EXERCISE 11.4 PAGE NO: 11.74

Find dy/dx in each of the following:

1. $xy = c^2$

Solution:

Given $xy = c^2$;

Now we have to find $\frac{dy}{dx}$ of given equation, so by differentiating the equation on both sides with respect to x, we get,

By using the product rule on the left hand side,

$$\frac{d(xy)}{dx} = \frac{dc^2}{dx}$$

x (dy/dx) + y (1) = 0

$$\frac{dy}{dx} = \frac{-y}{x}$$

We can further solve it by putting the value of y,

$$\frac{dy}{dx} = \frac{-c^2}{x^2}$$

2. $y^3 - 3xy^2 = x^3 + 3x^2y$

Solution:

Given $y^3 - 3xy^2 = x^3 + 3x^2y$,

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\Rightarrow \frac{d}{dx}(y^3) - \frac{d}{dx}(3xy^2) = \frac{d}{dx}(x^3) + \frac{d}{dx}(3x^2y)$$

Now by using product rule we get,

$$\Rightarrow 3y^2\frac{dy}{dx} - 3\left[x\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(x)\right] = 3x^2 + 3\left[x^2\frac{d}{dx}(y) + y\frac{d}{dx}(x^2)\right]$$

$$\Rightarrow 3y^2\frac{dy}{dx} - 3\left[x(2y)\frac{dy}{dx} + y^2\right] = 3x^2 + 3\left[x^2\frac{dy}{dx} + y(2x)\right]$$

$$\Rightarrow 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3y^2 = 3x^2 + 3x^2\frac{dy}{dx} + 6xy$$

$$\Rightarrow 3y^2\frac{dy}{dx} - 6xy\frac{dy}{dx} - 3x^2\frac{dy}{dx} = 3x^2 + 6xy + 3y^2$$

$$\Rightarrow 3\frac{dy}{dx}\left(y^2 - 2xy - x^2\right) = 3\left(x^2 + 2xy + y^2\right)$$

Now by taking 3 as common we get,

$$\Rightarrow \frac{dy}{dx} = \frac{3(x+y)^2}{3\left(y^2 - 2xy - x^2\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{(x+y)^2}{y^2 - 2xy - x^2}$$

3. $x^{2/3} + y^{2/3} = a^{2/3}$

Solution:

Given $x^{2/3} + y^{2/3} = a^{2/3}$,

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{2}{3}\frac{1}{x^{1/3}} + \frac{2}{3}\frac{1}{y^{1/3}}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-y^{1/3}}{x^{1/3}}$$

Now by substituting the value, we get

$$\frac{dy}{dx} = \frac{-\sqrt{a^{2/3} - x^{2/3}}}{x^{1/3}}$$

4. 4x + 3y = log (4x – 3y)

Solution:
Given 4x + 3y = log (4x – 3y),
Now we have to find dy/dx of it, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{d}{dx}(4x)+\frac{d}{dx}(3y)=\frac{d}{dx}\{\log(4x-3y)\}$$

$$\Rightarrow 4+3\frac{dy}{dx}=\frac{1}{(4x-3y)}\frac{d}{dx}(4x-3y)$$

$$\Rightarrow 4+3\frac{dy}{dx}=\frac{1}{(4x-3y)}\left(4-3\frac{dy}{dx}\right)$$

$$\Rightarrow 3\frac{dy}{dx}+\frac{3}{(4x-3y)}\frac{dy}{dx}=\frac{4}{(4x-3y)}-4$$

$$\Rightarrow 3\frac{dy}{dx}\left\{1+\frac{1}{(4x-3y)}\right\}=4\left\{\frac{1}{(4x-3y)}-1\right\}$$

$$\Rightarrow 3\frac{dy}{dx}\left\{\frac{4x-3y+1}{(4x-3y)}\right\}=4\left\{\frac{1-4x+3y}{(4x-3y)}\right\}$$

$$\Rightarrow \frac{dy}{dx}=\frac{4}{3}\left\{\frac{1-4x+3y}{(4x-3y)}\right\}\left(\frac{4x-3y}{4x-3y+1}\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{4}{3}\left(\frac{1-4x+3y}{4x-3y+1}\right)$$

5. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

Solution:
Given $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$,

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{2x}{a^2} + \frac{2y}{b^2}\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-xb^2}{ya^2}$$

6. $x^5 + y^5 = 5xy$

Solution:

Given $x^5 + y^5 = 5xy$

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{d}{dx}(x^5) + \frac{d}{dx}(y^5) = \frac{d}{dx}(5xy)$$

Now by using product rule, we get

$$\Rightarrow 5x^4 + 5y^4\frac{dy}{dx} = 5\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right]$$

$$\Rightarrow 5x^4 + 5y^4\frac{dy}{dx} = 5\left[x\frac{dy}{dx} + y(1)\right]$$

$$\Rightarrow 5x^4 + 5y^4\frac{dy}{dx} = 5x\frac{dy}{dx} + 5y$$

$$\Rightarrow 5y^4\frac{dy}{dx} - 5x\frac{dy}{dx} = 5y - 5x^4$$

$$\Rightarrow 5\frac{dy}{dx}(y^4 - x) = 5(y - x^4)$$

$$\Rightarrow \frac{dy}{dx} = \frac{5(y - x^4)}{5(y^4 - x)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - x^4}{y^4 - x}$$

7. $(x + y)^2 = 2axy$

Solution:

Given $(x + y)^2 = 2axy$

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\Rightarrow \frac{d}{dx}(x+y)^2 = \frac{d}{dx}(2axy)$$

Now by using product rule, we get

$$\Rightarrow 2(x+y)\frac{d}{dx}(x+y) = 2a\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right]$$

$$\Rightarrow 2(x+y)\left[1 + \frac{dy}{dx}\right] = 2a\left[x\frac{dy}{dx} + y(1)\right]$$

$$\Rightarrow 2(x+y) + 2(x+y)\frac{dy}{dx} = 2ax\frac{dy}{dx} + 2ay$$

$$\Rightarrow \frac{dy}{dx}[2(x+y) - 2ax] = 2ay - 2(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2[ay - x - y]}{2[x + y - ax]}$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{ay - x - y}{x + y - ax}\right)$$

8. $(x^2 + y^2)^2 = xy$

Solution:

Given $(x + y)^2 = 2axy$

Now we have to find dy/dx of given equation, so by differentiating the equation on both sides with respect to x, we get,

$$\Rightarrow \frac{d}{dx}\left[\left(x^2+y^2\right)^2\right]=\frac{d}{dx}(xy)$$

Now by applying product rule we get,

$$\Rightarrow 2\left(x^2+y^2\right)\frac{d}{dx}\left(x^2+y^2\right)=x\frac{dy}{dx}+y\frac{d}{dx}(x)$$

$$\Rightarrow 2\left(x^2+y^2\right)\left(2x+2y\frac{dy}{dx}\right)=x\frac{dy}{dx}+y(1)$$

$$\Rightarrow 4x\left(x^2+y^2\right)+4y\left(x^2+y^2\right)\frac{dy}{dx}=x\frac{dy}{dx}+y$$

$$\Rightarrow 4y\left(x^2+y^2\right)\frac{dy}{dx}-x\frac{dy}{dx}=y-4x\left(x^2+y^2\right)$$

$$\Rightarrow \frac{dy}{dx}\left[4y\left(x^2+y^2\right)-x\right]=y-4x\left(x^2+y^2\right)$$

$$\Rightarrow \frac{dy}{dx}=\frac{y-4x\left(x^2+y^2\right)}{4y\left(x^2+y^2\right)-x}$$

$$\Rightarrow \frac{dy}{dx}=\frac{4x\left(x^2+y^2\right)-y}{x-4y\left(x^2+y^2\right)}$$

9. Tan^{-1} (x^2 + y^2)

Solution:

Given tan $^{-1}(x^2 + y^2) = a$,

Now we have to find dy/dx of given function, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{1}{x^2+y^2}\left(2x+2y\frac{dy}{dx}\right)=0$$

$$\frac{dy}{dx}=\frac{-x}{y}$$

10. $e^{x-y} = log\left(\frac{x}{y}\right)$

Solution:

Given $e^{x-y} = log\left(\frac{x}{y}\right)$

Now we have to find dy/dx of given function, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{d}{dx}\left(e^{x-y}\right) = \frac{d}{dx}\left\{\log\left(\frac{x}{y}\right)\right\}$$

$$\Rightarrow e^{(x-y)}\frac{d}{dx}(x-y) = \frac{1}{\left(\frac{x}{y}\right)} \times \frac{d}{dx}\left(\frac{x}{y}\right)$$

Now by applying quotient rule we get

$$\Rightarrow e^{(x-y)}\left(1-\frac{dy}{dx}\right) = \frac{y}{x}\left[\frac{y\frac{d}{dx}(x) - x\frac{dy}{dx}}{y^2}\right]$$

$$\Rightarrow e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{xy}\left[y(1) - x\frac{dy}{dx}\right]$$

$$\Rightarrow e^{(x-y)} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{x} - \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} - e^{(x-y)}\frac{dy}{dx} = \frac{1}{x} - e^{(x-y)}$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{1}{y} - \frac{e^{(x-y)}}{1}\right] = \frac{1}{x} - \frac{e^{(x-y)}}{1}$$

$$\Rightarrow \frac{dy}{dx}\left[\frac{1-ye^{(x-y)}}{y}\right] = \frac{1-xe^{(x-y)}}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left[\frac{1 - xe^{(x-y)}}{1 - ye^{(x-y)}}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{-x}\left[\frac{xe^{(x-y)} - 1}{ye^{(x-y)} - 1}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{x}\left[\frac{xe^{(x-y)} - 1}{ye^{(x-y)} - 1}\right]$$

11. Sin xy + cos (x + y) = 1

Solution:

Given Sin x y + cos (x + y) = 1

Now we have to find dy/dx of given function, so by differentiating the equation on both sides with respect to x, we get,

$$\frac{d}{dx}(\sin xy) + \frac{d}{dx}\cos(x+y) = \frac{d}{dx}(1)$$

$$\Rightarrow \cos xy \frac{d}{dx}(xy) - \sin(x+y)\frac{d}{dx}(x+y) = 0$$

$$\Rightarrow \cos xy\left[x\frac{dy}{dx} + y\frac{d}{dx}(x)\right] - \sin(x+y)\left[1 + \frac{dy}{dx}\right] = 0$$

$$\Rightarrow \cos xy\left[x\frac{dy}{dx} + y(1)\right] - \sin(x+y) - \sin(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow x\cos xy\frac{dy}{dx} + y\cos xy - \sin(x+y) - \sin(x+y)\frac{dy}{dx} = 0$$

$$\Rightarrow [x\cos xy - \sin(x+y)]\frac{dy}{dx} = [\sin(x+y) - y\cos xy]$$

$$\Rightarrow \frac{dy}{dx} = \left[\frac{\sin(x+y) - y\cos xy}{x\cos xy - \sin(x+y)}\right]$$

12. $If\ \sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y),\ prove\ that\ \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}.$

Solution:

Given $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$

Let x = sin A and y = sin B

Then given equation becomes,

$$\Rightarrow \sqrt{1-\sin^2 A}+\sqrt{1-\sin^2 B}=a(\sin A-\sin B)$$

$$\Rightarrow \cos A+\cos B=a(\sin A-\sin B)$$

$$\Rightarrow a=\frac{\cos A+\cos B}{\sin A-\sin B}$$

Now by applying the formula we get,

$$\Rightarrow a=\frac{2\cos\frac{A+B}{2}\cos\frac{A-B}{2}}{2\cos\frac{A+B}{2}\sin\frac{A-B}{2}}$$

$$\Rightarrow a=\cot\left(\frac{A-B}{2}\right)$$

$$\Rightarrow \cot^{-1}a=\frac{A-B}{2}$$

$$\Rightarrow 2\cot^{-1}a=A-B$$

$$\Rightarrow 2\cot^{-1}a=\sin^{-1}x-\sin^{-1}y.$$

Now by differentiating with respect to x we get,

$$\frac{d}{dx}(2cot^{-1}a)=\frac{d}{dx}(\sin^{-1}x)-\frac{d}{dx}(\sin^{-1}y)$$

$$\Rightarrow 0=\frac{1}{\sqrt{1-x^2}}-\frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}$$

$$\Rightarrow \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx}=\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx}=\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$$

13. $If\ y=\sqrt{1-x^2}+x\sqrt{1-y^2}=1,\ prove\ that\ \frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}.$

Solution:

Given, $y=\sqrt{1-x^2}+x\sqrt{1-y^2}=1$

Let x = sin A and y = sin B

Then given equation becomes,

$$\Rightarrow \sin B\sqrt{1-\sin^2 A}+\sin A\sqrt{1-\sin^2 B}=1$$

Now by applying the identity, we get

$$\Rightarrow \sin B\cos A+\sin A\cos B=1$$

$$\Rightarrow \sin(A+B)=1$$

$$\Rightarrow A+B=\sin^{-1}(1)$$

Now by substituting the values of A and B, we get

$$\Rightarrow \sin^{-1}x+\sin^{-1}y=\frac{\pi}{2}$$

Now by differentiating with respect to x, we get

$$\Rightarrow \frac{d}{dx}(\sin^{-1} x) + \frac{d}{dx}(\sin^{-1} y) = \frac{d}{dx}\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$14.\ If\ xy = 1,\ prove\ that\ \frac{dy}{dx} + y^2 = 0.$$

Solution:

Given x y = 1

Differentiating with respect to x, we get

$$\frac{d}{dx}(xy) = \frac{d}{dx}(1)$$

By using product rule,

$$\Rightarrow x\frac{dy}{dx} + y\frac{d}{dx}(x) = 0.$$

$$\Rightarrow x\frac{dy}{dx} + y(1) = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{x}$$

We have x y = 1, therefore x = 1/y

$$\Rightarrow \frac{dy}{dx} = -\frac{y}{\frac{1}{y}}$$

$$\Rightarrow \frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} + y^2 = 0$$

15. $If\ xy^2 = 1,\ prove\ that\ 2\frac{dy}{dx} + y^3 = 0.$

Solution:

Given $xy^2 = 1$

Now differentiating given equation with respect to x, we get

$$\frac{d}{dx}(xy^2) = \frac{d}{dx}(1)$$

$$\Rightarrow x\frac{d}{dx}(y^2) + y^2\frac{d}{dx}(x) = 0$$

$$\Rightarrow x(2y)\frac{dy}{dx} + y^2(1) = 0$$

$$\Rightarrow 2xy\frac{dy}{dx} = -y^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y^2}{2xy}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2x}$$

Now by substituting $x = 1/y^2$ in above equation we get

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{2\left(\frac{1}{y^2}\right)}$$

$$\Rightarrow 2\frac{dy}{dx} = -y^3$$

$$\Rightarrow 2\frac{dy}{dx} + y^3 = 0$$