EXERCISE 29B

1. A bag contains 17 tickets, numbered from 1 to 17. A ticket is drawn and then another ticket is drawn without replacing the first one. Find the probability that both the tickets may show even numbers.

Solution:

It is given that a bag contains 17 tickets numbered from 1 to 17 where each trial is independent of the other.

Sample space \( S = \{1, 2, 3, \ldots, 17\} \)

Consider success ticket drawn is even = \( 8/17 \)

So the probability of success in the first trial

\[ P_1(\text{success}) = \frac{8}{17} \]

Consider the probability of success in the second trial without replacement of the first draw as

\[ P_2(\text{success}) = \frac{7}{16} \]

So the probability that both the tickets may show even numbers with each trial being independent is given by

\[ P_1 \times P_2 = \frac{8}{17} \times \frac{7}{16} = \frac{7}{34} \]

2. Two marbles are drawn successively from a box containing 3 black and 4 white marbles. Find the probability that both the marbles are black, if the first marble is not replaced before the second draw.

Solution:

It is given that a box contains 3 black and 4 white marbles where each trial is independent of the other.

Sample space \( S = \{1B, 2B, 3B, 1W, 2W, 3W, 4W\} \)

Consider success marble drawn is black = \( 3/7 \)

So the probability of success in the first trial

\[ P_1(\text{success}) = \frac{3}{7} \]

Consider probability of success in the second trial without replacement of the first draw

\[ P_2(\text{success}) = \frac{2}{6} \]

So the probability that both the marbles drawn are black, with each trial being independent

\[ P_1 \times P_2 = \frac{3}{7} \times \frac{2}{6} = \frac{1}{7} \]

3. A card is drawn from a well-shuffled deck of 52 cards and without replacing this card, a second card is drawn. Find the probability that the first card is a club and the second card is a spade.

Solution:

It is given that a well-shuffled deck of 52 cards where each draw is independent of the other.

Consider success for the first trial be getting a club

So the probability of success in the first trial

\[ P_1(\text{success}) = \frac{13}{52} \]

Consider success for the second trial be getting a spade

So the probability of success in the second trial without replacement of the first draw

\[ P_2(\text{success}) = \frac{13}{51} \]
So the probability that the first card drawn is a club and the second card is a spade with each trial being independent
\[ P_1 \times P_2 = \frac{13}{52} \times \frac{13}{51} = \frac{13}{204} \]

4. There is a box containing 30 bulbs of which 5 are defective. If two bulbs are chosen at random from the box in succession without replacing the first, what is the probability that both the bulbs chosen are defective?
Solution:

It is given that a box containing 30 bulbs of which 5 are defective with each trial independent of other trial
Consider success bulb chosen is defective = 5/30

So the probability of success in the first trial
\[ P_1 (success) = \frac{5}{30} \]

So the probability of success in the second trial without replacement of the first draw
\[ P_2 (success) = \frac{4}{29} \]

Probability that both the bulbs chosen are defective, with each trial being independent
\[ P_1 \times P_2 = \frac{5}{30} \times \frac{4}{29} = \frac{2}{87} \]

5. A bag contains 10 white and 15 black balls. Two balls are drawn in succession without replacement. What is the probability that the first ball is white and the second is black?
Solution:

It is given that a bag contains 10 white and 15 black balls with each trial independent of the other.
Consider success in the first draw be getting a white ball
So the probability of success in the first trial
\[ P_1 (success) = \frac{10}{25} \]

Consider success in the second draw be getting a black ball
So the probability of success in the second trial without replacement of the first draw
\[ P_2 (success) = \frac{15}{24} \]

Here the probability that first ball drawn is white and the second ball drawn is black with each trial being independent
\[ P_1 \times P_2 = \frac{10}{25} \times \frac{15}{24} = \frac{1}{4} \]

6. An urn contains 5 white and 8 black balls. Two successive drawings of 3 balls at a time are made such that the balls drawn in the first draw are not replaced before the second draw. Find the probability that the first draw gives 3 white balls and the second draw gives 3 black balls.
Solution:

It is given that an urn contains 5 white and 8 black balls with each trial being independent of the other.
Consider success in the first draw be getting 3 white balls
So the probability of success in the first trial
Consider the success in the second draw be getting 3 black balls
So the probability of success in the second trial without replacement of the first draw

Here the probability that the first draw gives 3 white and the second draw gives 3 black balls with each trial being independent

7. Let \( E_1 \) and \( E_2 \) be the events such that \( P (E_1) = \frac{1}{3} \) and \( P (E_2) = \frac{3}{5} \).

Find:
(i) \( P (E_1 \cup E_2) \), when \( E_1 \) and \( E_2 \) are mutually exclusive,
(ii) \( P (E_1 \cap E_2) \), when \( E_1 \) and \( E_2 \) are independent.

Solution:
(i) \( P (E_1 \cup E_2) \), when \( E_1 \) and \( E_2 \) are mutually exclusive

We know that when two events are mutually exclusive
\[ P (E_1 \cap E_2) = 0 \]
So \( P (E_1 \cup E_2) = P (E_1) + P (E_2) \)
By substituting the values
\[ = \frac{1}{3} + \frac{3}{5} \]
\[ = \frac{14}{15} \]

Hence, \( P (E_1 \cup E_2) = \frac{14}{15} \), when \( E_1 \) and \( E_2 \) are mutually exclusive.

(ii) \( P (E_1 \cap E_2) \), when \( E_1 \) and \( E_2 \) are independent

We know that when two events are independent
\[ P (E_1 \cap E_2) = P (E_1) \times P (E_2) \]
By substituting the values
\[ = \frac{1}{3} \times \frac{3}{5} \]
\[ = \frac{1}{5} \]

Hence, \( P (E_1 \cap E_2) = \frac{1}{5} \) when \( E_1 \) and \( E_2 \) are independent.

8. If \( E_1 \) and \( E_2 \) are the two events such that \( P (E_1) = \frac{1}{4} \), \( P (E_2) = \frac{1}{3} \) and \( P (E_1 \cup E_2) = \frac{1}{2} \), show that \( E_1 \) and \( E_2 \) are independent events.

Solution:

It is given that \( E_1 \) and \( E_2 \) are the two events such that \( P (E_1) = \frac{1}{4} \), \( P (E_2) = \frac{1}{3} \) and \( P (E_1 \cup E_2) = \frac{1}{2} \)
We know that
\[ P (E_1 \cap E_2) = P (E_1) + P (E_2) - P (E_1 \cup E_2) \]
By substituting the values
\[ = \frac{1}{4} + \frac{1}{3} - \frac{1}{2} \]
\[ = \frac{1}{12} \]

Condition: Two events are independent
9. If \( E_1 \) and \( E_2 \) are independent events such that \( P(E_1) = 0.3 \) and \( P(E_2) = 0.4 \), find

(i) \( P(E_1 \cap E_2) \)

(ii) \( P(E_1 \cup E_2) \)

(iii) \( P(\overline{E_1} \cap \overline{E_2}) \)

(iv) \( P(\overline{E_1} \cap E_2) \)

Solution:

It is given that \( E_1 \) and \( E_2 \) are independent events such that \( P(E_1) = 0.3 \) and \( P(E_2) = 0.4 \).

(i) \( P(E_1 \cap E_2) \)

We know that \( E_1 \) and \( E_2 \) are independent events.

\[ P(E_1 \cap E_2) = P(E_1) \times P(E_2) \]

By substituting the values,

\[= 0.3 \times 0.4 \]

\[= 0.12 \]

Hence, \( P(E_1 \cap E_2) = 0.12 \) when \( E_1 \) and \( E_2 \) are independent events.

(ii) \( P(E_1 \cup E_2) \)

when \( E_1 \) and \( E_2 \) are independent events

\[ P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2) \]

By substituting the values,

\[= 0.3 + 0.4 - 0.12 \]

\[= 0.58 \]

Hence, \( P(E_1 \cup E_2) = 0.58 \) when \( E_1 \) and \( E_2 \) are independent events.

(iii) \( P(\overline{E_1} \cap \overline{E_2}) = P(\overline{E_1}) \times P(\overline{E_2}) \)

Here \( P(E_1) = 0.3 \) and \( P(E_2) = 0.4 \).

\[ P(\overline{E_1}) = 1 - P(E_1) = 0.7 \]

\[ P(\overline{E_2}) = 1 - P(E_2) = 0.6 \]

It is given that \( E_1 \) and \( E_2 \) are independent events, so even \( \overline{E_1} \) and \( \overline{E_2} \) are also independent.

\[ P(\overline{E_1} \cap \overline{E_2}) = 0.7 \times 0.6 = 0.42 \]

Hence, \( P(\overline{E_1} \cap \overline{E_2}) = 0.42 \).

(iv) \( P(\overline{E_1} \cap E_2) = P(\overline{E_1}) \times P(E_2) \)

By substituting the values,
10. Let A and B be the events such that
P (A) = 1/2, P (B) = 7/12 and P (not A or not B) = 1/4.
State whether A and B are
(i) mutually exclusive,
(ii) independent.
Solution:

(i) If A and B are mutually exclusive
P (not A or not B) = 1/4 i.e. \(P(\overline{A} \cup \overline{B}) = \frac{1}{4}\)
Here we get
\[
P(\overline{A} \cup \overline{B}) = P(A \cap B)' = 1 - P(A \cap B) = \frac{1}{4}
\]
By substituting the values
\[
P(A \cap B) = 1 - \frac{1}{4} = \frac{3}{4}
\] … (1)
For two mutually exclusive events
P (A \cap B) = 0 but here P (A \cap B) ≠ 0
Hence, A and B are not mutually exclusive.

(ii) If A and B are independent
Two events is independent is written as
P (E_1 \cap E_2) = P (E_1) \times P (E_2)
By substituting the values
\[
= \frac{1}{2} \times \frac{7}{12} = \frac{7}{24}
\] … (2)
Here equation (1) is not equal to equation (2)
Hence, A and B are not independent.

11. Kamal and Vimal appeared for an interview for two vacancies. The probability of Kamal’s selection is 1/3 and that of Vimal’s selection is 1/5. Find the probability that only one of them will be selected.
Solution:

Consider A as the event ‘Kamal is selected’ and B as the event ‘Vimal is selected’
So P (A) = 1/3 and P (B) = 1/5
Here
P (only one of them is selected) = P (A and not B or B and not A)
It can be written as
\[
= P(A \cap \overline{B}) + (B \cap \overline{A})
\]
We get
\[
= P(A) \times P(\overline{B}) + P(B) \times P(\overline{A})
\]
Substituting the values
\[
= \frac{1}{3} \times \left(1 - \frac{1}{5}\right) + \frac{1}{5} \times \left(1 - \frac{1}{3}\right)
\]
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\[ \frac{1}{3} \left(1 - \frac{1}{5}\right) + \frac{1}{5} \left(1 - \frac{1}{3}\right) \]

On further calculation
\[ = \frac{4}{15} + \frac{2}{15} \]
\[ = \frac{2}{5} \]

Hence, the probability that only one of them will be selected is \( \frac{2}{5} \).

12. Arun and Ved appeared for an interview for two vacancies. The probability of Arun’s selection is \( \frac{1}{4} \) and that of Ved’s rejection is \( \frac{2}{3} \). Find the probability that at least one of them will be selected.

Solution:

Consider A as the event ‘Arun is selected’ and B as the event ‘Ved is selected’

Here
\[ \begin{align*}
P(A) &= \frac{1}{4} \quad \text{and } P(\overline{B}) = \frac{2}{3} \\
P(B) &= \frac{1}{3} \quad \text{and } P(\overline{A}) = \frac{3}{4}
\end{align*} \]

We know that
\[ P(\text{atleast one of them getting selected}) = P(\text{selecting only Arun}) + P(\text{selecting only Ved}) + P(\text{selecting both}) \]
\[ = P(A \text{ and not } B) + P(B \text{ and not } A) + P(A \text{ and } B) \]
So we get
\[ = P(A) \times P(\overline{B}) + P(B) \times P(\overline{A}) + P(A) \times P(B) \]

By substituting the values
\[ = \left(\frac{1}{4} \times \frac{2}{3}\right) + \left(\frac{1}{3} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{1}{3}\right) \]

We get
\[ = \frac{2}{12} + \frac{3}{12} + \frac{1}{12} \]
\[ = \frac{1}{2} \]

Hence, the probability that at least one of them will get selected is \( \frac{1}{2} \).

13. A and B appear for an interview for two vacancies in the same post. The probability of A’s selection is \( \frac{1}{6} \) and that of B’s selection is \( \frac{1}{4} \). Find the probability that

(i) both of them are selected
(ii) only one of them is selected
(iii) none is selected
(iv) at least one of them is selected.

Solution:

(i) Probability that both of them are selected
\[ P(\text{both of them are selected}) = P(A \cap B) = P(A) \times P(B) \]
Substituting the values
\[ = \frac{1}{6} \times \frac{1}{4} \]
\[ = \frac{1}{24} \]

Hence, the probability that both of them are selected is \( \frac{1}{24} \).

(ii) Probability that only one of them is selected
\[ P(\text{only one of them is selected}) = P(A \text{ and not } B) \]
\[ = \frac{1}{6} \times \frac{3}{4} \]
\[ = \frac{1}{8} \]

Hence, the probability that only one of them will be selected is \( \frac{1}{8} \).

(iii) Probability that none is selected
\[ P(\text{none is selected}) = P(\overline{A}) \times P(\overline{B}) \]
\[ = \frac{3}{4} \times \frac{2}{3} \]
\[ = \frac{1}{2} \]

Hence, the probability that none is selected is \( \frac{1}{2} \).

(iv) Probability that at least one of them is selected
\[ P(\text{atleast one of them getting selected}) = 1 - P(\text{none is selected}) \]
\[ = 1 - \frac{1}{2} \]
\[ = \frac{1}{2} \]

Hence, the probability that at least one of them will get selected is \( \frac{1}{2} \).
= P (A and not B) + P (B and not A)
We get
= P(A ∩ B̅) + P(B ∩ A̅)
= P(A) × P(B̅) + P(B) × P(A̅)
Substituting the values
= (1/6 × 3/4) + (1/4 × 5/6)
= 3/24 + 5/24
So we get
= 1/3
Hence, the probability that only one of them is selected is 1/3.

(iii) Probability that none is selected
P (none is selected)
= P(A̅ ∩ B̅) = P(A̅) × P(B̅)
Substituting the values
= 5/6 × 3/4
= 5/8
Hence, the probability that none is selected is 5/8.

(iv) Probability that at least one of them is selected
P (at least one of them is selected) = P (selecting only A) + P (selecting only B) + P (selecting both)
We know that
= P (A and not B) + P (B and not A) + P (A and B)
It can be written as
= P(A ∩ B̅) + P(B ∩ A̅) + P(A ∩ B)
= P(A) × P(B̅) + P(B) × P(A̅) + P(A) × P(B)
Substituting the values
= (1/6 × 3/4) + (1/4 × 5/6) + (1/6 × 1/4)
On further calculation
= 3/24 + 5/24 + 1/24
= 3/8
Hence, the probability that at least one of them is selected is 3/8.