1. Let $A$ and $B$ be the events such that $P(A) = \frac{7}{12}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$.

Find (i) $P(A/B)$ (ii) $P(B/A)$ (iii) $P(A \cup B)$ (iv) $P(\overline{B}/A)$

**Solution:**

(i) $P(A/B) = \frac{P(A \cap B)}{P(B)}$

Substituting the values

$$= \frac{\frac{4}{13}}{\frac{9}{13}} = \frac{4}{9}$$

(ii) $P(B/A) = \frac{P(A \cap B)}{P(A)}$

Substituting the values

$$= \frac{\frac{4}{13}}{\frac{7}{13}} = \frac{4}{7}$$

(iii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Substituting the values

$$= \frac{7}{13} + \frac{9}{13} - \frac{4}{13} = \frac{12}{13}$$

(iv) $P(\overline{B}/A) = \frac{P(A \cap \overline{B})}{P(A)}$

Using De-Morgan’s Law

$(A \cup B)^C = A^C \cap B^C$

We know that

$P(A \cap B) = P(A \cup B)$

It can be written as

$$\frac{P(A \cap \overline{B})}{P(A)} = \frac{P(A \cup B)}{P(A)}$$

So we get
2. Let A and B be the events such that
\[ P(A) = \frac{5}{11}, \ P(B) = \frac{6}{11}, \ P(A \cup B) = \frac{7}{11}. \]
Find (i) \( P(A \cap B) \) (ii) \( P(A/B) \) (ii) \( P(B/A) \) (iv) \( P(A/B) \).

Solution:

(i) \[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

We know that

\[ P(A \cap B) = P(A) + P(B) - P(A \cup B) \]

Substituting the values

\[ = \frac{5}{11} + \frac{6}{11} - \frac{7}{11} = \frac{4}{11} \]

(ii) \[ P(A/B) = \frac{P(A \cap B)}{P(B)} \]

Substituting the values

\[ = \frac{4}{11} \div \frac{6}{11} \]

So we get

\[ = \frac{4}{6} = \frac{2}{3} \]

(iii) \[ P(B/A) = \frac{P(A \cap B)}{P(A)} \]

Substituting the values
3. Let A and B be the events such that
\[ P(A) = \frac{3}{10}, \quad P(B) = \frac{1}{2}, \quad \text{and} \quad P(B/A) = \frac{2}{5}. \]
Find (i) \( P(A \cap B) \) (ii) \( P(A \cup B) \) (iii) \( P(A/B) \).

Solution:

(i) \( P(B/A) = \frac{P(A \cap B)}{P(A)} \)

Using De-Morgan's Law
\[ (A \cup B)^c = A^c \cap B^c \]
We know that
\[ P(A \cap B) = P(A \cup B) \]
It can be written as
\[ \frac{P(A \cap B)}{P(A)} = \frac{P(A \cup B)}{P(A)} \]
So we get
\[ = 1 - P(A \cup B) \]
By substituting the values
\[ = 1 - \frac{7}{11} \]
\[ = \frac{4}{5} \]

(ii) \( P(A \cup B) = \frac{3}{10} \times \frac{2}{5} = \frac{3}{25} \)

(iii) \( P(A/B) \)
Let A and B be the events such that
\[2 \cdot P(A) = P(B) = \frac{5}{13}\] and \[P(A/B) = \frac{2}{5}.

Find (i) \(P(A \cap B)\) (ii) \(P(A \cup B)\)

**Solution:**

(ii) 
\[P(A \cup B) = P(A) + P(B) - P(A \cap B)\]
Substituting the values
\[= \frac{3}{10} + \frac{1}{2} - \frac{3}{25}\]
So we get
\[= \frac{15 + 25 - 6}{50} = \frac{34}{50}\]
By further calculation
\[= \frac{17}{25}\]

(iii) 
\[P(A/B) = \frac{P(A \cap B)}{P(B)}\]
Substituting the values
\[= \frac{3}{25} \div \frac{1}{2} = \frac{6}{25}\]

4. Let A and B be the events such that
\[2 \cdot P(A) = P(B) = \frac{5}{13}\] and \[P(A/B) = \frac{2}{5}.

Find (i) \(P(A \cap B)\) (ii) \(P(A \cup B)\)

**Solution:**

(i) 
\[P(A/B) = \frac{P(A \cap B)}{P(B)}\]
We can write it as
\[P(A \cap B) = P(B) \cdot P(A/B)\]
Substituting the values
\[= \frac{5}{13} \times \frac{2}{5} = \frac{2}{13}\]

(ii) 
\[P(A \cup B) = P(A) + P(B) - P(A \cap B)\]
Substituting the values
\[= \frac{5}{26} + \frac{5}{13} - \frac{2}{13}\]
5. A die is rolled. If the outcome is an even number, what is the probability that it is a number greater than 2?

Solution:

We know that a die has 6 spaces
Sample space \( S = \{1, 2, 3, 4, 5, 6\} \)
Total no. of outcomes = 6

Consider \( P(A) \) as the probability of getting an even number
So the sample space \( A = \{2, 4, 6\} \)
\[ P(A) = \frac{3}{6} = \frac{1}{2} \]

Consider \( P(B) \) as the probability of getting a number whose value is greater than 2
So the sample space \( B = \{3, 4, 5, 6\} \)
\((A \cap B) = \{4, 6\}\)
We get
\[ P(A \cap B) = \frac{2}{6} = \frac{1}{3} \]

Here the probability of getting a number greater than 2 given that the outcomes is even is written as
\[ P(\frac{B}{A}) = \frac{P(A \cap B)}{P(A)} \]

Substituting the values
\[ = \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} \]

6. A coin is tossed twice. If the outcome is at most one tail, what is the probability that both head and tail have appeared?

Solution:

We know that a coin has two sides
Sample space \( S = \{H, T\} \)
Total no. of outcomes = 2
It is given that a coin is tossed twice.

Consider \( P(A) \) as the probability of getting at most 1 tail
Sample space of \( A = \{(H, H), (H, T), (T, H)\} \)

Consider \( P(B) \) as the probability of getting a head
Sample space of \( B = \{H\} \)
\[ P (B) = \frac{1}{2} \]

So the probability of getting at most one tail and a head
\[ (A \cap B) = \{(H, H)\} \]
\[ P (A \cap B) = \frac{1}{3} \]

Here the probability that both head and tail have appeared can be written as
\[ P(A/B) = \frac{P(A \cap B)}{P(B)} \]

Substituting the values
\[ = \frac{1/3}{1/2} \]
\[ = \frac{2}{3} \]

7. Three coins are tossed simultaneously. Find the probability that all coins show heads if at least one of the coins shows a head.

Solution:

We know that when three coins are tossed simultaneously the total number of outcomes = \( 2^3 = 8 \)
Sample space \( S = \{(H, H, H), (H, H, T), (H, T, T), (H, T, H), (T, H, T), (T, T, H), (T, H, H), (T, T, T)\} \)

Consider \( P (A) \) as the probability of getting 3 heads
Sample space of \( A = \{(H, H, H)\} \)
\[ P (A) = \frac{1}{8} \]

Consider \( P (B) \) as the probability of getting at least one head
Probability of one head = 1 – probability of no heads
\[ = 1 - \frac{1}{8} = \frac{7}{8} \]
\[ P (B) = \frac{7}{8} \]

Here probability that the throw is either all head or at least one head
\[ P (A \cup B) = \frac{7}{8} \]

We know that
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B) \]

Similarly
\[ P(A \cap B) = P(A) + P(B) - P(A \cup B) \]

Substituting the values
\[ = \frac{1}{8} + \frac{7}{8} - \frac{1}{8} = \frac{7}{8} \]

Here the probability that all coins show heads if at least one of the coins showed a head
8. Two unbiased dice are thrown. Find the probability that the sum of the numbers appearing is 8 or greater, if 4 appears on the first die.
Solution:

It is given that two unbiased dice are thrown
So the outcome = 6 × 6 = 36

Consider P (A) as the probability of getting a sum greater than 8
Consider P (B) as the probability of getting 4 on the first die
Sample space of B = {(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)}
P (B) = 6/36 = 1/6

Consider P (A ∩ B) as the probability of getting 4 on the first die and the sum greater than or equal to 8
Sample space (A ∩ B) = {(4, 4), (4, 5), (4, 6)}
P (A ∩ B) = 3/36 = 1/12

Here the probability that sum of the numbers is greater than or equal to 8 given that 4 was thrown first

\[ P(A/B) = \frac{P(A \cap B)}{P(B)} \]

Substituting the values

\[ = \frac{1/12}{1/6} = \frac{1}{2} \]

9. A die is thrown twice and the sum of the numbers appearing is observed to be 8. What is the conditional probability that the number 5 has appeared at least once?
Solution:

It is given that a die is thrown twice
So the total outcome = 6 × 6 = 36

Consider P (A) as the probability of getting the number 5 at least once
Consider \( P(B) \) as the probability of getting the sum = 8
Sample space of \( B = \{(2, 6), (3, 5), (4, 4), (5, 3), (6, 2)\} \)
\[ P(B) = \frac{5}{36} \]

Consider \( P(A \cap B) \) as the probability of getting the number 5 at least once and the sum equal to 8
Sample space of \( A \cap B = \{(3, 5), (5, 3)\} \)
\[ P(A \cap B) = \frac{2}{36} = \frac{1}{18} \]

Here the probability that the number 5 have appeared at least once given that the sum is 8
\[ P(A/B) = \frac{P(A \cap B)}{P(B)} \]
Substituting the values
\[ \frac{1/18}{5/36} = \frac{2}{5} \]