EXERCISE 17.1 PAGE NO: 17.10

1. Prove that the function $f(x) = \log_e x$ is increasing on $(0, \infty)$.

Solution:

Let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$\Rightarrow \log_e x_1 < \log_e x_2$

$\Rightarrow f(x_1) < f(x_2)$

So, f(x) is increasing in $(0, \infty)$

2. Prove that the function $f(x) = \log_a x$ is increasing on $(0, \infty)$ if $a > 1$ and decreasing on $(0, \infty)$, if $0 < a < 1$.

Solution:

Case I

When $a > 1$

Let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$\Rightarrow \log_e x_1 < \log_e x_2$

$\Rightarrow f(x_1) < f(x_2)$

So, f(x) is increasing in $(0, \infty)$

Case II

When $0 < a < 1$

$f(x) = \log_a x = \frac{\log x}{\log a}$

When $a < 1 \Rightarrow \log a < 0$

Let $x_1 < x_2$

$\Rightarrow \log x_1 < \log x_2$

$\Rightarrow \frac{\log x_1}{\log a} > \frac{\log x_2}{\log a}$ $[\because \log a < 0]$

$\Rightarrow f(x_1) > f(x_2)$

So, f(x) is decreasing in $(0, \infty)$

3. Prove that f(x) = ax + b, where a, b are constants and a > 0 is an increasing function on R.

Solution:

Given,

$f(x) = ax + b, a > 0$

Let $x_1, x_2 \in R$ and $x_1 > x_2$

$\Rightarrow ax_1 > ax_2$ for some $a > 0$

$\Rightarrow ax_1 + b > ax_2 + b$ for some b

$\Rightarrow f(x_1) > f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

So, f(x) is increasing function of R

4. Prove that f(x) = ax + b, where a, b are constants and a < 0 is a decreasing function on R.

Solution:

Given,

$f(x) = ax + b, a < 0$

Let $x_1, x_2 \in R$ and $x_1 > x_2$

$\Rightarrow ax_1 < ax_2$ for some $a > 0$

$\Rightarrow ax_1 + b < ax_2 + b$ for some b

$\Rightarrow f(x_1) < f(x_2)$

Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$

So, f(x) is decreasing function of R