EXERCISE 11.6

1. $If\ y=\sqrt{x+\sqrt{x+\sqrt{x+.....\ to\ \infty}}},\ prove\ that\ \frac{dy}{dx}=\frac{1}{2y-1}.$

Solution:

Given,

$$y=\sqrt{x+\sqrt{x+\sqrt{x+\cdots\text{to }\infty}}}$$

$$y=\sqrt{x+y}$$

Where $y=\sqrt{x+\sqrt{x+\cdots\text{to }\infty}}$

On squaring both sides,

$y^2 = x + y$

Differentiating both sides with respect to x,

$$2y\frac{dy}{dx}=1+\frac{dy}{dx}$$

$$\frac{dy}{dx}(2y-1)=1$$

$$\frac{dy}{dx}=\frac{1}{2y-1}$$

Hence proved.

2. $If\ y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x+.....\ to\ \infty}}},\ prove\ that\ \frac{dy}{dx}=\frac{\sin x}{1-2y}.$

Solution:

Given,

$$y=\sqrt{\cos x+\sqrt{\cos x+\sqrt{\cos x+\cdots\text{to }\infty}}}$$

$y = \sqrt{\cos x + y}$

Where $y = \sqrt{\cos x + \sqrt{\cos x + \cdots \text{to } \infty}}$

Squaring on both sides,

$y^2 = \cos x + y$

Differentiating both sides with respect to x,

$$2y\frac{dy}{dx} = -\sin x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y - 1) = -\sin x$$

$$\frac{dy}{dx} = -\frac{\sin x}{2y - 1}$$

$$\frac{dy}{dx} = \frac{\sin x}{1 - 2y}$$

Hence proved.

3. $If\ y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \ldots.. \text{ to } \infty}}},\ prove\ that\ (2y - 1)\frac{dy}{dx} = \frac{1}{x}.$

Solution:

Given

$$y = \sqrt{\log x + \sqrt{\log x + \sqrt{\log x + \cdots \text{to } \infty}}}$$

$y = \sqrt{\log x + y}$

Where $y = \sqrt{\log x + \sqrt{\log x + \cdots \text{to } \infty}}$

Squaring on both sides,

$y^2 = \text{Log } x + y$

Differentiating both sides with respect to x,

$$2y\frac{dy}{dx} = \frac{1}{x} + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y-1) = \frac{1}{x}$$

$$\frac{dy}{dx} = \frac{1}{x(2y-1)}$$

Hence proved.

4. *If* $y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \ldots.. \text{ to } \infty}}}$, *prove that* $\frac{dy}{dx} = \frac{\sec^2 x}{2y-1}$.

Solution:

Given,

$$y = \sqrt{\tan x + \sqrt{\tan x + \sqrt{\tan x + \cdots \text{to } \infty}}}$$

$$y = \sqrt{\tan x + y}$$

On squaring both sides,

$y^2 = \tan x + y$

Differentiating both sides with respect to x,

$$2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\frac{dy}{dx}(2y-1) = \sec^2 x$$

$$\frac{dy}{dx} = \frac{\sec^2 x}{(2y-1)}$$

Hence proved.