EXERCISE 8.2 PAGE NO: 8.20

Solve the following systems of homogeneous linear equations by matrix method:

1. 2x – y + z = 0
3x + 2y – z = 0
x + 4y + 3z = 0

Solution:

Given
2x – y + z = 0
3x + 2y – z = 0
X + 4y + 3z = 0
The system can be written as

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & -1 \\ 1 & 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0
Now, |A| = 2(6 + 4) + 1(9 + 1) + 1(12 – 2)
|A| = 2(10) + 10 + 10
|A| = 40 ≠ 0
Since, |A|≠ 0, hence x = y = z = 0 is the only solution of this homogeneous equation.

2. 2x – y + 2z = 0
5x + 3y – z = 0
X + 5y – 5z = 0

Solution:

Given 2x – y + 2z = 0
5x + 3y – z = 0
X + 5y – 5z = 0
The system can be written as

$$\begin{bmatrix} 2 & -1 & 2 \\ 5 & 3 & -1 \\ 1 & 5 & -5 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0
Now, |A| = 2(– 15 + 5) + 1(– 25 + 1) + 2(25 – 3)

|A| = – 20 – 24 + 44
|A| = 0
Hence, the system has infinite solutions
Let z = k
2x – y = – 2k
5x + 3y = k

$$\begin{bmatrix} 2 & -1 \\ 5 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ k \end{bmatrix}$$

A X = B

$|A| = 6 + 5 = 11 \neq 0$ So, A^{-1} exist

Now adj A = $\begin{bmatrix} 3 & -5 \\ 1 & 2 \end{bmatrix}^T = \begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}$

$X = A^{-1}B = \frac{1}{|A|}(\text{adj } A)B = \frac{1}{11}\begin{bmatrix} 3 & 1 \\ -5 & 2 \end{bmatrix}\begin{bmatrix} -2k \\ k \end{bmatrix}$

$X = \begin{bmatrix} \frac{-5k}{11} \\ \frac{12k}{11} \end{bmatrix}$

Hence, $x = \frac{-5k}{11}$, $y = \frac{12k}{11}$ and z = k

3. 3x – y + 2z = 0
4x + 3y + 3z = 0
5x + 7y + 4z = 0

Solution:
Given 3x – y + 2z = 0
4x + 3y + 3z = 0
5x + 7y + 4z = 0
The system can be written as

$$\begin{bmatrix} 3 & -1 & 2 \\ 4 & 3 & 3 \\ 5 & 7 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, $|A| = 3(12 - 21) + 1(16 - 15) + 2(28 - 15)$

$|A| = -27 + 1 + 26$

$|A| = 0$

Hence, the system has infinite solutions

Let $z = k$

$3x - y = -2k$

$4x + 3y = -3k$

$$\begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2k \\ -3k \end{bmatrix}$$

$A X = B$

$|A| = 9 + 4 = 13 \neq 0$ So, A^{-1} exist

Now adj A = $\begin{bmatrix} 3 & -1 \\ 4 & 3 \end{bmatrix}^T = \begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}$

$X = A^{-1} B = \frac{1}{|A|}(\text{adj } A)B = \frac{1}{13}\begin{bmatrix} 3 & 1 \\ -4 & 3 \end{bmatrix}\begin{bmatrix} -2k \\ -3k \end{bmatrix}$

$X = \begin{bmatrix} \frac{-9k}{13} \\ \frac{-k}{13} \end{bmatrix}$

Hence, $x = \frac{-9k}{13}$, $y = \frac{-k}{13}$ and $z = k$

4. $x + y - 6z = 0$

$x - y + 2z = 0$

$-3x + y + 2z = 0$

Solution:

Given $x + y - 6z = 0$

$x - y + 2z = 0$

$-3x + y + 2z = 0$

The system can be written as

$$\begin{bmatrix} 1 & 1 & -6 \\ 1 & -1 & 2 \\ -3 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

A X = 0

Now, |A| = 1(– 2 – 2) – 1(2 + 6) – 6(1 – 3)

|A| = – 4 – 8 + 12

|A| = 0

Hence, the system has infinite solutions

Let z = k

x + y = 6k

x – y = – 2k

$$\begin{bmatrix}1 & 1\\1 & -1\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}6k\\-2k\end{bmatrix}$$

A X = B

|A| = – 1 – 1 = – 2 ≠0 So, A^{-1} exist

Now adj A = $\begin{bmatrix}-1 & -1\\-1 & 1\end{bmatrix}^T = \begin{bmatrix}-1 & -1\\-1 & 1\end{bmatrix}$

X = A^{-1} B = $\frac{1}{|A|}$ (adj A)B $= \frac{1}{-2}\begin{bmatrix}-1 & -1\\-1 & 1\end{bmatrix}\begin{bmatrix}6k\\-2k\end{bmatrix}$

X = $\frac{1}{-2}\begin{bmatrix}-6k + 2k\\-6k - 2k\end{bmatrix}$

X = $\begin{bmatrix}-4k\\-8k\end{bmatrix}$

Hence, x = 2k, y = 4k and z = k