EXERCISE 7.2 PAGE NO: 7.26

1. Find the maximum and minimum values of each of the following trigonometrical expressions:
(i) 12 sin x – 5 cos x
(ii) 12 cos x + 5 sin x + 4
(iii) 5 cos x + 3 sin (π/6 - x) + 4
(iv) sin x – cos x + 1
Solution:
We know that the maximum value of A cos α + B sin α + C is C + $\sqrt{(A^2 + B^2)}$,
And the minimum value is C - $\sqrt{(a^2 + B^2)}$.

(i) 12 sin x – 5 cos x
Given: f(x) = 12 sin x – 5 cos x
Here, A = -5, B = 12 and C = 0
$-\sqrt{((-5)^2 + 12^2)} \leq 12 \sin x - 5 \cos x \leq \sqrt{((-5)^2 + 12^2)}$
$-\sqrt{(25+144)} \leq 12 \sin x - 5 \cos x \leq \sqrt{(25+144)}$
$-\sqrt{169} \leq 12 \sin x - 5 \cos x \leq \sqrt{169}$
$-13 \leq 12 \sin x - 5 \cos x \leq 13$
Hence, the maximum and minimum values of f(x) are 13 and -13 respectively.

(ii) 12 cos x + 5 sin x + 4
Given: f(x) = 12 cos x + 5 sin x + 4
Here, A = 12, B = 5 and C = 4
$4 - \sqrt{(12^2 + 5^2)} \leq 12 \cos x + 5 \sin x + 4 \leq 4 + \sqrt{(12^2 + 5^2)}$
$4 - \sqrt{(144+25)} \leq 12 \cos x + 5 \sin x + 4 \leq 4 + \sqrt{(144+25)}$
$4 - \sqrt{169} \leq 12 \cos x + 5 \sin x + 4 \leq 4 + \sqrt{169}$
$-9 \leq 12 \cos x + 5 \sin x + 4 \leq 17$
Hence, the maximum and minimum values of f(x) are -9 and 17 respectively.

(iii) 5 cos x + 3 sin (π/6 - x) + 4
Given: f(x) = 5 cos x + 3 sin (π/6 - x) + 4
We know that, sin (A - B) = sin A cos B – cos A sin B
f(x) = 5 cos x + 3 sin (π/6 - x) + 4
= 5 cos x + 3 (sin π/6 cos x – cos π/6 sin x) + 4
= 5 cos x + 3/2 cos x - 3√3/2 sin x + 4
= 13/2 cos x - 3√3/2 sin x + 4
So, here A = 13/2, B = - 3√3/2, C = 4
$4 - \sqrt{[(13/2)^2 + (-3\sqrt{3}/2)^2]} \leq 13/2 \cos x - 3\sqrt{3}/2 \sin x + 4 \leq 4 + \sqrt{[(13/2)^2 + (-3\sqrt{3}/2)^2]}$

$4 - \sqrt{[(169/4) + (27/4)]} \leq 13/2 \cos x - 3\sqrt{3}/2 \sin x + 4 \leq 4 + \sqrt{[(169/4) + (27/4)]}$
$4 – 7 \leq 13/2 \cos x - 3\sqrt{3}/2 \sin x + 4 \leq 4 + 7$
$-3 \leq 13/2 \cos x - 3\sqrt{3}/2 \sin x + 4 \leq 11$
Hence, the maximum and minimum values of f(x) are -3 and 11 respectively.

(iv) sin x – cos x + 1
Given: f(x) = sin x – cos x + 1
So, here A = -1, B = 1 And c = 1
$1 - \sqrt{[(-1)^2 + 1^2]} \leq \sin x – \cos x + 1 \leq 1 + \sqrt{[(-1)^2 + 1^2]}$
$1 - \sqrt{(1+1)} \leq \sin x – \cos x + 1 \leq 1 + \sqrt{(1+1)}$
$1 - \sqrt{2} \leq \sin x – \cos x + 1 \leq 1 + \sqrt{2}$
Hence, the maximum and minimum values of f(x) are $1 - \sqrt{2}$ and $1 + \sqrt{2}$ respectively.

2. Reduce each of the following expressions to the Sine and Cosine of a single expression:
(i) √3 sin x – cos x
(ii) cos x – sin x
(iii) 24 cos x + 7 sin x
Solution:
(i) √3 sin x – cos x
Let f(x) = √3 sin x – cos x
Dividing and multiplying by $\sqrt{((\sqrt{3})^2 + 1^2)}$ i.e. by 2
f(x) = 2(√3/2 sin x – 1/2 cos x)

Sine expression:
f(x) = 2(cos π/6 sin x – sin π/6 cos x) (since, √3/2 = cos π/6 and 1/2 = sin π/6)
We know that, sin A cos B – cos A sin B = sin (A – B)
f(x) = 2 sin (x - π/6)

Again,
f(x) = 2(√3/2 sin x – 1/2 cos x)
Cosine expression:
f(x) = 2(sin π/3 sin x – cos π/3 cos x)
We know that, cos A cos B – sin A sin B = cos (A + B)
f(x) = -2 cos(π/3 + x)

(ii) cos x – sin x
Let f(x) = cos x – sin x
Dividing and multiplying by $\sqrt{(1^2 + 1^2)}$ i.e. by √2,

f(x) = √2(1/√2 cos x – 1/√2 sin x)

Sine expression:
f(x) = √2(sin π/4 cos x – cos π/4 sin x) (since, 1/√2 = sin π/4 and 1/√2 = cos π/4)
We know that sin A cos B – cos A sin B = sin (A – B)
f(x) = √2 sin (π/4 - x)

Again,
f(x) = √2(1/√2 cos x – 1/√2 sin x)
Cosine expression:
f(x) = 2(cos π/4 cos x – sin π/4 sin x)
We know that cos A cos B – sin A sin B = cos (A + B)
f(x) = √2 cos (π/4 + x)

(iii) 24 cos x + 7 sin x
Let f(x) = 24 cos x + 7 sin x
Dividing and multiplying by $\sqrt{((\sqrt{24})^2 + 7^2)}$ = √625 i.e. by 25,
f(x) = 25(24/25 cos x + 7/25 sin x)

Sine expression:
f(x) = 25(sin α cos x + cos α sin x) where, sin α = 24/25 and cos α = 7/25
We know that sin A cos B + cos A sin B = sin (A + B)
f(x) = 25 sin (α + x)

Cosine expression:
f(x) = 25(cos α cos x + sin α sin x) where, cos α = 24/25 and sin α = 7/25
We know that cos A cos B + sin A sin B = cos (A - B)
f(x) = 25 cos (α - x)

3. Show that Sin 100^o – Sin 10^o is positive.
Solution:
Let f(x) = sin 100° – sin 10°
Dividing And multiplying by $\sqrt{(1^2 + 1^2)}$ i.e. by √2,
f(x) = √2(1/√2 sin 100^o – 1/√2 sin 10^o)
f(x) = √2(cos π/4 sin (90+10)^o – sin π/4 sin 10^o) (since, 1/√2 = cos π/4 and 1/√2 = sin π/4)
f(x) = √2(cos π/4 cos 10^o – sin π/4 sin 10^o)
We know that cos A cos B – sin A sin B = cos (A + B)
f(x) = √2 cos (π/4 + 10^o)
∴ f(x) = √2 cos 55°

4. Prove that (2√3 + 3) sin x + 2√3 cos x lies between – (2√3 + √15) and (2√3 + √15).

Solution:

Let f(x) = (2√3 + 3) sin x + 2√3 cos x

Here, A = 2√3, B = 2√3 + 3 and C = 0

$- \sqrt{[(2\sqrt{3})^2 + (2\sqrt{3} + 3)^2]} \leq (2\sqrt{3} + 3) \sin x + 2\sqrt{3} \cos x \leq \sqrt{[(2\sqrt{3})^2 + (2\sqrt{3} + 3)^2]}$

- √[12+12+9+12√3] ≤ (2√3 + 3) sin x + 2√3 cos x ≤ √[12+12+9+12√3]

- √[33+12√3] ≤ (2√3 + 3) sin x + 2√3 cos x ≤ √[33+12√3]

- √[15+12+6+12√3] ≤ (2√3 + 3) sin x + 2√3 cos x ≤ √[15+12+6+12√3]

We know that (12√3 + 6 < 12√5) because the value of √5 - √3 is more than 0.5

So if we replace, (12√3 + 6 with 12√5) the above inequality still holds.

So by rearranging the above expression √(15+12+12√5)we get, 2√3 + √15

- 2√3 + √15 ≤ (2√3 + 3) sin x + 2√3 cos x ≤ 2√3 + √15

Hence proved.