EXERCISE 9.3 PAGE NO: 9.42

Prove that:

1. $\sin^2 2\pi/5 - \sin^2 \pi/3 = (\sqrt{5} - 1)/8$

Solution:

Let us consider LHS:

$\sin^2 2\pi/5 - \sin^2 \pi/3 = \sin^2 (\pi/2 - \pi/10) - \sin^2 \pi/3$

we know, sin (90°– A) = cos A

So, $\sin^2 (\pi/2 - \pi/10) = \cos^2 \pi/10$

Sin $\pi/3 = \sqrt{3}/2$

Then the above equation becomes,

$= \cos^2 \pi/10 - (\sqrt{3}/2)^2$

We know, $\cos \pi/10 = \sqrt{(10+2\sqrt{5})}/4$

the above equation becomes,

$= [\sqrt{(10+2\sqrt{5})}/4]^2 - 3/4$

$= [10 + 2\sqrt{5}]/16 - 3/4$

$= [10 + 2\sqrt{5} - 12]/16$

$= [2\sqrt{5} - 2]/16$

$= [\sqrt{5} - 1]/8$

= RHS

Hence proved.

2. $\sin^2 24^o - \sin^2 6^o = (\sqrt{5} - 1)/8$

Solution:

Let us consider LHS:

$\sin^2 24^o - \sin^2 6^o$

we know, $\sin (A + B) \sin (A - B) = \sin^2 A - \sin^2 B$

Then the above equation becomes,

$\sin^2 24^o - \sin^2 6^o = \sin (24^o + 6^o) - \sin (24^o - 6^o)$

$= \sin 30^o - \sin 18^o$

$= \sin 30^o - (\sqrt{5} - 1)/4$ [since, $\sin 18^o = (\sqrt{5} - 1)/4$]

$= 1/2 \times (\sqrt{5} - 1)/4$

$= (\sqrt{5} - 1)/8$

= RHS

Hence proved.

3. $\sin^2 42^o - \cos^2 78^o = (\sqrt{5} + 1)/8$

Solution:

Let us consider LHS:

$\sin^2 42^o - \cos^2 78^o = \sin^2 (90^o - 48^o) - \cos^2 (90^o - 12^o)$

$= \cos^2 48^o - \sin^2 12^o$ [since, sin (90 - A) = cos A and cos (90 - A) = sin A]

We know, $\cos (A + B) \cos (A - B) = \cos^2 A - \sin^2 B$

Then the above equation becomes,

$= \cos^2 (48^o + 12^o) \cos (48^o - 12^o)$

$= \cos 60^o \cos 36^o$ [since, $\cos 36^o = (\sqrt{5} + 1)/4$]

$= 1/2 \times (\sqrt{5} + 1)/4$

$= (\sqrt{5} + 1)/8$

= RHS

Hence proved.

4. $\cos 78^o \cos 42^o \cos 36^o = 1/8$

Solution:

Let us consider LHS:

$\cos 78^o \cos 42^o \cos 36^o$

Let us multiply and divide by 2 we get,

$\cos 78^o \cos 42^o \cos 36^o = 1/2 (2 \cos 78^o \cos 42^o \cos 36^o)$

We know, $2 \cos A \cos B = \cos (A + B) + \cos (A - B)$

Then the above equation becomes,

$= 1/2 (\cos (78^o + 42^o) + \cos (78^o - 42^o)) \times \cos 36^o$

$= 1/2 (\cos 120^o + \cos 36^o) \times \cos 36^o$

$= 1/2 (\cos (180^o - 60^o) + \cos 36^o) \times \cos 36^o$

$= 1/2 (-\cos (60^o) + \cos 36^o) \times \cos 36^o$ [since, $\cos(180° - A) = - A$]

$= 1/2 (-1/2 + (\sqrt{5} + 1)/4) ((\sqrt{5} + 1)/4)$ [since, $\cos 36^o = (\sqrt{5} + 1)/4$]

$= 1/2 (\sqrt{5} + 1 - 2)/4 ((\sqrt{5} + 1)/4)$

$= 1/2 (\sqrt{5} - 1)/4) ((\sqrt{5} + 1)/4)$

$= 1/2 ((\sqrt{5})^2 - 1^2)/16$

$= 1/2 (5-1)/16$

$= 1/2 (4/16)$

$= 1/8$

= RHS

Hence proved.

5. $\cos \pi/15 \cos 2\pi/15 \cos 4\pi/15 \cos 7\pi/15 = 1/16$

Solution:

Let us consider LHS:

$\cos \pi/15 \cos 2\pi/15 \cos 4\pi/15 \cos 7\pi/15$

Let us multiply and divide by $2 \sin \pi/15$, we get,

$= [2 \sin \pi/15 \cos \pi/15] \cos 2\pi/15 \cos 4\pi/15 \cos 7\pi/15] / 2 \sin \pi/15$

We know, 2sin A cos A = sin 2A

Then the above equation becomes,

$= [(\sin 2\pi/15) \cos 2\pi/15 \cos 4\pi/15 \cos 7\pi/15] / 2 \sin \pi/15$

Now, multiply and divide by 2 we get,

$= [(2 \sin 2\pi/15 \cos 2\pi/15) \cos 4\pi/15 \cos 7\pi/15] / 2 \times 2 \sin \pi/15$

We know, 2sin A cos A = sin 2A

Then the above equation becomes,

$= [(\sin 4\pi/15) \cos 4\pi/15 \cos 7\pi/15] / 4 \sin \pi/15$

Now, multiply and divide by 2 we get,

$= [(2 \sin 4\pi/15 \cos 4\pi/15) \cos 7\pi/15] / 2 \times 4 \sin \pi/15$

We know, 2sin A cos A = sin 2A

Then the above equation becomes,

$= [(\sin 8\pi/15) \cos 7\pi/15] / 8 \sin \pi/15$

Now, multiply and divide by 2 we get,

$= [2 \sin 8\pi/15 \cos 7\pi/15] / 2 \times 8 \sin \pi/15$

We know, 2sin A cos B = sin (A+B) + sin (A–B)

Then the above equation becomes,

$= [\sin (8\pi/15 + 7\pi/15) + \sin (8\pi/15 - 7\pi/15)] / 16 \sin \pi/15$

$= [\sin (\pi) + \sin (\pi/15)] / 16 \sin \pi/15$

$= [0 + \sin (\pi/15)] / 16 \sin \pi/15$

$= \sin (\pi/15) / 16 \sin \pi/15$

$= 1/16$

= RHS

Hence proved.