EXERCISE 11.5 PAGE NO: 11.88

Differentiate the following functions with respect to x:

1. $x^{1/x}$

Solution:

Let $y = x^{\frac{1}{x}}$

Taking log both the sides:

$$\Rightarrow \log y = \log x^{\frac{1}{x}}$$

We know that $\log x^a = a \log x$, substituting this in above equation we get

$$\Rightarrow \log y = \frac{1}{x}\log x$$

Differentiating with respect to x, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d\left(\frac{1}{x}\log x\right)}{dx}$$

Now by using the product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{1}{x} \times \frac{d(\log x)}{dx} + \log x \times \frac{d(x^{-1})}{dx}$$

We have $\left\{\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}; \frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}\right\}$, by using this we get,

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} \times \frac{1}{x}\frac{dx}{dx} + \log x\left(\frac{-1}{x^2}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x^2} - \frac{1}{x^2}\log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1 - \log x}{x^2}$$

$$\Rightarrow \frac{dy}{dx} = y\left(\frac{1 - \log x}{x^2}\right)$$

Put the value of $y = x^{\frac{1}{x}}$

$$\Rightarrow \frac{dy}{dx} = x^{\frac{1}{x}}\left(\frac{1-\log x}{x^2}\right)$$

2. $x^{\sin x}$

Solution:

Let $y = x^{\sin x}$

Taking log both the sides

$\log y = \log (x^{\sin x})$

$\log y = \sin x \log x$ {$\log x^a = a \log x$}

Differentiating with respect to x, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\sin x \log x)}{dx}$$

Now by using product rule, we can write as

$$\Rightarrow \frac{d(\log y)}{dx} = \sin x \times \frac{d(\log x)}{dx} + \log x \times \frac{d(\sin x)}{dx}$$

Again we have, $\left\{\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx} \;\&\; \frac{d(\sin x)}{dx} = \cos x\right\}$, by using this we can write as

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin x \times \frac{1}{x}\frac{dx}{dx} + \log x(\cos x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\sin x}{x} + \log x \cos x$$

$$\Rightarrow \frac{dy}{dx} = y\left(\frac{\sin x}{x} + \log x \cos x\right)$$

Put the value of $y = x^{\sin x}$

$$\Rightarrow \frac{dy}{dx} = x^{\sin x}\left(\frac{\sin x}{x} + \log x \cos x\right)$$

3. (1 + cos x)^x

Solution:

Let y = (1 + cos x)^x

Taking log on both the sides

$\Rightarrow$ log y = log (1 + cos x)^x

$\Rightarrow$ log y = x log (1+ cos x) {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d[x \log(1+\cos x)]}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = x \times \frac{d[\log(1+\cos x)]}{dx} + \log(1+\cos x) \times \frac{dx}{dx}$$

Again we have, $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{(1+\cos x)}\frac{d(1+\cos x)}{dx} + \log(1+\cos x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{(1+\cos x)}(-\sin x) + \log(1+\cos x)$$

$$\left\{\frac{d(1+\cos x)}{dx} = \frac{d(1)}{dx} + \frac{d(\cos x)}{dx} = 0 + (-\sin x)\frac{dx}{dx} = -\sin x\right\}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{-x\sin x}{1+\cos x} + \log(1+\cos x)$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{-x\sin x}{1+\cos x} + \log(1+\cos x)\right\}$$

Put the value of y = (1 + cos x)^x

$$\Rightarrow \frac{dy}{dx} = (1+\cos x)^x\left\{\frac{-x\sin x}{1+\cos x} + \log(1+\cos x)\right\}$$

4. $x^{\cos^{-1} x}$

Solution:

Let $y = x^{\cos^{-1} x}$

Taking log both the sides

$\Rightarrow \log y = \log x^{\cos^{-1} x}$

$\Rightarrow \log y = \cos^{-1} x \log x$ {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\cos^{-1} x \log x)}{dx}$$

By using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \cos^{-1} x \times \frac{d(\log x)}{dx} + \log x \times \frac{d(\cos^{-1} x)}{dx}$$

Again we have, $\left\{\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx} \,\&\, \frac{d(\cos^{-1} x)}{dx} = \frac{-1}{\sqrt{1-x^2}}\right\}$, from this we can write as

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\cos^{-1} x}{x} + \log x\left(\frac{-1}{\sqrt{1-x^2}}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right\}$$

Put the value of $y = x^{\cos^{-1} x}$

$$\Rightarrow \frac{dy}{dx} = x^{\cos^{-1} x}\left\{\frac{\cos^{-1} x}{x} - \frac{\log x}{\sqrt{1-x^2}}\right\}$$

5. (log x)^x

Solution:

Let y = $(\log x)^x$

Taking log both the sides

$\Rightarrow \log y = \log (\log x)^x$

$\Rightarrow \log y = x \log (\log x)$ $\{\log x^a = a \log x\}$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(x \log \log x)}{dx}$$

By product rule, we have

$$\Rightarrow \frac{d(\log y)}{dx} = x \times \frac{d(\log \log x)}{dx} + \log \log x \times \frac{dx}{dx}$$

We know that $\left\{\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}\right\}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{\log x}\frac{d(\log x)}{dx} + \log \log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\log x} \times \frac{1}{x} + \log \log x$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{1}{\log x} + \log \log x\right\}$$

Put the value of $y = (\log x)^x$

$$\Rightarrow \frac{dy}{dx} = (\log x)^x \left\{\frac{1}{\log x} + \log \log x\right\}$$

6. $(\log x)^{\cos x}$

Solution:

Let y = $(\log x)^{\cos x}$

Taking log both the sides, we get

⇒ Log y = log (log x)$^{\cos x}$

⇒ Log y = cos x log (log x) {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\cos x \log \log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \cos x \times \frac{d(\log \log x)}{dx} + \log \log x \times \frac{d(\cos x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$ & $\frac{d(\cos x)}{dx} = -\sin x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos x \times \frac{1}{\log x}\frac{d(\log x)}{dx} + \log \log x\ (-\sin x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\cos x}{\log x} \times \frac{1}{x} - \sin x \log \log x$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{\cos x}{x \log x} - \sin x \log \log x\right\}$$

Put the value of y = (log x)$^{\cos x}$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{\cos x}\left\{\frac{\cos x}{x \log x} - \sin x \log \log x\right\}$$

7. (Sin x)$^{\cos x}$

Solution:

Let y = (sin x)$^{\cos x}$

Taking log both the sides

⇒ Log y = log (sin x)$^{\cos x}$

⇒ Log y = cos x log sin x {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\cos x \log \sin x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \cos x \times \frac{d(\log \sin x)}{dx} + \log \sin x \times \frac{d(\cos x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\cos x)}{dx} = -\sin x$; $\frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cos x \times \frac{1}{\sin x}\frac{d(\sin x)}{dx} + \log \sin x\,(-\sin x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \cot x\,(\cos x) - \sin x \log \sin x$$

$$\Rightarrow \frac{dy}{dx} = y\{\cos x \cot x - \sin x \log \sin x\}$$

Put the value of $y = (\sin x)^{\cos x}$

$$\Rightarrow \frac{dy}{dx} = (\sin x)^{\cos x}\{\cos x \cot x - \sin x \log \sin x\}$$

8. $e^{x \log x}$

Solution:

Let $y = e^{x \log x}$

Taking log both the sides, we get

$\Rightarrow \text{Log } y = \log (e)^{x \log x}$

$\Rightarrow \text{Log } y = x \log x \log e$ $\{\log x^a = a \log x\}$

$\Rightarrow \text{Log } y = x \log x$ $\{\log e = 1\}$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(x \log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = x \times \frac{d(\log x)}{dx} + \log x \times \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x}\frac{dx}{dx} + \log x$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{x} + \log x$$

$$\Rightarrow \frac{dy}{dx} = y\{1 + \log x\}$$

Put the value of $y = e^{x \log x}$

$$\Rightarrow \frac{dy}{dx} = e^{x \log x}\{1 + \log x\}$$

$$\Rightarrow \frac{dy}{dx} = e^{\log x^x}\{1 + \log x\}$$ $\{e^{\log a} = a;\ a \log x = x^a\}$

$$\Rightarrow \frac{dy}{dx} = x^x\{1 + \log x\}$$

9. $(\text{Sin } x)^{\log x}$

Solution:

Let $y = (\sin x)^{\log x}$

Taking log both the sides

$\Rightarrow \text{Log } y = \log(\sin x)^{\log x}$

$\Rightarrow \text{Log } y = \log x \log \sin x$ $\{\log x^a = a \log x\}$

Differentiating with respect to x, then we get

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\log x \log \sin x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \log x \times \frac{d(\log \sin x)}{dx} + \log \sin x \times \frac{d(\log x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log x \times \frac{1}{\sin x}\frac{d(\sin x)}{dx} + \log \sin x \left(\frac{1}{x}\frac{dx}{dx}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\log x}{\sin x}(\cos x) + \frac{\log \sin x}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\log x \cot x + \frac{\log \sin x}{x}\right\}$$

Put the value of $y = (\sin x)^{\log x}$

$$\Rightarrow \frac{dy}{dx} = (\sin x)^{\log x}\left\{\log x \cot x + \frac{\log \sin x}{x}\right\}$$

10. $10^{\log \sin x}$

Solution:

Let $y = 10^{\log \sin x}$

Taking log both the sides

$\Rightarrow$ Log y = log $10^{\log \sin x}$

$\Rightarrow$ Log y = log sin x log 10 {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\log 10 \log \sin x)}{dx}$$

Now by using chain rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \log 10 \times \frac{d(\log \sin x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\sin x)}{dx} = \cos x$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log 10 \times \frac{1}{\sin x}\frac{d(\sin x)}{dx}$

$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\log 10}{\sin x}(\cos x)$

$\Rightarrow \frac{dy}{dx} = y\{\log 10 \cot x\}$

Put the value of $y = 10^{\log \sin x}$

$\Rightarrow \frac{dy}{dx} = 10^{\log \sin x}\{\log 10 \cot x\}$

11. $(\log x)^{\log x}$

Solution:

Let $y = (\log x)^{\log x}$

Taking log both the sides

$\Rightarrow$ Log y = log $(\log x)^{\log x}$

$\Rightarrow$ Log y = log x log (log x) $\{\log x^a = a \log x\}$

Differentiating with respect to x, then we get

$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\log x \log(\log x))}{dx}$

Now by using product rule, we get

$\Rightarrow \frac{d(\log y)}{dx} = \log x \times \frac{d(\log(\log x))}{dx} + \log(\log x) \times \frac{d(\log x)}{dx}$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \log x \times \frac{1}{\log x}\frac{d(\log x)}{dx} + \log\log x\left(\frac{1}{x}\frac{dx}{dx}\right)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\log x}{\log x}\left(\frac{1}{x}\frac{dx}{dx}\right) + \frac{\log(\log x)}{x}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{1}{x} + \frac{\log(\log x)}{x}\right\}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{1 + \log(\log x)}{x}\right\}$$

Put the value of $y = (\log x)^{\log x}$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{\log x}\left\{\frac{1 + \log(\log x)}{x}\right\}$$

12. $10^{(10^x)}$

Solution:

Let $y = 10^{(10^x)}$

Taking log both the sides

$\Rightarrow$ Log y = log $10^{(10^x)}$

$\Rightarrow$ Log y = 10 x log 10 {log x^a = a log x}

$\Rightarrow$ Log y = (10 log 10) x

Differentiating with respect to x,

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d\{(10\log 10)x\}}{dx}$$

Here 10 log 10 is a constant term, therefore by using chain rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = 10 \times \log(10) \times \frac{d(x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 10\log(10)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = 10\log(10)$$

$$\Rightarrow \frac{dy}{dx} = y\{10\log(10)\}$$

Put the value of y = $10^{(10^x)}$

$$\Rightarrow \frac{dy}{dx} = 10^{10x}\{10\log(10)\}$$

13. Sin (x^x)

Solution:

Let y = sin (x^x)

Take sin inverse both sides

⇒ $\sin^{-1} y = \sin^{-1} (\sin x^x)$

⇒ $\sin^{-1} y = x^x$

Taking log both the sides

⇒ Log ($\sin^{-1}$ y) = log x^x

⇒ Log ($\sin^{-1}$ y) = x log x {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log(\sin^{-1} y))}{dx} = \frac{d(x\log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log(\sin^{-1} y))}{dx} = x \times \frac{d(\log x)}{dx} + \log x \times \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{\sin^{-1} y} \frac{d(\sin^{-1} y)}{dx} = x \times \frac{1}{x}\frac{dx}{dx} + \log x$$

Again we have, $\frac{d(\sin^{-1} u)}{dx} = \frac{1}{\sqrt{1-u^2}} \frac{du}{dx}$ by using this result we get

$$\Rightarrow \frac{1}{\sin^{-1} y} \times \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = \frac{x}{x} + \log x$$

$$\Rightarrow \frac{1}{\sin^{-1} y(\sqrt{1-y^2})}\frac{dy}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dy}{dx} = \sin^{-1} y\left(\sqrt{1-y^2}\right)(1+\log x)$$

Put the value of y = sin (x^x)

$$\Rightarrow \frac{dy}{dx} = \sin^{-1} (\sin x^x)\left(\sqrt{1-\sin^2(x^x)}\right)(1+\log x)$$

From $\sin^2 x + \cos^2 x=1$, we can write as

$$\Rightarrow \frac{dy}{dx} = x^x\left(\sqrt{\cos^2(x^x)}\right)(1+\log x)$$

$$\Rightarrow \frac{dy}{dx} = x^x \cos x^x (1+\log x)$$

14. $(\text{Sin}^{-1} x)^x$

Solution:

Let $y = (\sin^{-1} x)^x$

Taking log both the sides

$\Rightarrow \text{Log } y = \log (\sin^{-1} x)^x$

$\Rightarrow \text{Log } y = x \log (\sin^{-1} x)$ $\{\log x^a = a \log x\}$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(x\log(\sin^{-1}x))}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = x \times \frac{d(\log(\sin^{-1}x))}{dx} + \log(\sin^{-1}x) \times \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{\sin^{-1}x}\frac{d(\sin^{-1}x)}{dx} + \log(\sin^{-1}x)$$

Again we have, $\frac{d(\sin^{-1}u)}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$ by using this result we get

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\sin^{-1}x} \times \frac{1}{\sqrt{1-x^2}}\frac{dx}{dx} + \log(\sin^{-1}x)$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{x}{\sin^{-1}x\sqrt{1-x^2}} + \log(\sin^{-1}x)$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{x}{\sin^{-1}x\sqrt{1-x^2}} + \log(\sin^{-1}x)\right\}$$

Put the value of $y = (\sin^{-1}x)^x$

$$\Rightarrow \frac{dy}{dx} = (\sin^{-1}x)^x\left\{\frac{x}{\sin^{-1}x\sqrt{1-x^2}} + \log(\sin^{-1}x)\right\}$$

15. $x^{\sin^{-1}x}$

Solution:

Let $y = x^{\sin^{-1}x}$

Taking log both the sides

$$\Rightarrow \log y = \log x^{\sin^{-1}x}$$

$\Rightarrow$ Log y = $\sin^{-1}x \log x$ $\{\log x^a = a \log x\}$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\sin^{-1}x\ \log x)}{dx}$$

By using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \sin^{-1}x \times \frac{d(\log x)}{dx} + \log x \times \frac{d(\sin^{-1}x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}\ ; \frac{d(\sin^{-1}u)}{dx} = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \sin^{-1}x \times \frac{1}{x}\frac{dx}{dx} + \log x \times \frac{1}{\sqrt{1-x^2}}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right\}$$

Put the value of $y = x^{\sin^{-1}x}$:

$$\Rightarrow \frac{dy}{dx} = x^{\sin^{-1}x}\left\{\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right\}$$

16. $(\tan x)^{1/x}$

Solution:

Let $y = (\tan x)^{\frac{1}{x}}$

Taking log both the sides, we get

$$\Rightarrow \log y = \log(\tan x)^{\frac{1}{x}}$$

$$\Rightarrow \log y = \frac{1}{x}\log\tan x \quad \{\log x^a = a\log x\}$$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d\left(\frac{1}{x}\log\tan x\right)}{dx}$$

By using product rule, we can write as

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{1}{x} \times \frac{d(\log\tan x)}{dx} + \log\tan x \times \frac{d(x^{-1})}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}; \frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x} \times \frac{1}{\tan x}\frac{d(\tan x)}{dx} + \log\tan x\,(-x^{-2})$$

Again we have $\frac{d(\tan x)}{dx} = \sec^2 x$ by using this result in the above expression

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{x\tan x}(\sec^2 x) - \frac{\log\tan x}{x^2}$$

$$\frac{dy}{dx} = y\left\{\frac{\sec^2 x}{x\tan x} - \frac{\log\tan x}{x^2}\right\}$$

Put the value of $y = (\tan x)^{\frac{1}{x}}$

$$\frac{dy}{dx} = (\tan x)^{\frac{1}{x}}\left\{\frac{\sec^2 x}{x\tan x} - \frac{\log\tan x}{x^2}\right\}$$

17. $x^{\tan^{-1} x}$

Solution:

Let $y = x^{\tan^{-1}x}$

Taking log both the sides

$\Rightarrow \log y = \log x^{\tan^{-1}x}$

$\Rightarrow$ Log y = $\tan^{-1}$ x log x {log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d(\tan^{-1}x\ \log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \tan^{-1}x \times \frac{d(\log x)}{dx} + \log x \times \frac{d(\tan^{-1}x)}{dx}$$

Again we know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\tan^{-1}u)}{dx} = \frac{1}{u^2+1}\frac{du}{dx}$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \tan^{-1}x \times \frac{1}{x}\frac{dx}{dx} + \log x \times \frac{1}{x^2+1}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{\tan^{-1}x}{x} + \frac{\log x}{x^2+1}$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{\tan^{-1}x}{x} + \frac{\log x}{x^2+1}\right\}$$

Put the value of $y = x^{\tan^{-1}x}$

$$\Rightarrow \frac{dy}{dx} = x^{\tan^{-1}x}\left\{\frac{\tan^{-1}x}{x} + \frac{\log x}{x^2+1}\right\}$$

18. (i) $(x^x)\sqrt{x}$

Solution:

Let $y = (x)^x\sqrt{x}$

Taking log both the sides

$$\Rightarrow \log y = \log(x)^x\sqrt{x}$$

$$\Rightarrow \log y = \log(x)^x + \log\sqrt{x}$$ {Log (ab) = log a +log b}

$$\Rightarrow \log y = \log(x)^x + \log x^{\frac{1}{2}}$$

$$\Rightarrow \log y = x\log x + \frac{1}{2}\log x$$ {Log x^a = a log x}

$$\Rightarrow \log y = \left(x+\frac{1}{2}\right)\log x$$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log y)}{dx} = \frac{d\left(\left(x+\frac{1}{2}\right)\log x\right)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log y)}{dx} = \left(x+\frac{1}{2}\right)\times\frac{d(\log x)}{dx} + \log x \times \frac{d\left(x+\frac{1}{2}\right)}{dx}$$

Again we have to use chain rule for the above expression,

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \left(x+\frac{1}{2}\right)\times\frac{1}{x}\frac{dx}{dx} + \log x\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{(2x+1)}{2}\times\frac{1}{x} + \log x$$

$$\Rightarrow \frac{dy}{dx} = y\left\{\frac{(2x+1)}{2x} + \log x\right\}$$

Put the value of $y = (x)^x\sqrt{x}$

$$\Rightarrow \frac{dy}{dx} = (x)^x\sqrt{x}\left\{\frac{(2x+1)}{2x} + \log x\right\}$$

$$\Rightarrow \frac{dy}{dx} = (x)^x\sqrt{x}\left\{\frac{2x}{2x} + \frac{1}{2x} + \log x\right\}$$

$$\Rightarrow \frac{dy}{dx} = (x)^x\sqrt{x}\left\{1 + \frac{1}{2x} + \log x\right\}$$

18. (ii) $x^{(\sin x - \cos x)} + \frac{x^2-1}{x^2+1}$

Solution:

Let $y = x^{(\sin x - \cos x)} + \frac{x^2-1}{x^2+1}$

$\Rightarrow y = a + b$

where $a = x^{(\sin x - \cos x)}; b = \dfrac{x^2 - 1}{x^2 + 1}$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$a = x^{(\sin x - \cos x)}$

Taking log both the sides to the above expressions we get

$$\Rightarrow \log a = \log x^{(\sin x - \cos x)}$$

$\Rightarrow \log a = (\sin x - \cos x) \log x$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d((\sin x - \cos x)\log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log a)}{dx} = (\sin x - \cos x) \times \frac{d(\log x)}{dx} + \log x \times \frac{d(\sin x - \cos x)}{dx}$$

To the above expression we have to use chain rule,

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = (\sin x - \cos x) \times \frac{1}{x}\frac{dx}{dx} + \log x\left(\frac{d(\sin x)}{dx} - \frac{d(\cos x)}{dx}\right)$$

We know that $\dfrac{d(\cos x)}{dx} = -\sin x; \dfrac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{(\sin x - \cos x)}{x} + \log x(\cos x - (-\sin x))$$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{(\sin x - \cos x)}{x} + \log x(\cos x + \sin x)$$

$$\Rightarrow \frac{da}{dx} = a\left\{\frac{\sin x - \cos x}{x} + \log x\,(\cos x + \sin x)\right\}$$

Put the value of a = $x^{(\sin x - \cos x)}$

$$\Rightarrow \frac{da}{dx} = x^{(\sin x - \cos x)}\left\{\frac{\sin x - \cos x}{x} + \log x\,(\cos x + \sin x)\right\}$$

$$b = \frac{x^2 - 1}{x^2 + 1}$$

To differentiate above expression with respect to x we have to use quotient rule,

$$\Rightarrow \frac{db}{dx} = \frac{(x^2+1)\frac{d(x^2-1)}{dx} - (x^2-1)\,\frac{d(x^2+1)}{dx}}{(x^2+1)^2}$$

Now by using chain rule, we get

$$\Rightarrow \frac{db}{dx} = \frac{(x^2+1)(2x) - (x^2-1)\,(2x)}{(x^2+1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{(2x^3+2x) - (2x^3-2x)}{(x^2+1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{(2x^3+2x-2x^3+2x)}{(x^2+1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{4x}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

Now by substituting all the values in above expressions we get

$$\Rightarrow \frac{dy}{dx} = x^{(\sin x - \cos x)}\left\{\frac{\sin x - \cos x}{x} + \log x\,(\cos x + \sin x)\right\} + \frac{4x}{(x^2+1)^2}$$

$$18.(iii)\ x^{x\cos x} + \frac{x^2+1}{x^2-1}$$

Solution:

$$\text{Let } y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$$

$\Rightarrow y = a + b$

$$\text{where } a = x^{x\cos x};\ b = \frac{x^2+1}{x^2-1}$$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$a = x^{x\cos x}$

Taking log both the sides to the above equation we get

$\Rightarrow \log a = \log x^{x\cos x}$

$\Rightarrow \log a = x\cos x\log x$

$\{\text{Log } x^a = a \log x\}$

Differentiating with respect to x,

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d(x\cos x\log x)}{dx}$$

Now by using product rule, we can write as

$$\Rightarrow \frac{d(\log a)}{dx} = x\cos x \times \frac{d(\log x)}{dx} + \log x \times \frac{d(x\cos x)}{dx}$$

$$\Rightarrow \frac{d(\log a)}{dx} = x\cos x \times \frac{d(\log x)}{dx} + \log x\left\{x\frac{d(\cos x)}{dx} + \cos x\right\}$$

Again we have, $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$ by using this result in the above expressions we get

$\Rightarrow \frac{1}{a}\frac{da}{dx} = x\cos x \times \frac{1}{x}\frac{dx}{dx} + \log x\{x(-\sin x) + \cos x\}$

We know that $\frac{d(\cos x)}{dx} = -\sin x\,;\ \frac{d(\sin x)}{dx} = \cos x$

$\Rightarrow \frac{1}{a}\ \frac{da}{dx} = \frac{x\cos x}{x} + \log x(\cos x - x\sin x)$

$\Rightarrow \frac{da}{dx} = a\{\cos x + \log x(\cos x - x\sin x)\}$

Put the value of a = $x^{x\cos x}$:

$\Rightarrow \frac{da}{dx} = x^{x\cos x}\{\cos x + \log x(\cos x - x\sin x)\}$

$\Rightarrow \frac{da}{dx} = x^{x\cos x}\{\cos x + \log x\cos x - x\sin x\log x\}$

$\Rightarrow \frac{da}{dx} = x^{x\cos x}\{\cos x\,(1 + \log x) - x\sin x\log x\}$

$b = \frac{x^2+1}{x^2-1}$

Now we have to differentiate above expression using quotient rule, then we get

$$\Rightarrow \frac{db}{dx} = \frac{(x^2-1)\frac{d(x^2+1)}{dx} - (x^2+1)\ \frac{d(x^2-1)}{dx}}{(x^2-1)^2}$$

Now apply chain rule for the above equation,

$$\Rightarrow \frac{db}{dx} = \frac{(x^2-1)(2x) - (x^2+1)\ (2x)}{(x^2+1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{(2x^3-2x) - (2x^3+2x)}{(x^2+1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{(2x^3-2x-2x^3-2x)}{(x^2+1)^2}$$

$$\Rightarrow \frac{db}{dx} = \frac{-4x}{(x^2+1)^2}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

By substituting all values in the above expression we get

$$\Rightarrow \frac{dy}{dx} = x^{x\cos x}\{\cos x\,(1+\log x) - x\sin x\log x\} - \frac{4x}{(x^2+1)^2}$$

18. $(iv)\ (x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

Solution:

Let $y = (x\cos x)^x + (x\sin x)^{\frac{1}{x}}$

$\Rightarrow y = a + b$

where $a = (x\cos x)^x; b = (x\sin x)^{\frac{1}{x}}$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$a = (x\cos x)^x$

Taking log both the sides, we get

$\Rightarrow \log a = \log(x\cos x)^x$

$\Rightarrow \log a = x\log(x\cos x)$

$\{\text{Log } x^a = a \log x\}$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d(x\log(x\cos x))}{dx}$$

By using product rule, we get

$$\Rightarrow \frac{d(\log a)}{dx} = x \times \frac{d(\log(x\cos x))}{dx} + \log(x\cos x) \times \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = x \times \frac{1}{x\cos x}\frac{d(x\cos x)}{dx} + \log(x\cos x)$$

Again by using product rule, we can write as

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{x}{x\cos x}\left\{x\frac{d(\cos x)}{dx} + \cos x\right\} + \log(x\cos x)$$

We have $\frac{d(\cos x)}{dx} = -\sin x$ using this result we can write as

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{1}{\cos x}\{x(-\sin x) + \cos x\} + \log(x\cos x)$$

$$\Rightarrow \frac{da}{dx} = a\left\{\frac{\cos x - x\sin x}{\cos x} + \log(x\cos x)\right\}$$

Put the value of $a = (x\cos x)^x$:

$$\Rightarrow \frac{da}{dx} = (x\cos x)^x\left\{\frac{\cos x - x\sin x}{\cos x} + \log(x\cos x)\right\}$$

$$\Rightarrow \frac{da}{dx} = (x\cos x)^x\{1 - x\tan x + \log(x\cos x)\}$$

$$b = (x\sin x)^{\frac{1}{x}}$$

Taking log both the sides

$$\Rightarrow \log b = \log(x\sin x)^{\frac{1}{x}}$$

$$\Rightarrow \log b = \frac{1}{x}\log(x\sin x)$$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d\left(\frac{1}{x}\log(x\sin x)\right)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{1}{x} \times \frac{d(\log(x\sin x))}{dx} + \log(x\sin x) \times \frac{d(x^{-1})}{dx}$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{1}{x} \times \frac{1}{x\sin x}\frac{d(x\sin x)}{dx} + \log(x\sin x)\,(-x^{-2})$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{1}{x^2\sin x}\left(x\frac{d(\sin x)}{dx} + \sin x\frac{dx}{dx}\right) - \frac{\log(x\sin x)}{x^2}$$

We know that $\frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{x\cos x + \sin x}{x^2\sin x} - \frac{\log(x\sin x)}{x^2}\right\}$$

Put the value of $b = (x\sin x)^{\frac{1}{x}}$:

$$\Rightarrow \frac{db}{dx} = (x\sin x)^{\frac{1}{x}}\left\{\frac{x\cos x + \sin x}{x^2\sin x} - \frac{\log(x\sin x)}{x^2}\right\}$$

$$\Rightarrow \frac{db}{dx} = (x\sin x)^{\frac{1}{x}}\left\{\frac{x\cot x + 1}{x^2} - \frac{\log(x\sin x)}{x^2}\right\}$$

$$\Rightarrow \frac{db}{dx} = (x\sin x)^{\frac{1}{x}}\left\{\frac{x\cot x + 1 - \log(x\sin x)}{x^2}\right\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

Now by substituting all the values in above expression we get

$$\Rightarrow \frac{dy}{dx} = (x\cos x)^x\{1 - x\tan x + \log(x\cos x)\} + (x\sin x)^{\frac{1}{x}}\left\{\frac{x\cot x + 1 - \log(x\sin x)}{x^2}\right\}$$

18. (v) $\left(x + \frac{1}{x}\right)^x + x^{\left(1+\frac{1}{x}\right)}$

Solution:

Let $y = \left(x+\frac{1}{x}\right)^{x} + x^{\left(1+\frac{1}{x}\right)}$

$\Rightarrow y = a + b$

where $a = \left(x+\frac{1}{x}\right)^{x}; b = x^{\left(1+\frac{1}{x}\right)}$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$a = \left(x+\frac{1}{x}\right)^{x}$$

Taking log both the sides, we get

$$\Rightarrow \log a = \log\left(x+\frac{1}{x}\right)^{x}$$

$$\Rightarrow \log a = x\log\left(x+\frac{1}{x}\right) \quad \{\text{Log } x^a = a \log x\}$$

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d\left(x\log\left(x+\frac{1}{x}\right)\right)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log a)}{dx} = x \times \frac{d\left(\log\left(x+\frac{1}{x}\right)\right)}{dx} + \log\left(x+\frac{1}{x}\right) \times \frac{dx}{dx}$$

Again we know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$.

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = x \times \frac{1}{x+\frac{1}{x}}\frac{d\left(x+\frac{1}{x}\right)}{dx} + \log\left(x+\frac{1}{x}\right)$$

Again by using chain rule in the above expression we get

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{x}{\frac{x^2+1}{x}}\left\{\frac{dx}{dx} + \frac{d\left(\frac{1}{x}\right)}{dx}\right\} + \log\left(x+\frac{1}{x}\right)$$

By using $\frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{x^2}{x^2+1}\left\{1 + \left(-\frac{1}{x^2}\right)\right\} + \log\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \frac{da}{dx} = a\left\{\frac{x^2}{x^2+1}\left\{1 - \frac{1}{x^2}\right\} + \log\left(x+\frac{1}{x}\right)\right\}$$

Put the value of $a = \left(x+\frac{1}{x}\right)^x$:

$$\Rightarrow \frac{da}{dx} = \left(x+\frac{1}{x}\right)^x\left\{\frac{x^2}{x^2+1}\left\{1 - \frac{1}{x^2}\right\} + \log\left(x+\frac{1}{x}\right)\right\}$$

$$\Rightarrow \frac{da}{dx} = \left(x+\frac{1}{x}\right)^x\left\{\frac{x^2}{x^2+1} - \frac{1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right\}$$

$$\Rightarrow \frac{da}{dx} = \left(x+\frac{1}{x}\right)^x\left\{\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right\}$$

$b = x^{\left(1+\frac{1}{x}\right)}$

Taking log both the sides

$$\Rightarrow \log b = \log x^{\left(1+\frac{1}{x}\right)}$$

$$\Rightarrow \log b = \left(1+\frac{1}{x}\right)\log x$$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d\left(\left(1+\frac{1}{x}\right)\log x\right)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log b)}{dx} = \left(1+\frac{1}{x}\right) \times \frac{d(\log x)}{dx} + \log x \times \frac{d\left(1+\frac{1}{x}\right)}{dx}$$

Again for the above expression we have to apply chain rule,

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x+1}{x} \times \frac{1}{x}\frac{dx}{dx} + \log x \left(\frac{d(1)}{dx} + \frac{d\left(\frac{1}{x}\right)}{dx}\right)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x+1}{x^2} + \log x \left(-\frac{1}{x^2}\right)$$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{x+1}{x^2} - \frac{\log x}{x^2}\right\}$$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{x+1-\log x}{x^2}\right\}$$

Put the value of $b = x^{\left(1+\frac{1}{x}\right)}$:

$$\Rightarrow \frac{db}{dx} = x^{\left(1+\frac{1}{x}\right)}\left\{\frac{x+1-\log x}{x^2}\right\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

Now by substituting the all the values in above expression we get

$$\Rightarrow \frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left\{\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right\} + x^{\left(1+\frac{1}{x}\right)}\left\{\frac{x+1-\log x}{x^2}\right\}$$

18. (vi) $e^{\sin x} + (\tan x)^x$

Solution:

Let $y = e^{\sin x} + (\tan x)^x$

$\Rightarrow y = a + b$

Where $a = e^{\sin x}$; $b = (\tan x)^x$

Now we have to differentiate $y = a + b$ with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

a= $e^{\sin x}$

Taking log both the sides, we get

⇒ Log a= log $e^{\sin x}$

⇒ Log a= sin x log e {Log x^a = a log x}

⇒ Log a= sin x {log e =1}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d(\sin x)}{dx}$$

Again we have $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$; $\frac{d(\sin x)}{dx} = \cos x$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \cos x$$

$$\Rightarrow \frac{da}{dx} = a\,(\cos x)$$

Put the value of a = $e^{\sin x}$

$$\Rightarrow \frac{da}{dx} = e^{\sin x}\cos x$$

b = $(\tan x)^x$

Taking log both the sides:

⇒ Log b= log $(\tan x)^x$

⇒ Log b= x log (tan x) {Log x^a = a log x}

Differentiating with respect to x:

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d(x\log(\tan x))}{dx}$$

Again by using product rule,

$$\Rightarrow \frac{d(\log b)}{dx} = x \times \frac{d(\log(\tan x))}{dx} + \log(\tan x) \times \frac{dx}{dx}$$

We know that $\frac{d(\tan x)}{dx} = \sec^2 x$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = x \times \frac{1}{\tan x}\frac{d(\tan x)}{dx} + \log(\tan x)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x}{\tan x}(\sec^2 x) + \log(\tan x)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x\cos x}{\sin x}\left(\frac{1}{\cos^2 x}\right) + \log(\tan x)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x}{\sin x}\left(\frac{1}{\cos x}\right) + \log(\tan x)$$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{x}{\sin x\cos x} + \log(\tan x)\right\}$$

Put the value of b = $(\tan x)^x$

$$\Rightarrow \frac{db}{dx} = (\tan x)^x\left\{\frac{x}{\sin x\cos x} + \log(\tan x)\right\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$\Rightarrow \frac{dy}{dx} = e^{\sin x}\cos x + (\tan x)^x\left\{\frac{x}{\sin x\cos x} + \log(\tan x)\right\}$$

18. (vii) $(\cos x)^x + (\sin x)^{1/x}$

Solution:

Let $y = (\cos x)^x + (\sin x)^{\frac{1}{x}}$

$\Rightarrow y = a + b$

where $a = (\cos x)^x; b = (\sin x)^{\frac{1}{x}}$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$a = (\cos x)^x$$

Taking log both the sides

$$\Rightarrow \log a = \log(\cos x)^x$$

$$\Rightarrow \log a = x\log(\cos x)$$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d(x\log(\cos x))}{dx}$$

Now by using product rule, we have

$$\Rightarrow \frac{d(\log a)}{dx} = x \times \frac{d(\log(\cos x))}{dx} + \log(\cos x) \times \frac{dx}{dx}$$

Again we have $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = x \times \frac{1}{\cos x}\frac{d(\cos x)}{dx} + \log(\cos x)$$

We know that $\frac{d(\cos x)}{dx} = -\sin x$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{x}{\cos x}(-\sin x) + \log(\cos x)$$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{-x\sin x}{\cos x} + \log(\cos x)$$

$$\Rightarrow \frac{da}{dx} = a\{-x\tan x + \log(\cos x)\}$$

Put the value of a = $(\cos x)^{x}$:

$$\Rightarrow \frac{da}{dx} = (\cos x)^{x}\{-x\tan x + \log(\cos x)\}$$

$b = (\sin x)^{\frac{1}{x}}$

Taking log both the sides

$$\Rightarrow \log b = \log(\sin x)^{\frac{1}{x}}$$

$$\Rightarrow \log b = \frac{1}{x}\log(\sin x)$$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d\left(\frac{1}{x}\log(\sin x)\right)}{dx}$$

Again by product rule we have

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{1}{x} \times \frac{d(\log(\sin x))}{dx} + \log(\sin x) \times \frac{d(x^{-1})}{dx}$$

We know that $\frac{d(u^n)}{dx} = nu^{n-1}\frac{du}{dx}$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{1}{x} \times \frac{1}{\sin x}\frac{d(\sin x)}{dx} + \log(\sin x)\,(-x^{-2})$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{1}{x\sin x}(\cos x) - \frac{\log(\sin x)}{x^2}$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{\cos x}{x\sin x} - \frac{\log(\sin x)}{x^2}$$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right\}$$

Put the value of b = $(\sin x)^{\frac{1}{x}}$:

$$\Rightarrow \frac{db}{dx} = (\sin x)^{\frac{1}{x}}\left\{\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$\Rightarrow \frac{dy}{dx} = (\cos x)^x\{-x\tan x + \log(\cos x)\} + (\sin x)^{\frac{1}{x}}\left\{\frac{\cot x}{x} - \frac{\log(\sin x)}{x^2}\right\}$$

18. $(viii)\ x^{x^2-3} + (x-3)^{x^2}$

Solution:

Let $y = x^{x^2-3} + (x-3)^{x^2}$

$\Rightarrow y = a + b$

where $a = x^{x^2-3}; b = (x-3)^{x^2}$

Now we have to differentiate y = a + b with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$a = x^{x^2-3}$

Taking log both the sides

$\Rightarrow \log a = \log x^{x^2-3}$

$\Rightarrow \log a = (x^2-3)\log x$ {Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d((x^2-3)\log x)}{dx}$$

Now by using product rule,

$$\Rightarrow \frac{d(\log a)}{dx} = (x^2-3) \times \frac{d(\log x)}{dx} + \log x \times \frac{d(x^2-3)}{dx}$$

Again by using chain rule we get

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = (x^2 - 3) \times \frac{1}{x}\frac{dx}{dx} + \log x \times (2x)$$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{(x^2 - 3)}{x} + 2x \log x$$

$$\Rightarrow \frac{da}{dx} = a\left\{\frac{(x^2 - 3)}{x} + 2x \log x\right\}$$

Put the value of $a = x^{x^2-3}$:

$$\Rightarrow \frac{da}{dx} = x^{x^2-3}\left\{\frac{(x^2 - 3)}{x} + 2x \log x\right\}$$

$b = (x - 3)^{x^2}$

Taking log both the sides:

$$\Rightarrow \log b = (x - 3)^{x^2}$$

$\Rightarrow \log b = x^2 \log(x - 3)$ {Log x^a = a log x}

Differentiating with respect to x:

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d(x^2 \log(x - 3))}{dx}$$

Again by using product rule, we get

$$\Rightarrow \frac{d(\log b)}{dx} = x^2 \times \frac{d(\log\ (x - 3))}{dx} + \log(x - 3) \times \frac{d(x^2)}{dx}$$

For the above expression now we have to use chain rule,

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = x^2 \times \frac{1}{(x - 3)}\frac{d(x - 3)}{dx} + \log(x - 3) \times (2x)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x^2}{(x - 3)}\left(\frac{dx}{dx} - \frac{d(3)}{dx}\right) + 2x \log(x - 3)$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = \frac{x^2}{(x - 3)}(1) + 2x \log\ (x - 3)$$

$$\Rightarrow \frac{db}{dx} = b\left\{\frac{x^2}{(x-3)} + 2x\log(x-3)\right\}$$

Put the value of $b = (x-3)^{x^2}$:

$$\Rightarrow \frac{db}{dx} = (x-3)^{x^2}\left\{\frac{x^2}{(x-3)} + 2x\log(x-3)\right\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx}$$

$$\Rightarrow \frac{dy}{dx} = x^{x^2-3}\left\{\frac{(x^2-3)}{x} + 2x\log x\right\} + (x-3)^{x^2}\left\{\frac{x^2}{(x-3)} + 2x\log(x-3)\right\}$$

19. $y = e^x + 10^x + x^x$

Solution:

Let $y = e^x + 10^x + x^x$

$\Rightarrow y = a + b + c$

Where $a = e^x$; $b = 10^x$; $c = x^x$

Now we have to differentiate y = a + b + c with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx} + \frac{dc}{dx}$$

$a = e^x$

Taking log both the sides

$\Rightarrow$ Log a= Log e^x

$\Rightarrow$ Log a= x log e

{Log x^a = a log x}

$\Rightarrow$ Log a= x {log e =1}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{dx}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = 1$$

$$\Rightarrow \frac{da}{dx} = a$$

Put the value of $a = e^x$

$$\Rightarrow \frac{da}{dx} = e^x$$

$b = 10^x$

Taking log both the sides:

$\Rightarrow$ Log b= log 10^x

$\Rightarrow$ Log b= x log 10

{Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log b)}{dx} = \frac{d(x \log 10)}{dx}$$

Now by using chain rule,

$$\Rightarrow \frac{d(\log b)}{dx} = \log 10 \times \frac{dx}{dx}$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = b(\log 10)$$

$$\Rightarrow \frac{db}{dx} = b(\log 10)$$

Put the value of $b = 10^x$

$$\Rightarrow \frac{db}{dx} = 10^x(\log 10)$$

$c = x^x$

Taking log both the sides

$\Rightarrow$ Log c= log x^x

$\Rightarrow$ Log c= x log x

{Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log c)}{dx} = \frac{d(x \log x)}{dx}$$

By using product rule, we get

$$\Rightarrow \frac{d(\log c)}{dx} = x \times \frac{d(\log x)}{dx} + \log x \times \frac{dx}{dx}$$

$$\Rightarrow \frac{1}{c}\frac{dc}{dx} = x \times \frac{1}{x}\frac{dx}{dx} + \log x$$

$$\Rightarrow \frac{1}{c}\frac{dc}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dc}{dx} = c\{1 + \log x\}$$

Put the value of $c = x^x$

$$\Rightarrow \frac{dc}{dx} = x^x\{1 + \log x\}$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx} + \frac{dc}{dx}$$

$$\Rightarrow \frac{dy}{dx} = e^x + 10^x(\log 10) + x^x\{1 + \log x\}$$

20. $y = x^n + n^x + x^x + n^n$

Solution:

Let $y = x^n + n^x + x^x + n^n$

$\Rightarrow y = a + b + c + m$

Where $a= x^n$; $b = n^x$; $c = x^x$; $m= n^n$

Now we have to differentiate y = a + b + c + m with respect to x

By using chain rule, we can write as

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx} + \frac{dc}{dx} + \frac{dm}{dx}$$

$a= x^n$

Taking log both the sides

$\Rightarrow$ Log a= log x^n

$\Rightarrow$ Log a= n log x

{Log x^a = a log x}

$\Rightarrow$ Log a= n log x {log e =1}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log a)}{dx} = \frac{d(n \log x)}{dx}$$

Again by chain rule, we can write as

$$\Rightarrow \frac{d(\log a)}{dx} = n\frac{d(\log x)}{dx}$$

We know that $\frac{d(\log u)}{dx} = \frac{1}{u}\frac{du}{dx}$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = n \times \frac{1}{x}\frac{dx}{dx}$$

$$\Rightarrow \frac{1}{a}\frac{da}{dx} = \frac{n}{x}$$

$$\Rightarrow \frac{da}{dx} = \frac{an}{x}$$

Put the value of a = x^n

$$\frac{da}{dx} = \frac{nx^n}{x}$$

$$\frac{da}{dx} = nx^{n-1}$$

b = n^x

Taking log both the sides

⇒ Log b= log n^x

⇒ Log b= x log n {Log x^a = a log x}

Differentiating with respect to x using chain rule, we get

$$\Rightarrow \frac{d(\log b)}{dx} = \log n \times \frac{dx}{dx}$$

$$\Rightarrow \frac{1}{b}\frac{db}{dx} = b(\log n)$$

$$\Rightarrow \frac{db}{dx} = b(\log n)$$

Put the value of b = n^x

$$\Rightarrow \frac{db}{dx} = n^x(\log n)$$

c = x^x

Taking log both the sides

⇒ Log c= log x^x

⇒ Log c= x log x

{Log x^a = a log x}

Differentiating with respect to x

$$\Rightarrow \frac{d(\log c)}{dx} = \frac{d(x \log x)}{dx}$$

Now by using product rule, we get

$$\Rightarrow \frac{d(\log c)}{dx} = x \times \frac{d(\log x)}{dx} + \log x \times \frac{dx}{dx}$$

$$\Rightarrow \frac{1}{c}\frac{dc}{dx} = x \times \frac{1}{x}\frac{dx}{dx} + \log x$$

$$\Rightarrow \frac{1}{c}\frac{dc}{dx} = 1 + \log x$$

$$\Rightarrow \frac{dc}{dx} = c\{1 + \log x\}$$

Put the value of $c = x^x$

$$\Rightarrow \frac{dc}{dx} = x^x\{1 + \log x\}$$

$m = n^n$

$$\Rightarrow \frac{dm}{dx} = \frac{d(n^n)}{dx}$$

$$\Rightarrow \frac{dm}{dx} = 0$$

$$\frac{dy}{dx} = \frac{da}{dx} + \frac{db}{dx} + \frac{dc}{dx} + \frac{dm}{dx}$$

$$\Rightarrow \frac{dy}{dx} = nx^{n-1} + n^x(\log n) + x^x\{1 + \log x\} + 0$$

$$\Rightarrow \frac{dy}{dx} = nx^{n-1} + n^x(\log n) + x^x\{1 + \log x\}$$