

EXERCISE 17.2

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1. Find the intervals in which the following functions are increasing or decreasing.

(i) $f(x) = 10 - 6x - 2x^2$

Solution:

Given $f(x) = 10 - 6x - 2x^2$

By differentiating above equation we get,

$$\Rightarrow f'(x) = \frac{d}{dx}(10 - 6x - 2x^2)$$

$$\Rightarrow f'(x) = -6 - 4x$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow -6 - 4x > 0$$

$$\Rightarrow -4x > 6$$

$$\Rightarrow x < -\frac{6}{4}$$

$$\Rightarrow x < -\frac{3}{2}$$

$$\Rightarrow x \in \left(-\infty, -\frac{3}{2}\right)$$

Thus $f(x)$ is increasing on the interval $\left(-\infty, -\frac{3}{2}\right)$

Again, for $f(x)$ to be decreasing, we must have

$$f'(x) < 0$$

$$\Rightarrow -6 - 4x < 0$$

$$\Rightarrow -4x < 6$$

$$\Rightarrow x > -\frac{6}{4}$$

$$\Rightarrow x > -\frac{3}{2}$$

$$\Rightarrow x \in \left(-\frac{3}{2}, \infty\right)$$

Thus $f(x)$ is decreasing on interval $x \in \left(-\frac{3}{2}, \infty\right)$

$$(ii) f(x) = x^2 + 2x - 5$$

Solution:

$$\text{Given } f(x) = x^2 + 2x - 5$$

Now by differentiating the given equation we get,

$$\Rightarrow f'(x) = \frac{d}{dx}(x^2 + 2x - 5)$$

$$\Rightarrow f'(x) = 2x + 2$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow 2x + 2 > 0$$

$$\Rightarrow 2x < -2$$

$$\Rightarrow x < -\frac{2}{2}$$

$$\Rightarrow x < -1$$

$$\Rightarrow x \in (-\infty, -1)$$

Thus $f(x)$ is increasing on interval $(-\infty, -1)$

Again, for $f(x)$ to be decreasing, we must have

$$f'(x) < 0$$

$$\Rightarrow 2x + 2 < 0$$

$$\Rightarrow 2x > -2$$

$$\Rightarrow x > -\frac{2}{2}$$

$$\Rightarrow x > -1$$

$$\Rightarrow x \in (-1, \infty)$$

Thus $f(x)$ is decreasing on interval $x \in (-1, \infty)$

(iii) $f(x) = 6 - 9x - x^2$

Solution:

Given $f(x) = 6 - 9x - x^2$

$$\Rightarrow f'(x) = \frac{d}{dx}(6 - 9x - x^2)$$

$$\Rightarrow f'(x) = -9 - 2x$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow -9 - 2x > 0$$

$$\Rightarrow -2x > 9$$

$$\Rightarrow x < -\frac{9}{2}$$

$$\Rightarrow x < -\frac{9}{2}$$

$$\Rightarrow x \in \left(-\infty, -\frac{9}{2}\right)$$

Thus $f(x)$ is increasing on interval $\left(-\infty, -\frac{9}{2}\right)$

Again, for $f(x)$ to be decreasing, we must have

$$f'(x) < 0$$

$$\Rightarrow -9 - 2x < 0$$

$$\Rightarrow -2x < 9$$

$$\Rightarrow x > -\frac{9}{2}$$

$$\Rightarrow x \in \left(-\frac{9}{2}, \infty\right)$$

Thus $f(x)$ is decreasing on interval $x \in \left(-\frac{9}{2}, \infty\right)$

(iv) $f(x) = 2x^3 - 12x^2 + 18x + 15$

Solution:

Given $f(x) = 2x^3 - 12x^2 + 18x + 15$

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 12x^2 + 18x + 15)$$

$$\Rightarrow f'(x) = 6x^2 - 24x + 18$$

For $f(x)$ we have to find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 6x^2 - 24x + 18 = 0$$

$$\Rightarrow 6(x^2 - 4x + 3) = 0$$

$$\Rightarrow 6(x^2 - 3x - x + 3) = 0$$

$$\Rightarrow 6(x - 3)(x - 1) = 0$$

$$\Rightarrow (x - 3)(x - 1) = 0$$

$$\Rightarrow x = 3, 1$$

Clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$ and $f'(x) < 0$ if $1 < x < 3$

Thus, $f(x)$ increases on $(-\infty, 1) \cup (3, \infty)$ and $f(x)$ is decreasing on interval $x \in (1, 3)$

(v) $f(x) = 5 + 36x + 3x^2 - 2x^3$

Solution:

Given $f(x) = 5 + 36x + 3x^2 - 2x^3$

$$\Rightarrow f'(x) = \frac{d}{dx}(5 + 36x + 3x^2 - 2x^3)$$

$$\Rightarrow f'(x) = 36 + 6x - 6x^2$$

For $f(x)$ now we have to find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 36 + 6x - 6x^2 = 0$$

$$\Rightarrow 6(-x^2 + x + 6) = 0$$

$$\Rightarrow 6(-x^2 + 3x - 2x + 6) = 0$$

$$\Rightarrow -x^2 + 3x - 2x + 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow (x - 3)(x + 2) = 0$$

$$\Rightarrow x = 3, -2$$

Clearly, $f'(x) > 0$ if $-2 < x < 3$ and $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, $f(x)$ increases on $x \in (-2, 3)$ and $f(x)$ is decreasing on interval $(-\infty, -2) \cup (3, \infty)$

$$\text{(vi) } f(x) = 8 + 36x + 3x^2 - 2x^3$$

Solution:

$$\text{Given } f(x) = 8 + 36x + 3x^2 - 2x^3$$

Now differentiating with respect to x

$$\Rightarrow f'(x) = \frac{d}{dx}(8 + 36x + 3x^2 - 2x^3)$$

$$\Rightarrow f'(x) = 36 + 6x - 6x^2$$

For $f(x)$ we have to find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 36 + 6x - 6x^2 = 0$$

$$\Rightarrow 6(-x^2 + x + 6) = 0$$

$$\Rightarrow 6(-x^2 + 3x - 2x + 6) = 0$$

$$\Rightarrow -x^2 + 3x - 2x + 6 = 0$$

$$\Rightarrow x^2 - 3x + 2x - 6 = 0$$

$$\Rightarrow (x - 3)(x + 2) = 0$$

$$\Rightarrow x = 3, -2$$

Clearly, $f'(x) > 0$ if $-2 < x < 3$ and $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, $f(x)$ increases on $x \in (-2, 3)$ and $f(x)$ is decreasing on interval $(-\infty, -2) \cup (3, \infty)$

$$\text{(vii) } f(x) = 5x^3 - 15x^2 - 120x + 3$$

Solution:

Given $f(x) = 5x^3 - 15x^2 - 120x + 3$

Now by differentiating above equation with respect x , we get

$$\Rightarrow f'(x) = \frac{d}{dx}(5x^3 - 15x^2 - 120x + 3)$$

$$\Rightarrow f'(x) = 15x^2 - 30x - 120$$

For $f(x)$ we have to find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 15x^2 - 30x - 120 = 0$$

$$\Rightarrow 15(x^2 - 2x - 8) = 0$$

$$\Rightarrow 15(x^2 - 4x + 2x - 8) = 0$$

$$\Rightarrow x^2 - 4x + 2x - 8 = 0$$

$$\Rightarrow (x - 4)(x + 2) = 0$$

$$\Rightarrow x = 4, -2$$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 4$ and $f'(x) < 0$ if $-2 < x < 4$

Thus, $f(x)$ increases on $(-\infty, -2) \cup (4, \infty)$ and $f(x)$ is decreasing on interval $x \in (-2, 4)$

(viii) $f(x) = x^3 - 6x^2 - 36x + 2$

Solution:

Given $f(x) = x^3 - 6x^2 - 36x + 2$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 6x^2 - 36x + 2)$$

$$\Rightarrow f'(x) = 3x^2 - 12x - 36$$

For $f(x)$ we have to find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 3x^2 - 12x - 36 = 0$$

$$\Rightarrow 3(x^2 - 4x - 12) = 0$$

$$\Rightarrow 3(x^2 - 6x + 2x - 12) = 0$$

$$\Rightarrow x^2 - 6x + 2x - 12 = 0$$

$$\Rightarrow (x - 6)(x + 2) = 0$$

$$\Rightarrow x = 6, -2$$

Clearly, $f'(x) > 0$ if $x < -2$ and $x > 6$ and $f'(x) < 0$ if $-2 < x < 6$

Thus, $f(x)$ increases on $(-\infty, -2) \cup (6, \infty)$ and $f(x)$ is decreasing on interval $x \in (-2, 6)$

(ix) $f(x) = 2x^3 - 15x^2 + 36x + 1$

Solution:

Given $f(x) = 2x^3 - 15x^2 + 36x + 1$

Now by differentiating above equation with respect x , we get

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 15x^2 + 36x + 1)$$

$$\Rightarrow f'(x) = 6x^2 - 30x + 36$$

For $f(x)$ we have to find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 6x^2 - 30x + 36 = 0$$

$$\Rightarrow 6(x^2 - 5x + 6) = 0$$

$$\Rightarrow 3(x^2 - 3x - 2x + 6) = 0$$

$$\Rightarrow x^2 - 3x - 2x + 6 = 0$$

$$\Rightarrow (x - 3)(x - 2) = 0$$

$$\Rightarrow x = 3, 2$$

Clearly, $f'(x) > 0$ if $x < 2$ and $x > 3$ and $f'(x) < 0$ if $2 < x < 3$

Thus, $f(x)$ increases on $(-\infty, 2) \cup (3, \infty)$ and $f(x)$ is decreasing on interval $x \in (2, 3)$

$$(x) f(x) = 2x^3 + 9x^2 + 12x + 20$$

Solution:

$$\text{Given } f(x) = 2x^3 + 9x^2 + 12x + 20$$

Differentiating above equation we get

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 + 9x^2 + 12x + 20)$$

$$\Rightarrow f'(x) = 6x^2 + 18x + 12$$

For $f(x)$ we have to find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 6x^2 + 18x + 12 = 0$$

$$\Rightarrow 6(x^2 + 3x + 2) = 0$$

$$\Rightarrow 6(x^2 + 2x + x + 2) = 0$$

$$\Rightarrow x^2 + 2x + x + 2 = 0$$

$$\Rightarrow (x + 2)(x + 1) = 0$$

$$\Rightarrow x = -1, -2$$

Clearly, $f'(x) > 0$ if $-2 < x < -1$ and $f'(x) < 0$ if $x < -2$ and $x > -1$

Thus, $f(x)$ increases on $x \in (-2, -1)$ and $f(x)$ is decreasing on interval $(-\infty, -2) \cup (-1, \infty)$

2. Determine the values of x for which the function $f(x) = x^2 - 6x + 9$ is increasing or decreasing. Also, find the coordinates of the point on the curve $y = x^2 - 6x + 9$ where the normal is parallel to the line $y = x + 5$.

Solution:

Given $f(x) = x^2 - 6x + 9$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^2 - 6x + 9)$$

$$\Rightarrow f'(x) = 2x - 6$$

$$\Rightarrow f'(x) = 2(x - 3)$$

For $f(x)$ let us find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow 2(x - 3) = 0$$

$$\Rightarrow (x - 3) = 0$$

$$\Rightarrow x = 3$$

Clearly, $f'(x) > 0$ if $x > 3$ and $f'(x) < 0$ if $x < 3$

Thus, $f(x)$ increases on $(3, \infty)$ and $f(x)$ is decreasing on interval $x \in (-\infty, 3)$

Now, let us find coordinates of point

Equation of curve is $f(x) = x^2 - 6x + 9$

Slope of this curve is given by

$$\Rightarrow m_1 = \frac{dy}{dx}$$

$$\Rightarrow m_1 = \frac{d}{dx}(x^2 - 6x + 9)$$

$$\Rightarrow m_1 = 2x - 6$$

Equation of line is $y = x + 5$

Slope of this curve is given by

$$\Rightarrow m_2 = \frac{dy}{dx}$$

$$\Rightarrow m_2 = \frac{d}{dx}(x + 5)$$

$$\Rightarrow m_2 = 1$$

Since slope of curve is parallel to line

Therefore, they follow the relation

$$\Rightarrow \frac{-1}{m_1} = m_2$$

$$\Rightarrow \frac{-1}{2x-6} = 1$$

$$\Rightarrow 2x - 6 = -1$$

$$\Rightarrow x = \frac{5}{2}$$

Thus putting the value of x in equation of curve, we get

$$\Rightarrow y = x^2 - 6x + 9$$

$$\Rightarrow y = \left(\frac{5}{2}\right)^2 - 6\left(\frac{5}{2}\right) + 9$$

$$\Rightarrow y = \frac{25}{4} - 15 + 9$$

$$\Rightarrow y = \frac{25}{4} - 6$$

$$\Rightarrow y = \frac{1}{4}$$

Thus the required coordinates is $\left(\frac{5}{2}, \frac{1}{4}\right)$

3. Find the intervals in which $f(x) = \sin x - \cos x$, where $0 < x < 2\pi$ is increasing or decreasing.

Solution:

$$\text{Given } f(x) = \sin x - \cos x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\sin x - \cos x)$$

$$\Rightarrow f'(x) = \cos x + \sin x$$

For $f(x)$ let us find critical point, we must have

$$\Rightarrow f'(x) = 0$$

$$\Rightarrow \cos x + \sin x = 0$$

$$\Rightarrow \tan(x) = -1$$

$$\Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$$

Here these points divide the angle range from 0 to 2π since we have x as angle

Clearly, $f'(x) > 0$ if $0 < x < \frac{3\pi}{4}$ and $\frac{7\pi}{4} < x < 2\pi$ and $f'(x) < 0$ if $\frac{3\pi}{4} < x < \frac{7\pi}{4}$

Thus, $f(x)$ increases on $(0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$ and $f(x)$ is decreasing on interval $(\frac{3\pi}{4}, \frac{7\pi}{4})$

4. Show that $f(x) = e^{2x}$ is increasing on \mathbb{R} .

Solution:

Given $f(x) = e^{2x}$

$$\Rightarrow f'(x) = \frac{d}{dx}(e^{2x})$$

$$\Rightarrow f'(x) = 2e^{2x}$$

For $f(x)$ to be increasing, we must have

$$\Rightarrow f'(x) > 0$$

$$\Rightarrow 2e^{2x} > 0$$

$$\Rightarrow e^{2x} > 0$$

Since, the value of e lies between 2 and 3

So, whatever be the power of e (that is x in domain \mathbb{R}) will be greater than zero.

Thus $f(x)$ is increasing on interval \mathbb{R}

5. Show that $f(x) = e^{1/x}$, $x \neq 0$ is a decreasing function for all $x \neq 0$.

Solution:

Given $f(x) = e^{\frac{1}{x}}$

$$\Rightarrow f'(x) = \frac{d}{dx}\left(e^{\frac{1}{x}}\right)$$

$$\Rightarrow f'(x) = e^{\frac{1}{x}} \cdot \left(\frac{-1}{x^2}\right)$$

$$\Rightarrow f'(x) = -\frac{e^{\frac{1}{x}}}{x^2}$$

As given $x \in \mathbb{R}$, $x \neq 0$

$$\Rightarrow \frac{1}{x^2} > 0 \text{ and } e^{\frac{1}{x}} > 0$$

Their ratio is also greater than 0

$$\Rightarrow \frac{\frac{1}{e^x}}{x^2} > 0$$

$$\Rightarrow -\frac{\frac{1}{e^x}}{x^2} < 0; \text{ as by applying negative sign change in comparison sign}$$

$$\Rightarrow f'(x) < 0$$

Hence, condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing for all $x \neq 0$

6. Show that $f(x) = \log_a x$, $0 < a < 1$ is a decreasing function for all $x > 0$.

Solution:

Given $f(x) = \log_a x$, $0 < a < 1$

$$\Rightarrow f'(x) = \frac{d}{dx}(\log_a x)$$

$$\Rightarrow f'(x) = \frac{1}{x \log a}$$

As given $0 < a < 1$

$$\Rightarrow \log(a) < 0 \text{ and for } x > 0$$

$$\Rightarrow \frac{1}{x} > 0$$

Therefore $f'(x)$ is

$$\Rightarrow \frac{1}{x \log a} < 0$$

$$\Rightarrow f'(x) < 0$$

Hence, condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing for all $x > 0$

7. Show that $f(x) = \sin x$ is increasing on $(0, \pi/2)$ and decreasing on $(\pi/2, \pi)$ and neither

increasing nor decreasing in $(0, \pi)$.

Solution:

$$\text{Given } f(x) = \sin x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\sin x)$$

$$\Rightarrow f'(x) = \cos x$$

Taking different region from 0 to 2π

$$\text{Let } x \in (0, \frac{\pi}{2})$$

$$\Rightarrow \cos(x) > 0$$

$$\Rightarrow f'(x) > 0$$

Thus $f(x)$ is increasing in $(0, \frac{\pi}{2})$

$$\text{Let } x \in (\frac{\pi}{2}, \pi)$$

$$\Rightarrow \cos(x) < 0$$

$$\Rightarrow f'(x) < 0$$

Thus $f(x)$ is decreasing in $(\frac{\pi}{2}, \pi)$

Therefore, from above condition we find that

$$\Rightarrow f(x) \text{ is increasing in } (0, \frac{\pi}{2}) \text{ and decreasing in } (\frac{\pi}{2}, \pi)$$

Hence, condition for $f(x)$ neither increasing nor decreasing in $(0, \pi)$

8. Show that $f(x) = \log \sin x$ is increasing on $(0, \pi/2)$ and decreasing on $(\pi/2, \pi)$.

Solution:

$$\text{Given } f(x) = \log \sin x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\log \sin x)$$

$$\Rightarrow f'(x) = \frac{1}{\sin x} \times \cos x$$

$$\Rightarrow f'(x) = \cot(x)$$

Taking different region from 0 to π

$$\text{Let } x \in (0, \frac{\pi}{2})$$

$$\Rightarrow \cot(x) > 0$$

$$\Rightarrow f'(x) > 0$$

Thus $f(x)$ is increasing in $(0, \frac{\pi}{2})$

$$\text{Let } x \in (\frac{\pi}{2}, \pi)$$

$$\Rightarrow \cot(x) < 0$$

$$\Rightarrow f'(x) < 0$$

Thus $f(x)$ is decreasing in $(\frac{\pi}{2}, \pi)$

Hence proved

9. Show that $f(x) = x - \sin x$ is increasing for all $x \in \mathbb{R}$.

Solution:

$$\text{Given } f(x) = x - \sin x$$

$$\Rightarrow f'(x) = \frac{d}{dx} (x - \sin x)$$

$$\Rightarrow f'(x) = 1 - \cos x$$

Now, as given $x \in \mathbb{R}$

$$\Rightarrow -1 < \cos x < 1$$

$$\Rightarrow -1 > \cos x > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $x \in \mathbb{R}$

10. Show that $f(x) = x^3 - 15x^2 + 75x - 50$ is an increasing function for all $x \in \mathbb{R}$.

Solution:

$$\text{Given } f(x) = x^3 - 15x^2 + 75x - 50$$

$$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 15x^2 + 75x - 50)$$

$$\Rightarrow f'(x) = 3x^2 - 30x + 75$$

$$\Rightarrow f'(x) = 3(x^2 - 10x + 25)$$

$$\Rightarrow f'(x) = 3(x - 5)^2$$

Now, as given $x \in \mathbb{R}$

$$\Rightarrow (x - 5)^2 > 0$$

$$\Rightarrow 3(x - 5)^2 > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $x \in \mathbb{R}$

11. Show that $f(x) = \cos^2 x$ is a decreasing function on $(0, \pi/2)$.**Solution:**

$$\text{Given } f(x) = \cos^2 x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\cos^2 x)$$

$$\Rightarrow f'(x) = 2 \cos x (-\sin x)$$

$$\Rightarrow f'(x) = -2 \sin(x) \cos(x)$$

$$\Rightarrow f'(x) = -\sin 2x$$

Now, as given x belongs to $(0, \pi/2)$.

$$\Rightarrow 2x \in (0, \pi)$$

$$\Rightarrow \sin(2x) > 0$$

$$\Rightarrow -\sin(2x) < 0$$

$$\Rightarrow f'(x) < 0$$

Hence, condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing on interval $(0, \pi/2)$.

Hence proved

12. Show that $f(x) = \sin x$ is an increasing function on $(-\pi/2, \pi/2)$.**Solution:**

$$\text{Given } f(x) = \sin x$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\sin x)$$

$$\Rightarrow f'(x) = \cos x$$

Now, as given $x \in (-\pi/2, \pi/2)$.

That is 4th quadrant, where

$$\Rightarrow \cos x > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $(-\pi/2, \pi/2)$.

13. Show that $f(x) = \cos x$ is a decreasing function on $(0, \pi)$, increasing in $(-\pi, 0)$ and neither increasing nor decreasing in $(-\pi, \pi)$.

Solution:

Given $f(x) = \cos x$

$$\Rightarrow f'(x) = \frac{d}{dx}(\cos x)$$

$$\Rightarrow f'(x) = -\sin x$$

Taking different region from 0 to 2π

Let $x \in (0, \pi)$.

$$\Rightarrow \sin(x) > 0$$

$$\Rightarrow -\sin x < 0$$

$$\Rightarrow f'(x) < 0$$

Thus $f(x)$ is decreasing in $(0, \pi)$

Let $x \in (-\pi, 0)$.

$$\Rightarrow \sin(x) < 0$$

$$\Rightarrow -\sin x > 0$$

$$\Rightarrow f'(x) > 0$$

Thus $f(x)$ is increasing in $(-\pi, 0)$.

Therefore, from above condition we find that

$\Rightarrow f(x)$ is decreasing in $(0, \pi)$ and increasing in $(-\pi, 0)$.

Hence, condition for $f(x)$ neither increasing nor decreasing in $(-\pi, \pi)$

14. Show that $f(x) = \tan x$ is an increasing function on $(-\pi/2, \pi/2)$.

Solution:

Given $f(x) = \tan x$

$$\Rightarrow f'(x) = \frac{d}{dx}(\tan x)$$

$$\Rightarrow f'(x) = \sec^2 x$$

Now, as given

$$x \in (-\pi/2, \pi/2).$$

That is 4th quadrant, where

$$\Rightarrow \sec^2 x > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, Condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $(-\pi/2, \pi/2)$.

15. Show that $f(x) = \tan^{-1}(\sin x + \cos x)$ is a decreasing function on the interval $(\pi/4, \pi/2)$.

Solution:

$$\text{Given } f(x) = \tan^{-1}(\sin x + \cos x)$$

$$\Rightarrow f'(x) = \frac{d}{dx}(\tan^{-1}(\sin x + \cos x))$$

$$\Rightarrow f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$\Rightarrow f'(x) = \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now, as given

$$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$\Rightarrow \cos x - \sin x < 0$; as here cosine values are smaller than sine values for same angle

$$\Rightarrow \frac{\cos x - \sin x}{2(1 + \sin x \cos x)} < 0$$

$$\Rightarrow f'(x) < 0$$

Hence, Condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

16. Show that the function $f(x) = \sin(2x + \pi/4)$ is decreasing on $(3\pi/8, 5\pi/8)$.

Solution:

$$\text{Given, } f(x) = \sin\left(2x + \frac{\pi}{4}\right)$$

$$\Rightarrow f'(x) = \frac{d}{dx}\left\{\sin\left(2x + \frac{\pi}{4}\right)\right\}$$

$$\Rightarrow f'(x) = \cos\left(2x + \frac{\pi}{4}\right) \times 2$$

$$\Rightarrow f'(x) = 2\cos\left(2x + \frac{\pi}{4}\right)$$

$$\text{Now, as given } x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$$

$$\Rightarrow \frac{3\pi}{8} < x < \frac{5\pi}{8}$$

$$\Rightarrow \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$$

$$\Rightarrow \pi < 2x + \frac{\pi}{4} < \frac{3\pi}{2};$$

As here $2x + \frac{\pi}{4}$ lies in 3rd quadrant

$$\Rightarrow \cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow 2\cos\left(2x + \frac{\pi}{4}\right) < 0$$

$$\Rightarrow f'(x) < 0$$

Hence, condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing on the interval $(3\pi/8, 5\pi/8)$.

17. Show that the function $f(x) = \cot^{-1}(\sin x + \cos x)$ is decreasing on $(0, \pi/4)$ and increasing on $(\pi/4, \pi/2)$.

Solution:

$$\text{Given } f(x) = \cot^{-1}(\sin x + \cos x)$$

$$\Rightarrow f'(x) = \frac{d}{dx} \{ \cot^{-1}(\sin x + \cos x) \}$$

$$\Rightarrow f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1 + \sin^2 x + \cos^2 x + 2 \sin x \cos x}$$

$$\Rightarrow f'(x) = \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now, as given $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\Rightarrow \cos x - \sin x < 0$; as here cosine values are smaller than sine values for same angle

$$\Rightarrow \frac{\cos x - \sin x}{2(1 + \sin x \cos x)} < 0$$

$$\Rightarrow f'(x) < 0$$

Hence, condition for $f(x)$ to be decreasing

Thus $f(x)$ is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

18. Show that $f(x) = (x - 1) e^x + 1$ is an increasing function for all $x > 0$.

Solution:

Given $f(x) = (x - 1) e^x + 1$

Now differentiating the given equation with respect to x , we get

$$\Rightarrow f'(x) = \frac{d}{dx} ((x - 1) e^x + 1)$$

$$\Rightarrow f'(x) = e^x + (x - 1) e^x$$

$$\Rightarrow f'(x) = e^x(1 + x - 1)$$

$$\Rightarrow f'(x) = x e^x$$

As given $x > 0$

$$\Rightarrow e^x > 0$$

$$\Rightarrow x e^x > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, condition for $f(x)$ to be increasing
Thus $f(x)$ is increasing on interval $x > 0$

19. Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on $(0, 1)$.

Solution:

$$\text{Given } f(x) = x^2 - x + 1$$

Now by differentiating the given equation with respect to x , we get

$$\Rightarrow f'(x) = \frac{d}{dx}(x^2 - x + 1)$$

$$\Rightarrow f'(x) = 2x - 1$$

Taking different region from $(0, 1)$

$$\text{Let } x \in (0, \frac{1}{2})$$

$$\Rightarrow 2x - 1 < 0$$

$$\Rightarrow f'(x) < 0$$

Thus $f(x)$ is decreasing in $(0, \frac{1}{2})$

$$\text{Let } x \in (\frac{1}{2}, 1)$$

$$\Rightarrow 2x - 1 > 0$$

$$\Rightarrow f'(x) > 0$$

Thus $f(x)$ is increasing in $(\frac{1}{2}, 1)$

Therefore, from above condition we find that

$\Rightarrow f(x)$ is decreasing in $(0, \frac{1}{2})$ and increasing in $(\frac{1}{2}, 1)$

Hence, condition for $f(x)$ neither increasing nor decreasing in $(0, 1)$

20. Show that $f(x) = x^9 + 4x^7 + 11$ is an increasing function for all $x \in \mathbb{R}$.

Solution:

$$\text{Given } f(x) = x^9 + 4x^7 + 11$$

Now by differentiating above equation with respect to x , we get

$$\Rightarrow f'(x) = \frac{d}{dx}(x^9 + 4x^7 + 11)$$

$$\Rightarrow f'(x) = 9x^8 + 28x^6$$

$$\Rightarrow f'(x) = x^6(9x^2 + 28)$$

As given $x \in \mathbb{R}$

$$\Rightarrow x^6 > 0 \text{ and } 9x^2 + 28 > 0$$

$$\Rightarrow x^6(9x^2 + 28) > 0$$

$$\Rightarrow f'(x) > 0$$

Hence, condition for $f(x)$ to be increasing

Thus $f(x)$ is increasing on interval $x \in \mathbb{R}$

