EXERCISE 8.1 PAGE NO: 8.6

1. Express each of the following as the sum or difference of sines and cosines:
(i) 2 sin 3x cos x
(ii) 2 cos 3x sin 2x
(iii) 2 sin 4x sin 3x
(iv) 2 cos 7x cos 3x
Solution:
(i) 2 sin 3x cos x
By using the formula,
2 sin A cos B = sin (A + B) + sin (A – B)
2 sin 3x cos x = sin (3x + x) + sin (3x – x)
= sin (4x) + sin (2x)
= sin 4x + sin 2x

(ii) 2 cos 3x sin 2x
By using the formula,
2 cos A sin B = sin (A + B) – sin (A – B)
2 cos 3x sin 2x = sin (3x + 2x) – sin (3x – 2x)
= sin (5x) – sin (x)
= sin 5x – sin x

(iii) 2 sin 4x sin 3x
By using the formula,
2 sin A sin B = cos (A – B) – cos (A + B)
2 sin 4x sin 3x = cos (4x – 3x) – cos (4x + 3x)
= cos (x) – cos (7x)
= cos x – cos 7x

(iv) 2 cos 7x cos 3x
By using the formula,
2 cos A cos B = cos (A + B) + cos (A – B)
2 sin 3x cos x = cos (7x + 3x) + cos (7x – 3x)
= cos (10x) + cos (4x)
= cos 10x + cos 4x

2. Prove that:
(i) 2 sin 5π/12 sin π/12 = 1/2
(ii) 2 cos 5π/12 cos π/12 = 1/2

(iii) 2 sin 5π/12 cos π/12 = (√3 + 2)/2

Solution:

(i) 2 sin 5π/12 sin π/12 = 1/2

By using the formula,

2 sin A sin B = cos (A – B) – cos (A + B)

2 sin 5π/12 sin π/12 = cos (5π/12 - π/12) – cos (5π/12 + π/12)
= cos (4π/12) – cos (6π/12)
= cos (π/3) – cos (π/2)
= cos (180°/3) – cos (180°/2)
= cos 60° - cos 90°
= 1/2 – 0
= 1/2

Hence Proved.

(ii) 2 cos 5π/12 cos π/12 = 1/2

By using the formula,

2 cos A cos B = cos (A + B) + cos (A - B)

2 cos 5π/12 cos π/12 = cos (5π/12 + π/12) + cos (5π/12 - π/12)
= cos (6π/12) + cos (4π/12)
= cos (π/2) + cos (π/3)
= cos (180°/2) + cos (180°/3)
= cos 90° + cos 60°
= 0 + 1/2
= 1/2

Hence Proved.

(iii) 2 sin 5π/12 cos π/12 = (√3 + 2)/2

By using the formula,

2 sin A cos B = sin (A + B) + sin (A - B)

2 sin 5π/12 cos π/12 = sin (5π/12 + π/12) + sin (5π/12 - π/12)
= sin (6π/12) + sin (4π/12)
= sin (π/2) + sin (π/3)
= sin (180°/2) + sin (180°/3)
= sin 90° + sin 60°
= 1 + √3
= (2 + √3)/2
= (√3 + 2)/2

Hence Proved.

3. show that:
(i) sin 50° cos 85° = (1 - √2sin 35°)/2√2
(ii) sin 25° cos 115° = 1/2 {sin 140° - 1}
Solution:
(i) sin 50° cos 85° = (1 - √2sin 35°)/2√2
By using the formula,
2 sin A cos B = sin (A + B) + sin (A – B)
sin A cos B = [sin (A + B) + sin (A – B)] / 2
sin 50° cos 85° = [sin(50° + 85°) + sin (50° – 85°)] / 2
= [sin (135°) + sin (-35°)] / 2
= [sin (135°) - sin (35°)] / 2 (since, sin (-x) = -sin x)
= [sin (180° – 45°) – sin 35°] / 2
= [sin 45° – sin 35°] / 2
= [(1/√2) – sin 35°] / 2
= [(1 – sin 35°)/√2] / 2
= (1 – sin 35°) / 2√2
Hence proved.

(ii) sin 25° cos 115° = 1/2 {sin 140° - 1}
By using the formula,
2 sin A cos B = sin (A + B) + sin (A – B)
sin A cos B = [sin (A + B) + sin (A – B)] / 2
sin 20° cos 115° = [sin(25° + 115°) + sin (25° – 115°)] / 2
= [sin (140°) + sin (-90°)] / 2
= [sin (140°) - sin (90°)] / 2 (since, sin (-x) = -sin x)
= 1/2 {sin 140° - 1}
Hence proved.

4. Prove that:
4 cos x cos (π/3 + x) cos (π/3 - x) = cos 3x
Solution:
Let us consider LHS:
4 cos x cos (π/3 + x) cos (π/3 - x) = 2 cos x (2 cos (π/3 + x) cos (π/3 - x))
By using the formula,
2 cos A cos B = cos (A + B) + cos (A – B)
2 cos x (2 cos (π/3+x) cos (π/3 - x)) = 2 cos x (cos (π/3+x + π/3-x) + cos (π/3+x - π/3+ x))
= 2 cos x (cos (2π/3) + cos (2x))
= 2 cos x {cos 120° + cos 2x}
= 2 cos x {cos (180° - 60°) + cos 2x}

= 2 cos x (cos 2x - cos 60°) (since, {cos (180° - A) = - cos A})
= 2 cos 2x cos x – 2 cos x cos 60°
= (cos (x + 2x) + cos (2x - x)) – (2cos x)/2
= cos 3x + cos x – cos x
= cos 3x
= RHS

Hence Proved.