EXERCISE 6.5 PAGE NO: 6.89

Solve each of the following system of homogeneous linear equations:

1. x + y – 2z = 0
2x + y – 3z =0
5x + 4y – 9z = 0

Solution:

Given x + y – 2z = 0
2x + y – 3z =0
5x + 4y – 9z = 0
Any system of equation can be written in matrix form as AX = B

Now finding the Determinant of these set of equations,

$$D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$$

$$|A| = 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

= 1(1 × (– 9) – 4 × (– 3)) – 1(2 × (– 9) – 5 × (– 3)) – 2(4 × 2 – 5 × 1)

= 1(– 9 + 12) – 1(– 18 + 15) – 2(8 – 5)

= 1 × 3 –1 × (– 3) – 2 × 3

= 3 + 3 – 6

= 0

Since D = 0, so the system of equation has infinite solution.

Now let z = k

⇒ x + y = 2k

And 2x + y = 3k

Now using the Cramer's rule

$$x = \frac{D_1}{D}$$

$$x = \frac{\begin{vmatrix} 2k & 1 \\ 3k & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}}$$

$$x = \frac{-k}{-1}$$

x = k

Similarly,

$$y = \frac{D_2}{D}$$

$$y = \frac{\begin{vmatrix} 1 & 2k \\ 2 & 3k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}}$$

$$y = \frac{-k}{-1}$$

y = k

Hence, x = y = z = k.

2. 2x + 3y + 4z = 0
x + y + z = 0
2x + 5y – 2z = 0

Solution:
Given
2x + 3y + 4z = 0
x + y + z = 0
2x + 5y – 2z = 0
Any system of equation can be written in matrix form as AX = B
Now finding the Determinant of these set of equations,

$$D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & 5 & -2 \end{vmatrix}$$

$$|A| = 2\begin{vmatrix}1 & 1\\5 & -2\end{vmatrix} - 3\begin{vmatrix}1 & 1\\2 & -2\end{vmatrix} + 4\begin{vmatrix}1 & 1\\2 & 5\end{vmatrix}$$

= 2(1 × (– 2) – 1 × 5) – 3(1 × (– 2) – 2 × 1) + 4(1 × 5 – 2 × 1)

= 2(– 2 – 5) – 3(– 2 – 2) + 4(5 – 2)

= 1 × (– 7) – 3 × (– 4) + 4 × 3

= – 7 + 12 + 12

= 17

Since D ≠ 0, so the system of equation has infinite solution.

Therefore the system of equation has only solution as x = y = z = 0.