EXERCISE 17.1 PAGE NO: 17.10

1. Prove that the function f(x) = $\log_e x$ is increasing on (0, ∞).

Solution:
Let $x_1, x_2 \in (0, \infty)$
We have, $x_1 < x_2$
$\Rightarrow \log_e x_1 < \log_e x_2$
$\Rightarrow f(x_1) < f(x_2)$
So, f(x) is increasing in (0, ∞)

2. Prove that the function f(x) = $\log_a x$ is increasing on (0, ∞) if a > 1 and decreasing on (0, ∞), if 0 < a < 1.

Solution:
Case I

When a > 1

Let $x_1, x_2 \in (0, \infty)$

We have, $x_1 < x_2$

$\Rightarrow \log_e x_1 < \log_e x_2$

$\Rightarrow f(x_1) < f(x_2)$

So, f(x) is increasing in (0, ∞)

Case II

When 0 < a < 1

$f(x) = \log_a x = \frac{\log x}{\log a}$

When a < 1 $\Rightarrow$ log a < 0

Let $x_1 < x_2$

$\Rightarrow \log x_1 < \log x_2$

$\Rightarrow \frac{\log x_1}{\log a} > \frac{\log x_2}{\log a}$ $[\because \log a < 0]$

$\Rightarrow f(x_1) > f(x_2)$

So, f(x) is decreasing in $(0, \infty)$

3. Prove that f(x) = ax + b, where a, b are constants and a > 0 is an increasing function on R.

Solution:
Given,
$f(x) = ax + b, a > 0$
Let $x_1, x_2 \in R$ and $x_1 > x_2$
$\Rightarrow ax_1 > ax_2$ for some $a > 0$
$\Rightarrow ax_1 + b > ax_2 + b$ for some b
$\Rightarrow f(x_1) > f(x_2)$
Hence, $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$
So, f(x) is increasing function of R

4. Prove that f(x) = ax + b, where a, b are constants and a < 0 is a decreasing function on R.

Solution:
Given,
$f(x) = ax + b, a < 0$
Let $x_1, x_2 \in R$ and $x_1 > x_2$
$\Rightarrow ax_1 < ax_2$ for some $a > 0$
$\Rightarrow ax_1 + b < ax_2 + b$ for some b
$\Rightarrow f(x_1) < f(x_2)$
Hence, $x_1 > x_2 \Rightarrow f(x_1) < f(x_2)$
So, f(x) is decreasing function of R

EXERCISE 17.2 PAGE NO: 17.33

1. Find the intervals in which the following functions are increasing or decreasing.

(i) f (x) = 10 – 6x – 2x^2

Solution:

Given f (x) = 10 – 6x – 2x^2

By differentiating above equation we get,

$\Rightarrow f'(x) = \frac{d}{dx}(10 - 6x - 2x^2)$

$\Rightarrow f'(x) = -6 - 4x$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow -6 - 4x > 0$

$\Rightarrow -4x > 6$

$\Rightarrow x < -\frac{6}{4}$

$\Rightarrow x < -\frac{3}{2}$

$\Rightarrow x \in (-\infty, -\frac{3}{2})$

Thus f(x) is increasing on the interval $\left(-\infty, -\frac{3}{2}\right)$

Again, for f(x) to be increasing, we must have

$f'(x) < 0$

$\Rightarrow -6 - 4x < 0$

$\Rightarrow -4x < 6$

$\Rightarrow x > -\frac{6}{4}$

$\Rightarrow x > -\frac{3}{2}$

$\Rightarrow x \in (-\frac{3}{2}, \infty)$

Thus f(x) is decreasing on interval $x \in (-\frac{3}{2}, \infty)$

(ii) f (x) = x^2 + 2x – 5

Solution:

Given f (x) = $x^2 + 2x - 5$

Now by differentiating the given equation we get,

$\Rightarrow f'(x) = \frac{d}{dx}(x^2 + 2x - 5)$

$\Rightarrow f'(x) = 2x + 2$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow 2x + 2 > 0$

$\Rightarrow 2x < -2$

$\Rightarrow x < -\frac{2}{2}$

$\Rightarrow x < -1$

$\Rightarrow x \in (-\infty, -1)$

Thus f(x) is increasing on interval $(-\infty, -1)$

Again, for f(x) to be increasing, we must have

$f'(x) < 0$

$\Rightarrow 2x + 2 < 0$

$\Rightarrow 2x > -2$

$\Rightarrow x > -\frac{2}{2}$

$\Rightarrow x > -1$

$\Rightarrow x \in (-1, \infty)$

Thus f(x) is decreasing on interval $x \in (-1, \infty)$

(iii) f (x) = 6 – 9x – x^2

Solution:

Given f (x) = 6 – 9x – x^2

$\Rightarrow f'(x) = \frac{d}{dx}(6 - 9x - x^2)$

$\Rightarrow f'(x) = -9 - 2x$

For f(x) to be increasing, we must have

$\Rightarrow f'(x) > 0$

$\Rightarrow -9 - 2x > 0$

$\Rightarrow -2x > 9$

$\Rightarrow x < -\frac{9}{2}$

$\Rightarrow x < -\frac{9}{2}$

$\Rightarrow x \in (-\infty, -\frac{9}{2})$

Thus f(x) is increasing on interval $\left(-\infty, -\frac{9}{2}\right)$

Again, for f(x) to be decreasing, we must have

$f'(x) < 0$

$\Rightarrow -9 - 2x < 0$

$\Rightarrow -2x < 9$

$\Rightarrow x > -\frac{9}{2}$

$\Rightarrow x \in (-\frac{9}{2}, \infty)$

Thus f(x) is decreasing on interval $x \in (-\frac{9}{2}, \infty)$

(iv) $f(x) = 2x^3 - 12x^2 + 18x + 15$

Solution:

Given $f(x) = 2x^3 - 12x^2 + 18x + 15$

$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 12x^2 + 18x + 15)$

$\Rightarrow f'(x) = 6x^2 - 24x + 18$

For f(x) we have to find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 - 24x + 18 = 0$

$\Rightarrow 6(x^2 - 4x + 3) = 0$

$\Rightarrow 6(x^2 - 3x - x + 3) = 0$

$\Rightarrow 6(x - 3)(x - 1) = 0$

$\Rightarrow (x - 3)(x - 1) = 0$

$\Rightarrow x = 3, 1$

Clearly, $f'(x) > 0$ if $x < 1$ and $x > 3$ and $f'(x) < 0$ if $1 < x < 3$

Thus, f(x) increases on $(-\infty, 1) \cup (3, \infty)$ and f(x) is decreasing on interval $x \in (1, 3)$

(v) $f(x) = 5 + 36x + 3x^2 - 2x^3$

Solution:

Given $f(x) = 5 + 36x + 3x^2 - 2x^3$

$\Rightarrow f'(x) = \frac{d}{dx}(5 + 36x + 3x^2 - 2x^3)$

$\Rightarrow f'(x) = 36 + 6x - 6x^2$

For f(x) now we have to find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 36 + 6x - 6x^2 = 0$

$\Rightarrow 6(-x^2 + x + 6) = 0$

$\Rightarrow 6(-x^2 + 3x - 2x + 6) = 0$

$\Rightarrow -x^2 + 3x - 2x + 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow x = 3, -2$

Clearly, $f'(x) > 0$ if $-2 < x < 3$ and $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, f(x) increases on $x \in (-2, 3)$ and f(x) is decreasing on interval $(-\infty, -2) \cup (3, \infty)$

(vi) $f(x) = 8 + 36x + 3x^2 - 2x^3$

Solution:

Given $f(x) = 8 + 36x + 3x^2 - 2x^3$

Now differentiating with respect to x

$\Rightarrow f'(x) = \frac{d}{dx}(8 + 36x + 3x^2 - 2x^3)$

$\Rightarrow f'(x) = 36 + 6x - 6x^2$

For f(x) we have to find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 36 + 6x - 6x^2 = 0$

$\Rightarrow 6(-x^2 + x + 6) = 0$

$\Rightarrow 6(-x^2 + 3x - 2x + 6) = 0$

$\Rightarrow -x^2 + 3x - 2x + 6 = 0$

$\Rightarrow x^2 - 3x + 2x - 6 = 0$

$\Rightarrow (x - 3)(x + 2) = 0$

$\Rightarrow x = 3, -2$

Clearly, $f'(x) > 0$ if $-2 < x < 3$ and $f'(x) < 0$ if $x < -2$ and $x > 3$

Thus, f(x) increases on $x \in (-2, 3)$ and f(x) is decreasing on interval $(-\infty, 2) \cup (3, \infty)$

(vii) $f(x) = 5x^3 - 15x^2 - 120x + 3$

Solution:

Given $f(x) = 5x^3 – 15x^2 – 120x + 3$
Now by differentiating above equation with respect x, we get
$\Rightarrow f'(x) = \frac{d}{dx}(5x^3 - 15x^2 - 120x + 3)$
$\Rightarrow f'(x) = 15x^2 – 30x – 120$
For f(x) we have to find critical point, we must have
$\Rightarrow f'(x) = 0$
$\Rightarrow 15x^2 – 30x – 120 = 0$
$\Rightarrow 15(x^2 – 2x – 8) = 0$
$\Rightarrow 15(x^2 – 4x + 2x – 8) = 0$
$\Rightarrow x^2 – 4x + 2x – 8 = 0$
$\Rightarrow (x – 4) (x + 2) = 0$
$\Rightarrow x = 4, – 2$
Clearly, $f'(x) > 0$ if $x < –2$ and $x > 4$ and $f'(x) < 0$ if $–2 < x < 4$
Thus, f(x) increases on $(–\infty,–2) \cup (4, \infty)$ and f(x) is decreasing on interval $x \in (–2, 4)$

(viii) f(x) = $x^3 – 6x^2 – 36x + 2$

Solution:
Given $f (x) = x^3 – 6x^2 – 36x + 2$
$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 6x^2 - 36x + 2)$
$\Rightarrow f'(x) = 3x^2 – 12x – 36$
For f(x) we have to find critical point, we must have
$\Rightarrow f'(x) = 0$
$\Rightarrow 3x^2 – 12x – 36 = 0$
$\Rightarrow 3(x^2 – 4x – 12) = 0$
$\Rightarrow 3(x^2 – 6x + 2x – 12) = 0$
$\Rightarrow x^2 – 6x + 2x – 12 = 0$
$\Rightarrow (x – 6) (x + 2) = 0$
$\Rightarrow x = 6, – 2$
Clearly, $f'(x) > 0$ if $x < –2$ and $x > 6$ and $f'(x) < 0$ if $–2< x < 6$
Thus, f(x) increases on $(–\infty,–2) \cup (6, \infty)$ and f(x) is decreasing on interval $x \in (–2, 6)$

(ix) f(x) = $2x^3 – 15x^2 + 36x + 1$

Solution:
Given $f (x) = 2x^3 – 15x^2 + 36x + 1$

Now by differentiating above equation with respect x, we get

$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 - 15x^2 + 36x + 1)$

$\Rightarrow f'(x) = 6x^2 - 30x + 36$

For f(x) we have to find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 - 30x + 36 = 0$

$\Rightarrow 6 (x^2 - 5x + 6) = 0$

$\Rightarrow 3(x^2 - 3x - 2x + 6) = 0$

$\Rightarrow x^2 - 3x - 2x + 6 = 0$

$\Rightarrow (x - 3) (x - 2) = 0$

$\Rightarrow x = 3, 2$

Clearly, $f'(x) > 0$ if $x < 2$ and $x > 3$ and $f'(x) < 0$ if $2 < x < 3$

Thus, f(x) increases on $(-\infty, 2) \cup (3, \infty)$ and f(x) is decreasing on interval $x \in (2, 3)$

(x) f (x) = $2x^3 + 9x^2 + 12x + 20$

Solution:

Given f (x) = $2x^3 + 9x^2 + 12x + 20$

Differentiating above equation we get

$\Rightarrow f'(x) = \frac{d}{dx}(2x^3 + 9x^2 + 12x + 20)$

$\Rightarrow f'(x) = 6x^2 + 18x + 12$

For f(x) we have to find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 6x^2 + 18x + 12 = 0$

$\Rightarrow 6(x^2 + 3x + 2) = 0$

$\Rightarrow 6(x^2 + 2x + x + 2) = 0$

$\Rightarrow x^2 + 2x + x + 2 = 0$

$\Rightarrow (x + 2) (x + 1) = 0$

$\Rightarrow x = -1, -2$

Clearly, $f'(x) > 0$ if $-2 < x < -1$ and $f'(x) < 0$ if $x < -1$ and $x > -2$

Thus, f(x) increases on $x \in (-2,-1)$ and f(x) is decreasing on interval $(-\infty, -2) \cup (-2, \infty)$

2. Determine the values of x for which the function f(x) = $x^2 - 6x + 9$ is increasing or decreasing. Also, find the coordinates of the point on the curve y = $x^2 - 6x + 9$ where the normal is parallel to the line y = x + 5.

Solution:

Given $f(x) = x^2 - 6x + 9$

$\Rightarrow f'(x) = \frac{d}{dx}(x^2 - 6x + 9)$

$\Rightarrow f'(x) = 2x - 6$

$\Rightarrow f'(x) = 2(x - 3)$

For f(x) let us find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow 2(x - 3) = 0$

$\Rightarrow (x - 3) = 0$

$\Rightarrow x = 3$

Clearly, $f'(x) > 0$ if $x > 3$ and $f'(x) < 0$ if $x < 3$

Thus, f(x) increases on $(3, \infty)$ and f(x) is decreasing on interval $x \in (-\infty, 3)$

Now, let us find coordinates of point

Equation of curve is $f(x) = x^2 - 6x + 9$

Slope of this curve is given by

$\Rightarrow m_1 = \frac{dy}{dx}$

$\Rightarrow m_1 = \frac{d}{dx}(x^2 - 6x + 9)$

$\Rightarrow m_1 = 2x - 6$

Equation of line is $y = x + 5$

Slope of this curve is given by

$\Rightarrow m_2 = \frac{dy}{dx}$

$\Rightarrow m_2 = \frac{d}{dx}(x + 5)$

$\Rightarrow m_2 = 1$

Since slope of curve is parallel to line

Therefore, they follow the relation

$\Rightarrow \frac{-1}{m_1} = m_2$

$\Rightarrow \frac{-1}{2x-6} = 1$

$\Rightarrow 2x - 6 = -1$

$\Rightarrow x = \frac{5}{2}$

Thus putting the value of x in equation of curve, we get

$\Rightarrow y = x^2 - 6x + 9$

$\Rightarrow y = (\frac{5}{2})^2 - 6\left(\frac{5}{2}\right) + 9$

$\Rightarrow y = \frac{25}{4} - 15 + 9$

$\Rightarrow y = \frac{25}{4} - 6$

$\Rightarrow y = \frac{1}{4}$

Thus the required coordinates is $(\frac{5}{2}, \frac{1}{4})$

3. Find the intervals in which f(x) = sin x – cos x, where 0 < x < 2π is increasing or decreasing.

Solution:

Given f (x) = sin x – cos x

$\Rightarrow f'(x) = \frac{d}{dx}(\sin x - \cos x)$

$\Rightarrow f'(x) = \cos x + \sin x$

For f(x) let us find critical point, we must have

$\Rightarrow f'(x) = 0$

$\Rightarrow \text{Cos } x + \sin x = 0$

$\Rightarrow \text{Tan } (x) = -1$

$\Rightarrow x = \frac{3\pi}{4}, \frac{7\pi}{4}$

Here these points divide the angle range from 0 to 2 π since we have x as angle

Clearly, f'(x) > 0 if $0 < x < \frac{3\pi}{4}$ and $\frac{7\pi}{4} < x < 2\pi$ and f'(x) < 0 if $\frac{3\pi}{4} < x < \frac{7\pi}{4}$

Thus, f(x) increases on $(0, \frac{3\pi}{4}) \cup (\frac{7\pi}{4}, 2\pi)$ and f(x) is decreasing on interval $(\frac{3\pi}{4}, \frac{7\pi}{4})$

4. Show that f(x) = e^{2x} is increasing on R.

Solution:

Given f (x) = e^{2x}

$\Rightarrow f'(x) = \frac{d}{dx}(e^{2x})$

⇒ f'(x) = $2e^{2x}$

For f(x) to be increasing, we must have

⇒ f'(x) > 0

⇒ $2e^{2x} > 0$

⇒ $e^{2x} > 0$

Since, the value of e lies between 2 and 3

So, whatever be the power of e (that is x in domain R) will be greater than zero.

Thus f(x) is increasing on interval R

5. Show that f (x) = $e^{1/x}$, x ≠ 0 is a decreasing function for all x ≠ 0.

Solution:

Given $f(x) = e^{\frac{1}{x}}$

$\Rightarrow f'(x) = \frac{d}{dx}\left(e^{\frac{1}{x}}\right)$

$\Rightarrow f'(x) = e^{\frac{1}{x}}.\left(\frac{-1}{x^2}\right)$

$\Rightarrow f'(x) = -\frac{e^{\frac{1}{x}}}{x^2}$

As given x ∈ R, x ≠ 0

$\Rightarrow \frac{1}{x^2} > 0$ and $e^{\frac{1}{x}} > 0$

Their ratio is also greater than 0

$\Rightarrow \frac{e^{\frac{1}{x}}}{x^2} > 0$

$\Rightarrow -\frac{e^{\frac{1}{x}}}{x^2} < 0$; as by applying negative sign change in comparison sign

$\Rightarrow f'(x) < 0$

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing for all $x \neq 0$

6. Show that $f(x) = \log_a x$, $0 < a < 1$ is a decreasing function for all $x > 0$.

Solution:

Given $f(x) = \log_a x$, $0 < a < 1$

$\Rightarrow f'(x) = \frac{d}{dx}(\log_a x)$

$\Rightarrow f'(x) = \frac{1}{x \log a}$

As given $0 < a < 1$

$\Rightarrow \log(a) < 0$ and for $x > 0$

$\Rightarrow \frac{1}{x} > 0$

Therefore f'(x) is

$\Rightarrow \frac{1}{x \log a} < 0$

$\Rightarrow f'(x) < 0$

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing for all $x > 0$

7. Show that $f(x) = \sin x$ is increasing on $(0, \pi/2)$ and decreasing on $(\pi/2, \pi)$ and neither

increasing nor decreasing in (0, π).

Solution:

Given f(x) = sin x

$\Rightarrow f'(x) = \frac{d}{dx}(\sin x)$

$\Rightarrow f'(x) = \cos x$

Taking different region from 0 to 2π

Let $x \in (0, \frac{\pi}{2})$

$\Rightarrow \cos(x) > 0$

$\Rightarrow f'(x) > 0$

Thus f(x) is increasing in $(0, \frac{\pi}{2})$

Let $x \in (\frac{\pi}{2}, \pi)$

$\Rightarrow \cos(x) < 0$

$\Rightarrow f'(x) < 0$

Thus f(x) is decreasing in $(\frac{\pi}{2}, \pi)$

Therefore, from above condition we find that

⇒ f (x) is increasing in $(0, \frac{\pi}{2})$ and decreasing in $(\frac{\pi}{2}, \pi)$

Hence, condition for f(x) neither increasing nor decreasing in (0, π)

8. Show that f(x) = log sin x is increasing on (0, π/2) and decreasing on (π/2, π).

Solution:

Given f(x) = log sin x

$\Rightarrow f'(x) = \frac{d}{dx}(\log \sin x)$

$\Rightarrow f'(x) = \frac{1}{\sin x} \times \cos x$

⇒ f'(x) = cot(x)

Taking different region from 0 to π

Let $x \in (0, \frac{\pi}{2})$

⇒ Cot(x) > 0

⇒ f'(x) > 0

Thus f(x) is increasing in $(0, \frac{\pi}{2})$

Let $x \in (\frac{\pi}{2}, \pi)$

⇒ Cot (x) < 0

⇒ f'(x) < 0

Thus f(x) is decreasing in $(\frac{\pi}{2}, \pi)$

Hence proved

9. Show that f(x) = x – sin x is increasing for all x ϵ R.

Solution:
Given f (x) = x – sin x
$\Rightarrow f'(x) = \frac{d}{dx}(x - \sin x)$
⇒ f'(x) = 1 – cos x
Now, as given x ϵ R
⇒ –1 < cos x < 1
⇒ –1 > cos x > 0
⇒ f'(x) > 0
Hence, condition for f(x) to be increasing
Thus f(x) is increasing on interval x ∈ R

10. Show that f(x) = $x^3 - 15x^2 + 75x - 50$ is an increasing function for all x ϵ R.

Solution:

Given $f(x) = x^3 - 15x^2 + 75x - 50$

$\Rightarrow f'(x) = \frac{d}{dx}(x^3 - 15x^2 + 75x - 50)$

$\Rightarrow f'(x) = 3x^2 - 30x + 75$

$\Rightarrow f'(x) = 3(x^2 - 10x + 25)$

$\Rightarrow f'(x) = 3(x - 5)^2$

Now, as given $x \in R$

$\Rightarrow (x - 5)^2 > 0$

$\Rightarrow 3(x - 5)^2 > 0$

$\Rightarrow f'(x) > 0$

Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval $x \in R$

11. Show that $f(x) = \cos^2 x$ is a decreasing function on $(0, \pi/2)$.

Solution:

Given $f(x) = \cos^2 x$

$\Rightarrow f'(x) = \frac{d}{dx}(\cos^2 x)$

$\Rightarrow f'(x) = 2 \cos x (-\sin x)$

$\Rightarrow f'(x) = -2 \sin(x) \cos(x)$

$\Rightarrow f'(x) = -\sin 2x$

Now, as given x belongs to $(0, \pi/2)$.

$\Rightarrow 2x \in (0, \pi)$

$\Rightarrow \text{Sin}(2x) > 0$

$\Rightarrow -\text{Sin}(2x) < 0$

$\Rightarrow f'(x) < 0$

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing on interval $(0, \pi/2)$.

Hence proved

12. Show that $f(x) = \sin x$ is an increasing function on $(-\pi/2, \pi/2)$.

Solution:

Given $f(x) = \sin x$

$\Rightarrow f'(x) = \frac{d}{dx}(\sin x)$

⇒ f'(x) = cos x
Now, as given x ∈ (–π/2, π/2).
That is 4th quadrant, where
⇒ Cos x> 0
⇒ f'(x) > 0
Hence, condition for f(x) to be increasing
Thus f(x) is increasing on interval (–π/2, π/2).

13. Show that f(x) = cos x is a decreasing function on (0, π), increasing in (–π, 0) and neither increasing nor decreasing in (–π, π).

Solution:
Given f(x) = cos x
⇒ $f'(x) = \frac{d}{dx}(\cos x)$
⇒ f'(x) = –sin x
Taking different region from 0 to 2π
Let x ∈ (0, π).
⇒ Sin(x) > 0
⇒ –sin x < 0
⇒ f'(x) < 0
Thus f(x) is decreasing in (0, π)
Let x ∈ (–π, o).
⇒ Sin (x) < 0
⇒ –sin x > 0
⇒ f'(x) > 0
Thus f(x) is increasing in (–π, 0).
Therefore, from above condition we find that
⇒ f (x) is decreasing in (0, π) and increasing in (–π, 0).
Hence, condition for f(x) neither increasing nor decreasing in (–π, π)

14. Show that f(x) = tan x is an increasing function on (–π/2, π/2).

Solution:
Given f (x) = tan x
⇒ $f'(x) = \frac{d}{dx}(\tan x)$
⇒ f'(x) = $\sec^2 x$

Now, as given

$x \in (-\pi/2, \pi/2)$.

That is 4th quadrant, where

$\Rightarrow \sec^2 x > 0$

$\Rightarrow f'(x) > 0$

Hence, Condition for f(x) to be increasing

Thus f(x) is increasing on interval $(-\pi/2, \pi/2)$.

15. Show that $f(x) = \tan^{-1} (\sin x + \cos x)$ is a decreasing function on the interval $(\pi/4, \pi/2)$.

Solution:

Given $f(x) = \tan^{-1} (\sin x + \cos x)$

$$\Rightarrow f'(x) = \frac{d}{dx}(\tan^{-1}(\sin x + \cos x))$$

$$\Rightarrow f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$\Rightarrow f'(x) = \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now, as given

$$x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$\Rightarrow$ Cos x – sin x < 0; as here cosine values are smaller than sine values for same angle

$$\Rightarrow \frac{\cos x - \sin x}{2(1 + \sin x \cos x)} < 0$$

$\Rightarrow f'(x) < 0$

Hence, Condition for f(x) to be decreasing

Thus f(x) is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

16. Show that the function f (x) = sin (2x + π/4) is decreasing on (3π/8, 5π/8).

Solution:

Given, $f(x) = \sin(2x + \frac{\pi}{4})$

$\Rightarrow f'(x) = \frac{d}{dx}\{\sin(2x + \frac{\pi}{4})\}$

$\Rightarrow f'(x) = \cos\left(2x + \frac{\pi}{4}\right) \times 2$

$\Rightarrow f'(x) = 2\cos\left(2x + \frac{\pi}{4}\right)$

Now, as given $x \in \left(\frac{3\pi}{8}, \frac{5\pi}{8}\right)$

$\Rightarrow \frac{3\pi}{8} < x < \frac{5\pi}{8}$

$\Rightarrow \frac{3\pi}{4} < 2x < \frac{5\pi}{4}$

$\Rightarrow \pi < 2x + \frac{\pi}{4} < \frac{3\pi}{2};$

As here $2x + \frac{\pi}{4}$ lies in 3rd quadrant

$\Rightarrow \cos\left(2x + \frac{\pi}{4}\right) < 0$

$\Rightarrow 2\cos\left(2x + \frac{\pi}{4}\right) < 0$

$\Rightarrow f'(x) < 0$

Hence, condition for f(x) to be decreasing

Thus f (x) is decreasing on the interval (3π/8, 5π/8).

17. Show that the function f(x) = cot^{-1} (sin x + cos x) is decreasing on (0, π/4) and increasing on (π/4, π/2).

Solution:

Given f(x) = cot^{-1} (sin x + cos x)

$\Rightarrow f'(x) = \frac{d}{dx}\{\cot^{-1}(\sin x + \cos x)\}$

$$\Rightarrow f'(x) = \frac{1}{1 + (\sin x + \cos x)^2} \times (\cos x - \sin x)$$

$$\Rightarrow f'(x) = \frac{(\cos x - \sin x)}{1 + \sin^2 x + \cos^2 x + 2\sin x \cos x}$$

$$\Rightarrow f'(x) = \frac{\cos x - \sin x}{2(1 + \sin x \cos x)}$$

Now, as given $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

⇒ Cos x – sin x < 0; as here cosine values are smaller than sine values for same angle

$\Rightarrow \frac{\cos x - \sin x}{2(1+\sin x \cos x)} < 0$

⇒ f'(x) < 0

Hence, condition for f(x) to be decreasing

Thus f(x) is decreasing on interval $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

18. Show that f(x) = (x – 1) e^x + 1 is an increasing function for all x > 0.

Solution:

Given f (x) = (x – 1) e^x + 1

Now differentiating the given equation with respect to x, we get

$\Rightarrow f'(x) = \frac{d}{dx}((x - 1)e^x + 1)$

⇒ f'(x) = e^x + (x – 1) e^x

⇒ f'(x) = e^x(1+ x – 1)

⇒ f'(x) = x e^x

As given x > 0

⇒ e^x > 0

⇒ x e^x > 0

⇒ f'(x) > 0

Hence, condition for f(x) to be increasing
Thus f(x) is increasing on interval $x > 0$

19. Show that the function $x^2 - x + 1$ is neither increasing nor decreasing on (0, 1).

Solution:
Given $f(x) = x^2 - x + 1$
Now by differentiating the given equation with respect to x, we get
$\Rightarrow f'(x) = \frac{d}{dx}(x^2 - x + 1)$
$\Rightarrow f'(x) = 2x - 1$
Taking different region from (0, 1)
Let $x \in (0, ½)$
$\Rightarrow 2x - 1 < 0$
$\Rightarrow f'(x) < 0$
Thus f(x) is decreasing in (0, ½)
Let $x \in (½, 1)$
$\Rightarrow 2x - 1 > 0$
$\Rightarrow f'(x) > 0$
Thus f(x) is increasing in (½, 1)
Therefore, from above condition we find that
$\Rightarrow$ f (x) is decreasing in (0, ½) and increasing in (½, 1)
Hence, condition for f(x) neither increasing nor decreasing in (0, 1)

20. Show that $f(x) = x^9 + 4x^7 + 11$ is an increasing function for all $x \in R$.

Solution:
Given $f(x) = x^9 + 4x^7 + 11$
Now by differentiating above equation with respect to x, we get
$\Rightarrow f'(x) = \frac{d}{dx}(x^9 + 4x^7 + 11)$
$\Rightarrow f'(x) = 9x^8 + 28x^6$
$\Rightarrow f'(x) = x^6(9x^2 + 28)$
As given $x \in R$
$\Rightarrow x^6 > 0$ and $9x^2 + 28 > 0$
$\Rightarrow x^6 (9x^2 + 28) > 0$
$\Rightarrow f'(x) > 0$
Hence, condition for f(x) to be increasing

Thus f(x) is increasing on interval $x \in R$