EXERCISE 13.1 PAGE NO: 13.4

1. Find the rate of change of the total surface area of a cylinder of radius r and height h, when the radius varies.

Solution:

We know that total Surface Area of Cylinder = $2\pi r^2 + 2\pi r h$

Given that radius of the cylinder varies.

Therefore, we need to find $\frac{dS}{dr}$

Where S = Surface Area of Cylinder and r = radius of Cylinder.

$$\frac{dS}{dr} = 4\pi r + 2\pi h$$

Hence, Rate of change of total surface area of the cylinder when the radius is varying is given by $(4\pi r + 2\pi h)$.

2. Find the rate of change of the volume of a sphere with respect to its diameter.

Solution:

We know that the volume of a Sphere = $\frac{1}{6}\pi D^3$

Where D = diameter of the Sphere

We need to find, $\frac{dV}{dD}$ where V = Volume of the sphere and D = Diameter of the Sphere.

$$\frac{dV}{dD} = \frac{\pi D^2}{2}$$

Hence, Rate of change of volume of sphere with respect to the diameter of the sphere is given by $\frac{\pi D^2}{2}$.

3. Find the rate of change of the volume of a sphere with respect to its surface area

when the radius is 2 cm.

Solution:

We know that volume of Sphere = $\frac{4}{3}\pi r^3$

And surface Area of Sphere = $4\pi r^2$

We need to find, $\frac{dV}{dS}$ where V = Volume of the Sphere and S = Surface Area of the Sphere.

$$\frac{dV}{dS} = \frac{dV}{dr} \times \frac{dr}{dS}$$

$$\frac{dV}{dr} = 4\pi r^2$$

$$\frac{dS}{dr} = 8\pi r$$

$$\frac{dV}{dS} = \frac{4\pi r^2}{8\pi r}$$

$$\frac{dV}{dS} = \frac{r}{2}$$

$$\left(\frac{dV}{dS}\right)_{\text{at } r=2} = \frac{2}{2} = 1 \text{ cm}$$

4. Find the rate of change of the area of a circular disc with respect to its circumference when the radius is 3 cm.

Solution:

We know that area of a Circular disc = πr^2 and circumference of a Circular disc = $2\pi r$

Where r = radius of Circular Disc.

Now we have to find $\frac{dA}{dC}$ where A = Area of Circular disc and C = Circumference of the Circular disk.

$$\frac{dA}{dC} = \frac{dA}{dr} \times \frac{dr}{dC}$$

$$\frac{dA}{dr} = 2\pi r$$

$$\frac{dC}{dr} = 2\pi$$

$$\frac{dA}{dC} = \frac{2\pi r}{2\pi} = r$$

$$\left(\frac{dA}{dr}\right)_{\text{at } x=3} = 3 \text{ cm}$$

5. Find the rate of change of the volume of a cone with respect to the radius of its base.

Solution:

We know that the volume of Cone = $\frac{1}{3}\pi r^2 h$

Where r = radius of the cone

h = height of the cone

We have to find, $\frac{dV}{dr}$ where V = Volume of cone and r = radius of the cone.

$$\frac{dV}{dr} = \frac{2}{3}\pi rh$$