1. In a bulb factory, three machines, A, B, C, manufacture 60%, 25% and 15% of the total production respectively. Of their respective outputs, 1%, 2% and 1% are defective. A bulb is drawn at random from the total product and it is found to be defective. Find the probability that it was manufactured by machine C.

Solution:

Consider D as the defective bulb
Here we should determine

\[ P(C|D) = \frac{P(C).P(D|C)}{P(A).P(D|A) + P(B).P(D|B) + P(C).P(D|C)} \]

We know that

Probability that bulb is made by machine A
\[ P(A) = \frac{60}{100} \]

Probability that bulb is made by machine B
\[ P(B) = \frac{25}{100} \]

Probability that bulb is made by machine C
\[ P(C) = \frac{15}{100} \]

Probability of defective bulb from machine A
\[ P(D|A) = \frac{1}{100} \]

Probability of defective bulb from machine B
\[ P(D|B) = \frac{2}{100} \]

Probability of defective bulb from machine C
\[ P(D|C) = \frac{1}{100} \]

So we get

\[ P(C|D) = \frac{15}{60 + 50 + 15} \]

On further calculation

= \frac{15}{125}

= \frac{3}{25}

Hence, the probability of selected defective bulb is from machine C is \(\frac{3}{25}\).

2. A company manufactures scooters at two plants, A and B. Plant A produces 80% and plant B produces 20% of the total product. 85% of the scooters produced at plant A and 65% of the scooters produced at plant B are of standard quality. A scooter produced by the company is selected at random and it is found to be of standard quality. What is the probability that it was manufactured at plant A?

Solution:

Consider S as the standard quality
Here we should determine

\[ P(A|S) = \frac{P(A).P(S|A)}{P(A).P(S|A) + P(B).P(S|B)} \]

We know that

Probability that scooter is made by plant A
\[ P(A) = \frac{80}{100} \]

Probability that scooter is made by plant B
\[ P(B) = \frac{20}{100} \]

Probability of standard scooter from plant A
\[ P(S|A) = \frac{85}{100} \]

Probability of standard scooter from plant B
\[ P(S|B) = \frac{65}{100} \]

So we get

\[ P(A|S) = \frac{80 \times 85}{80 \times 85 + 20 \times 65} \]

On further calculation

= \frac{80 \times 85}{80 \times 85 + 20 \times 65}

= \frac{28000}{28000 + 1300}

= \frac{28000}{41000}

= \frac{28}{41}

Hence, the probability that it was manufactured at plant A is \(\frac{28}{41}\).
We know that
Probability that scooter is from A
\[ P(A) = \frac{80}{100} \]
Probability that scooter is from B
\[ P(B) = \frac{20}{100} \]
Probability that standard scooter from A
\[ P(S|A) = \frac{85}{100} \]
Probability that standard scooter from B
\[ P(S|B) = \frac{65}{100} \]

So we get
\[ P(A|S) = \frac{(80)(85)}{(80)(85) + (20)(65)} \]
On further calculation
\[ = \frac{6800}{6800 + 1300} \]
\[ = \frac{68}{81} \]
Hence, the probability of selected standard scooter is from plant A is \( \frac{68}{81} \).

3. In a certain college, 4\% of boys and 1\% of girls are taller than 1.75 metres. Furthermore, 60\% of the students are girls. If a student is selected at random and is taller than 1.75 metres, what is the probability that the selected student is a girl?

Solution:

Consider \( T \) as the students taller than 1.75m
\( B \) as the boys in class and \( G \) as the girls in class
Here we should determine
\[ P(G|T) \] – probability that selected taller is a girl
\[ \frac{P(G).P(T|G)}{P(G).P(T|G) + P(B).P(T|B)} \]
By substituting the values
\[ \frac{\frac{60}{100} \times \frac{1}{100}}{\frac{60}{100} \times \frac{1}{100} + \frac{40}{100} \times \frac{4}{100}} \]
So we get
\[ = \frac{60}{220} \]
\[ = \frac{3}{11} \]
Hence, the probability of selected taller student is a girl is \( \frac{3}{11} \).

4. In a class, 5\% of the boys and 10\% of the girls have an IQ of more than 150. In this class, 60\% of the students are boys. If a student is selected at random and found to have an IQ of more than 150, find the probability that the student is a boy.
Solution:

Consider I as the students having IQ more than 150
B as the boys in class and G as the girls in class
Here we should determine
P (B/I) – probability that selected student having IQ greater than 150 is a boy

\[ P(B|I) = \frac{P(B).P(I|B)}{P(G).P(I|G) + P(B).P(I|B)} \]

By substituting the values
\[ = \left( \frac{60}{100} \right) \left( \frac{5}{100} \right) + \left( \frac{40}{100} \right) \left( \frac{10}{100} \right) \]

So we get
\[ = \frac{300}{300 + 400} = \frac{300}{700} = \frac{3}{7} \]

Hence, the probability that selected student having IQ greater than 150 is a boy is 3/7.

5. Suppose 5% of men and 0.25% of women have grey hair. A grey-haired person is selected at random. What is the probability of this person being male? Assume that there are equal number of males and females.

Solution:

Assume that there are 1000 men and 1000 women
MG as the men having grey hair, WG as the women having grey hair and G as the event of choosing an orator

\[ P(MG) = \frac{1000}{2000} = \frac{1}{2} \]

\[ P(WG) = \frac{1000}{2000} = \frac{1}{2} \]

We know that
\[ P(G|MG) = \frac{50}{1000} = \frac{1}{20} \]

\[ P(G|WG) = \frac{25}{1000} = \frac{1}{40} \]

Here we should determine
P (MG/G) – probability of a randomly selected grey haired person to be male

\[ P(MG|G) = \frac{P(MG).P(G|MG)}{P(MG).P(G|MG) + P(WG).P(G|WG)} \]

Substituting the values
\[ = \left( \frac{1}{2} \right) \left( \frac{1}{20} \right) + \left( \frac{1}{2} \right) \left( \frac{1}{40} \right) \]

On further calculation
\[ = \frac{1}{40} \times \frac{80}{3} = \frac{2}{3} \]

Hence, the probability of a randomly selected grey haired person to be male is 2/3.
6. Two groups are competing for the positions on the board of directors of a corporation. The probabilities that the first and the second groups will win are 0.6 and 0.4 respectively. Further, if the first group wins, the probability of introducing a new product is 0.7 and when the second group wins, the corresponding probability is 0.3. Find the probability that the new product introduced was by the second group.
Solution:
Consider $F$ as first group, $S$ as second group and $N$ as introducing a new product
Here we should determine $P(S|N)$ – New product introduced by the second group
Substituting the values
$$P(S|N) = \frac{P(S).P(N|S)}{P(S).P(N|S) + P(F).P(N|F)}$$
Substituting the values
$$= \frac{(0.6)(0.7) + (0.4)(0.3)}{(0.6)(0.7) + (0.4)(0.3)}$$
So we get
$$= 0.12/0.54$$
$$= 2/9$$
Hence, the probability of the second group introduced a new product is $2/9$.

7. A bag A contains 1 white and 6 red balls. Another bag contains 4 white and 3 red balls. One of the bags is selected at random and a ball is drawn from it, which is found to be white. Find the probability that the ball drawn is from the bag A.
Solution:
Consider $R$ as the red ball, $W$ as the white ball
$A$ as the bag A and $B$ as the bag B
Assume selecting bags is of equal probability $= 1/2$
Here we should determine
$P(A/W)$ – the selected white ball is from bag A
Substituting the values
$$= 1/5$$
Hence, the probability of selected white ball is from bag A is $1/5$.

8. There are two bags I and II. Bag I contains 3 white and 4 black balls, and bag II contains 5 white and 6 black balls. One ball is drawn at random from one of the bags and is found to be white. Find the probability that it was drawn from bag I.
Solution:
Consider $W$ as the white ball, $B$ as the black ball
$X$ as the first bag and $Y$ as the second bag
Assume selecting bags is of equal probability = 1/2
Here we should determine
P (X/W) – Probability of selected white ball is from the first bag
\[ P(X|W) = \frac{P(X) \cdot P(W|X)}{P(X) \cdot P(W|X) + P(Y) \cdot P(W|Y)} \]
Substituting the values
\[ \frac{\left(\frac{1}{2}\right) \left(\frac{3}{7}\right)}{\left(\frac{1}{2}\right) \left(\frac{3}{7}\right) + \left(\frac{1}{2}\right) \left(\frac{5}{11}\right)} \]
So we get
\[ \frac{3}{7} \]
\[ = \frac{3}{7} + \frac{5}{11} \]
\[ = \frac{33}{68} \]
Hence, the probability of selected white ball is from the first bag is 33/68.

9. A box contains 2 gold and 3 silver coins. Another box contains 3 gold and 3 silver coins. A box is chosen at random, and a coin is drawn from it. If the selected coin is a gold coin, find the probability that it was drawn from the second box.
Solution:
Consider G as gold coins, S as silver coins
A as the first box and B as the second box
Assume selecting bags is of equal probability = 1/2
Here we should determine
P (B/G) – Probability of selected gold coin is from the second box
\[ P(B|G) = \frac{P(B) \cdot P(G|B)}{P(A) \cdot P(G|A) + P(B) \cdot P(G|B)} \]
Substituting the values
\[ \frac{\left(\frac{3}{6}\right) \left(\frac{1}{2}\right)}{\left(\frac{2}{5}\right) \left(\frac{1}{2}\right) + \left(\frac{3}{6}\right) \left(\frac{1}{2}\right)} \]
\[ = \frac{5}{9} \]
Hence, the probability of selected gold coin is from the second box is 5/9.