos 8^o + \sin 8^o) = (1 - \tan 8^o) / (1 + \tan 8^o)$

$= (1 - \tan 8^o) / (1 + 1 \times \tan 8^o)$

$= (\tan 45^o - \tan 8^o) / (1 + \tan 45^o \times \tan 8^o)$

We know that $\tan (A+B) = (\tan A + \tan B) / (1 - \tan A \tan B)$

So,

$(\tan 45^o - \tan 8^o) / (1 + \tan 45^o \times \tan 8^o) = \tan (45^o - 8^o)$

$= \tan 37^o$

= RHS

∴ LHS = RHS

Hence proved.

12. Prove that:

(i)

$$sin\left(\frac{\pi}{3}-x\right)cos(\frac{\pi}{6}+x)+cos(\frac{\pi}{3}-x)\,sin(\frac{\pi}{6}+x)=1$$

(ii)

$$sin\,(\frac{4\pi}{9}+7)\,cos\,(\frac{\pi}{9}+7)-cos\,(\frac{4\pi}{9}+7)\,sin\,(\frac{\pi}{9}+7)=\frac{\sqrt{3}}{2}$$

(iii)

$$sin\,(\frac{3\pi}{8}-5)\,cos\,(\frac{\pi}{8}+5)+cos\,(\frac{3\pi}{8}-5)\,sin\,(\frac{\pi}{8}+5)=1$$

Solution:

(i)

$$sin\left(\frac{\pi}{3}-x\right)cos(\frac{\pi}{6}+x)+cos(\frac{\pi}{3}-x)\,sin(\frac{\pi}{6}+x)=1$$

Let us consider LHS:

$$sin\left(\frac{\pi}{3}-x\right)cos(\frac{\pi}{6}+x)+cos(\frac{\pi}{3}-x)\,sin(\frac{\pi}{6}+x)$$

We know that sin (A +B) = sin A cos B + cos A sin B

$$sin\left(\frac{\pi}{3}-x\right)cos(\frac{\pi}{6}+x)+cos(\frac{\pi}{3}-x)\,sin(\frac{\pi}{6}+x)=sin\,(\frac{\pi}{3}-x+\frac{\pi}{6}+x)$$

$$=sin\left(\frac{2\pi+\pi}{6}\right)$$

$$=sin\left(\frac{\pi}{2}\right)$$

= sin 90°

= 1

= RHS

∴ LHS = RHS

Hence proved.

(ii)

$$sin\,(\frac{4\pi}{9}+7)\,cos\,(\frac{\pi}{9}+7)-cos\,(\frac{4\pi}{9}+7)\,sin\,(\frac{\pi}{9}+7)=\frac{\sqrt{3}}{2}$$

Let us consider LHS:

$$sin\,(\frac{4\pi}{9}+7)\,cos\,(\frac{\pi}{9}+7)-cos\,(\frac{4\pi}{9}+7)\,sin\,(\frac{\pi}{9}+7)$$

We know that sin (A - B) = sin A cos B – cos A sin B

So,

$$\sin\left(\frac{4\pi}{9}+7\right)\cos\left(\frac{\pi}{9}+7\right)-\cos\left(\frac{4\pi}{9}+7\right)\sin\left(\frac{\pi}{9}+7\right)=\sin\left(\frac{4\pi}{9}+7-\frac{\pi}{9}-7\right)$$

$$=\sin\left(\frac{3\pi}{9}\right)$$

$$=\sin\left(\frac{\pi}{3}\right)$$

= sin 60°

= √3/2

= RHS

∴ LHS = RHS

Hence proved.

(iii)

$$\sin\left(\frac{3\pi}{8}-5\right)\cos\left(\frac{\pi}{8}+5\right)+\cos\left(\frac{3\pi}{8}-5\right)\sin\left(\frac{\pi}{8}+5\right)=1$$

Let us consider LHS:

$$\sin\left(\frac{3\pi}{8}-5\right)\cos\left(\frac{\pi}{8}+5\right)+\cos\left(\frac{3\pi}{8}-5\right)\sin\left(\frac{\pi}{8}+5\right)$$

We know that sin (A + B) = sin A cos B + cos A sin B

$$\sin\left(\frac{3\pi}{8}-5\right)\cos\left(\frac{\pi}{8}+5\right)+\cos\left(\frac{3\pi}{8}-5\right)\sin\left(\frac{\pi}{8}+5\right)=\sin\left(\frac{3\pi}{8}-5+\frac{\pi}{8}+5\right)$$

$$=\sin\left(\frac{3\pi+\pi}{8}\right)$$

$$=\sin\left(\frac{4\pi}{8}\right)$$

$$=\sin\left(\frac{\pi}{2}\right)$$

= sin 90°

= 1

= RHS

∴ LHS = RHS

Hence proved.

13. Prove that: (tan 69° + tan 66°) / (1 – tan 69° tan 66°) = -1

Solution:

Let us consider LHS:

$(\tan 69^o + \tan 66^o) / (1 – \tan 69^o \tan 66^o)$

We know that, $\tan (A + B) = (\tan A + \tan B) / (1 – \tan A \tan B)$

Here, $A = 69^o$ and $B = 66^o$

So,

$(\tan 69^o + \tan 66^o) / (1 – \tan 69^o \tan 66^o) = \tan (69 + 66)^o$

$= \tan 135^o$

$= - \tan 45^o$

$= - 1$

= RHS

∴ LHS = RHS

Hence proved.

14. (i) If tan A = 5/6 and tan B = 1/11, prove that A + B = π/4

(ii) If tan A = m/(m–1) and tan B = 1/(2m – 1), then prove that A – B = π/4

Solution:

(i) If tan A = 5/6 and tan B = 1/11, prove that A + B = π/4

Given:

tan A = 5/6 and tan B = 1/11

We know that, $\tan (A + B) = (\tan A + \tan B) / (1 – \tan A \tan B)$

$= [(5/6) + (1/11)] / [1 – (5/6) \times (1/11)]$

$= (55+6) / (66-5)$

$= 61/61$

$= 1$

$= \tan 45^o$ or $\tan \pi/4$

So, $\tan (A + B) = \tan \pi/4$

∴ $(A + B) = \pi/4$

Hence proved.

(ii) If tan A = m/(m–1) and tan B = 1/(2m – 1), then prove that A – B = π/4

Given:

tan A = m/(m–1) and tan B = 1/(2m – 1)

We know that, $\tan (A - B) = (\tan A - \tan B) / (1 + \tan A \tan B)$

$$= \frac{\frac{m}{m-1} - \frac{1}{2m-1}}{1 + \frac{m}{m-1} \times \frac{1}{2m-1}}$$

$= (2m^2 – m – m + 1) / (2m^2 – m – 2m + 1 + m)$

$= (2m^2 – 2m + 1) / (2m^2 – 2m + 1)$

$= 1$

$= \tan 45^o$ or $\tan \pi/4$

So, $\tan (A - B) = \tan \pi/4$

∴ $(A - B) = \pi/4$
Hence proved.

15. prove that:
(i) $\cos^2 \pi/4 - \sin^2 \pi/12 = \sqrt{3}/4$
(ii) $\sin^2 (n + 1) A - \sin^2 nA = \sin (2n + 1) A \sin A$
Solution:
(i) $\cos^2 \pi/4 - \sin^2 \pi/12 = \sqrt{3}/4$
Let us consider LHS:
$\cos^2 \pi/4 - \sin^2 \pi/12$
We know that, $\cos^2 A - \sin^2 B = \cos (A + B) \cos (A - B)$
So,
$\cos^2 \pi/4 - \sin^2 \pi/12 = \cos (\pi/4 + \pi/12) \cos (\pi/4 - \pi/12)$
$= \cos 4\pi/12 \cos 2\pi/12$
$= \cos \pi/3 \cos \pi/6$
$= 1/2 \times \sqrt{3}/2$
$= \sqrt{3}/4$
= RHS
∴ LHS = RHS
Hence proved.

(ii) $\sin^2 (n + 1) A - \sin^2 nA = \sin (2n + 1) A \sin A$
Let us consider LHS:
$\sin^2 (n + 1) A - \sin^2 nA$
We know that, $\sin^2 A - \sin^2 B = \sin (A + B) \sin (A - B)$
Here, $A = (n + 1) A$ and $B = nA$
So,
$\sin^2 (n + 1) A - \sin^2 n A = \sin ((n + 1) A + nA) \sin ((n + 1) A - nA)$
$= \sin (nA + A + nA) \sin (nA + A - nA)$
$= \sin (2nA + A) \sin (A)$
$= \sin (2n + 1) A \sin A$
= RHS
∴ LHS = RHS
Hence proved.

16. Prove that:
(i)
$$\frac{\sin (A + B) + \sin (A - B)}{\cos (A + B) + \cos (A - B)} = \tan A$$

(ii)
$$\frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}=0$$
(iii)
$$\frac{\sin(A-B)}{\sin A\sin B}+\frac{\sin(B-C)}{\sin B\sin C}+\frac{\sin(C-A)}{\sin C\sin A}=0$$

(iv) $\sin^2 B = \sin^2 A + \sin^2 (A-B) - 2\sin A \cos B \sin (A - B)$
(v) $\cos^2 A + \cos^2 B - 2 \cos A \cos B \cos (A + B) = \sin^2 (A + B)$
(vi)
$$\frac{\tan(A+B)}{\cot(A-B)}=\frac{\tan^2 A-\tan^2 B}{1-\tan^2 A\tan^2 B}$$
Solution:
(i)
$$\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}=\tan A$$
Let us consider LHS:
$$\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$$
We know that $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ and $\cos(A \pm B) = \cos A \cos B \pm \sin A \sin B$
$$\frac{\sin(A+B)+\sin(A-B)}{\cos(A+B)+\cos(A-B)}$$
$$=\frac{\sin A\cos B+\cos A\sin B+\sin A\cos B-\cos A\sin B}{\cos A\cos B-\sin A\sin B+\cos A\cos B+\sin A\sin B}$$
$$=\frac{2\sin A\cos B}{2\cos A\cos B}$$
= tan A
= RHS
∴ LHS = RHS
Hence proved.

(ii)
$$\frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}=0$$
Let us consider LHS:

$$\frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}$$

We know that, sin (A – B) = sin A cos B – cos A sin B

$$\frac{\sin(A-B)}{\cos A\cos B}+\frac{\sin(B-C)}{\cos B\cos C}+\frac{\sin(C-A)}{\cos C\cos A}$$

$$=\frac{\sin A\cos B-\cos A\sin B}{\cos A\cos B}+\frac{\sin B\cos C-\cos B\sin C}{\cos B\cos C}+\frac{\sin C\cos A-\cos C\sin A}{\cos C\cos A}$$

$$=\frac{\sin A\cos B}{\cos A\cos B}-\frac{\cos A\sin B}{\cos A\cos B}+\frac{\sin B\cos C}{\cos B\cos C}-\frac{\cos B\sin C}{\cos B\cos C}+\frac{\sin C\cos A}{\cos C\cos A}-\frac{\cos C\sin A}{\cos C\cos A}$$

= tan A – tan B + tan B – tan C + tan C – tan A

= 0

= RHS

∴ LHS = RHS

Hence proved.

(iii)

$$\frac{\sin(A-B)}{\sin A\sin B}+\frac{\sin(B-C)}{\sin B\sin C}+\frac{\sin(C-A)}{\sin C\sin A}=0$$

Let us consider LHS:

$$\frac{\sin(A-B)}{\sin A\sin B}+\frac{\sin(B-C)}{\sin B\sin C}+\frac{\sin(C-A)}{\sin C\sin A}$$

We know that, sin (A – B) = sin A cos B – cos A sin B

$$\frac{\sin(A-B)}{\sin A\sin B}+\frac{\sin(B-C)}{\sin B\sin C}+\frac{\sin(C-A)}{\sin C\sin A}$$

$$=\frac{\sin A\cos B-\cos A\sin B}{\sin A\sin B}+\frac{\sin B\cos C-\cos B\sin C}{\sin B\sin C}+\frac{\sin C\cos A-\cos C\sin A}{\sin C\sin A}$$

$$=\frac{\sin A\cos B}{\sin A\sin B}-\frac{\cos A\sin B}{\sin A\sin B}+\frac{\sin B\cos C}{\sin B\sin C}-\frac{\cos B\sin C}{\sin B\sin C}+\frac{\sin C\cos A}{\sin C\sin A}-\frac{\cos C\sin A}{\sin C\sin A}$$

= cotB – cotA + cot C – cotB + cotA – cot C

= 0

= RHS

∴ LHS = RHS

Hence proved.

(iv) $\sin^2 B = \sin^2 A + \sin^2 (A-B) – 2\sin A \cos B \sin (A - B)$

Let us consider RHS:

$\sin^2 A + \sin^2 (A - B) – 2 \sin A \cos B \sin (A - B)$

$\sin^2 A + \sin (A - B) [\sin (A – B) – 2 \sin A \cos B]$

We know that, $\sin (A – B) = \sin A \cos B – \cos A \sin B$

So,

$\sin^2 A + \sin (A - B) [\sin A \cos B – \cos A \sin B – 2 \sin A \cos B]$

$\sin^2 A + \sin (A - B) [-\sin A \cos B – \cos A \sin B]$

$\sin^2 A – \sin (A - B) [\sin A \cos B + \cos A \sin B]$

We know that, $\sin (A + B) = \sin A \cos B + \cos A \sin B$

So,

$\sin^2 A – \sin (A – B) \sin (A + B)$

$\sin^2 A – \sin^2 A + \sin^2 B$

$\sin^2 B$

= LHS

∴ LHS = RHS

Hence proved.

(v) $\cos^2 A + \cos^2 B – 2 \cos A \cos B \cos (A + B) = \sin^2 (A + B)$

Let us consider LHS:

$\cos^2 A + \cos^2 B – 2 \cos A \cos B \cos (A + B)$

$\cos^2 A + 1 – \sin^2 B - 2 \cos A \cos B \cos (A + B)$

$1 + \cos^2 A – \sin^2 B - 2 \cos A \cos B \cos (A + B)$

We know that, $\cos^2 A – \sin^2 B = \cos (A + B) \cos (A – B)$

So,

$1 + \cos (A + B) \cos (A – B) - 2 \cos A \cos B \cos (A + B)$

$1 + \cos (A + B) [\cos (A – B) – 2 \cos A \cos B]$

We know that, $\cos (A - B) = \cos A \cos B + \sin A \sin B$.

So,

$1 + \cos (A + B) [\cos A \cos B + \sin A \sin B – 2 \cos A \cos B]$

$1 + \cos (A + B) [-\cos A \cos B + \sin A \sin B]$

$1 – \cos (A + B) [\cos A \cos B – \sin A \sin B]$

We know that, $\cos (A + B) = \cos A \cos B – \sin A \sin B$.

So,

$1 – \cos^2 (A + B)$

$\sin^2 (A + B)$

= RHS

∴ LHS = RHS

Hence proved.

(vi)

$$\frac{tan\ (A+B)}{cot\ (A-B)} = \frac{tan^2 A - tan^2 B}{1 - tan^2 A\ tan^2 B}$$

Let us consider LHS:

$$\frac{tan\ (A+B)}{cot\ (A-B)}$$

We know that,

$$tan\,(A \pm B) = \frac{tan\,A \pm tan\,B}{1 \pm tan\,A\,tan\,B}$$

$$\frac{tan\,(A+B)}{\frac{1}{tan\,(A-B)}} = \frac{\frac{tan\,A + tan\,B}{1 - tan\,A\,tan\,B}}{\frac{1}{\frac{tan\,A - tan\,B}{1 + tan\,A\,tan\,B}}}$$

$$= \frac{tan\,A + tan\,B}{1 - tan\,A\,tan\,B} \times \frac{tan\,A - tan\,B}{1 + tan\,A\,tan\,B}$$

We know that, $(x + y)(x - y) = x^2 - y^2$

So,

$$\frac{tan\,A + tan\,B}{1 - tan\,A\,tan\,B} \times \frac{tan\,A - tan\,B}{1 + tan\,A\,tan\,B} = \frac{tan^2 A - tan^2 B}{1 - tan^2 A\,tan^2 B}$$

= RHS

∴ LHS = RHS

Hence proved.

17. Prove that:

(i) tan 8x – tan 6x – tan 2x = tan 8x tan 6x tan 2x

(ii) tan π/12 + tan π/6 + tan π/12 tan π/6 = 1

(iii) tan 36º + tan 9º + tan 36º tan 9º = 1

(iv) tan 13x – tan 9x – tan 4x = tan 13x tan 9x tan 4x

Solution:

(i) tan 8x – tan 6x – tan 2x = tan 8x tan 6x tan 2x

Let us consider LHS:

tan 8x – tan 6x – tan 2x

tan 8x = tan(6x + 2x)

We know that, tan (A + B) = (tan A + tan B) / (1 – tan A tan B)

So,

tan 8x = (tan 6x + tan 2x) / (1 – tan 6x tan 2x)

By cross-multiplying we get,
tan 8x (1 – tan 6x tan 2x) = tan 6x + tan 2x
tan 8x – tan 8x tan 6x tan2x = tan 6x + tan 2x
Upon rearranging we get,
tan 8x – tan 6x – tan 2x = tan 8x tan 6x tan 2x
= RHS
∴ LHS = RHS
Hence proved.

(ii) tan π/12 + tan π/6 + tan π/12 tan π/6 = 1
We know,
π/12 = 15° and π/6 = 30°
So, we have 15° + 30° = 45°
Tan (15° + 30°) = tan 45°
We know that, tan (A + B) = (tan A + tan B) / (1 – tan A tan B)
So,
(tan 15° + tan 30°) / (1 – tan 15° tan 30°) = 1
tan 15° + tan 30° = 1 – tan 15° tan 30°
Upon rearranging we get,
tan15° + tan30° + tan15° tan30° = 1
Hence proved.

(iii) tan 36° + tan 9° + tan 36° tan 9° = 1
We know 36° + 9° = 45°
So we have,
tan (36° + 9°) = tan 45°
We know that, tan (A + B) = (tan A + tan B) / (1 – tan A tan B)
So,
(tan 36° + tan 9°) / (1 – tan 36° tan 9°) = 1
tan 36° + tan 9° = 1 – tan 36° tan 9°
Upon rearranging we get,
tan 36° + tan 9° + tan 36° tan 9° = 1
Hence proved.

(iv) tan 13x – tan 9x – tan 4x = tan 13x tan 9x tan 4x
Let us consider LHS:
tan 13x – tan 9x – tan 4x
tan 13x = tan (9x + 4x)
We know that, tan (A + B) = (tan A + tan B) / (1 – tan A tan B)

So,

tan 13x = (tan 9x + tan 4x) / (1 – tan 9x tan 4x)

By cross-multiplying we get,

tan 13x (1 – tan 9x tan 4x) = tan 9x + tan 4x

tan 13x – tan 13x tan 9x tan 4x = tan 9x + tan 4x

Upon rearranging we get,

tan 13x – tan 9x – tan 4x = tan 13x tan 9x tan 4x

= RHS

∴ LHS = RHS

Hence proved.